The Arte of vulgar arithmeticke, both in Integers and Fractions, *deuided into two Bookes: whereof the first is called* *Nomodidactus Numerorum*, and the second *Portus Proportionum*: with certeine Demonstrations, reduced into so plaine and perfect Method, *as the like hath not hetherto beene published in English. Whereunto* is added a third Booke, entituled *Musa Mercatorum*: Comprehending all the most necessarie and profitable Rules *vsed in the trade of Merchandise.*

In all which three Bookes: the Rules, Precepts, and Maxims, are not *onely composed in meeter for the better retaining of them in memorie,* but also the operations, examples, demonstrations, and questions, *are in most easie wise expounded and explaned, in the forme* of a Dialogue, for the Readers more deere vnderstanding.

A knowledge pleasant for Gentlemen, commendable for Capteines and Soldiers, profitable for Merchants, and generally *necessarie for all estates and degrees.*

Newly collected, digested, and in some part deuised by *a well willer to the Mathematicals.*

Ecclesiasticus. cap. 19.

Learning vnto fooles is as fetters on their feete and Manicles vpon their right hand: but to the wise it is a Iewell of golde, and like a Bracelet vpon his right arme.

Boetius libr. 1. Arith. cap. 2.

Omnia quecunque a primeua natura constructa sunt, Numerorum videntur ratione formata. Hoc enim fuit principale in animo conditoris examplar.



Imprinted at London by *Gabriel Simson*, dwelling in Fleete-lane, 1600.

The Booke ſpeaketh.

I am a Booke, whoſe doctrine conſiſteth,
In working with numbers according to Arte:
The knowledge whereof, to learne who ſo liſteth,
I promiſe frankly, to giue and imparte.
So plainely, ſo eaſly, and eke in ſuch ſort,
That thereof the learning ſhall ſeeme but a ſport.

Arithmaticke ſpeaketh.

Yee that peruſe this preſent Booke, looke not (I hope) to finde,
Some high or haughtie rowling verſe, contrary to my kinde:
Sithe that my Rules are ſuch in truth, as no wiſe can affoord,
To be enlar'gde by eloquence, nor well to want a woord.
For Phaers *phraſe, nor* Virgils *verſe, cannot my Rules declare,*
So well as plaine and ſimple truth, in naked ſpeach and bare:
Wherefore good Reader take in woorth, my barrein verſes here,
which do contein vndouted truth, thogh rude they do appeare.
And I of force from ſtately ſtile, both muſt and do refraine,
for truth hath neuer greater grace, thē when ſhe is moſt plaine.

TO THE RIGHT HONO-rable ſir *Thomas Sackuill* knight, Baron *of Buckhurſt, Lord Treaſurer of England,* Lieutenant of her Maieſtie, within the Countie of *Suſſex, Knight of the honorable order of the Garter,* and of her Highneſſe moſt honorable *priuie Counſell.*



Although (right honorable) the excellencie and dignitie of your Lordſhips perſon, the ſuperiment, eſtate, and place, whereunto your Lordſhip is moſt woorthely called, the manifold gifts of learning, knowledge, wiſedome, and iudgement in all liberall ſciences, wherwith your Lordſhip is indued, are cauſes ſufficient to abaſh me from preſſing into your Lordſhips preſence, with this ſo rude and baſe a preſent: yet the honorable report which Fame proclaymeth of your Lordſhips ſinguler affabilitie, benignitie, and bountie towards all men, together with the experience thereof, practiſed by your good Lordſhip vpon my ſelfe, ceaſeth not to animate and ſpur me forward in this my enterpriſe, (which though it may ſeeme ouer bold) is yet not without reuerend baſhfulneſſe attempted.

For hauing heretofore enioyed the fruite of your honorable fauour and beneuolence, and that ſo bountifully as I acknowledge my ſelfe neuer able to deſerue, to the end the ſame might not ſeeme (as it were) vtterly caſt away, vpon a perſon ingratefull or voyd of the remembrance thereof, I haue often times wiſhed and deſired in my minde, that I could by ſome meanes ſhew my ſelfe at the leaſt mindefull of your Lordſhips honorable fauour, and vndeſerued beneuolence, wherein finding abilitie not correſpondent to will, I humbly offer vnto your good Lordſhip ſuch as I can, which is the dedication and conſideration of this ſimple treatiſe (ſuch as it is) as a poore token, and teſtimonie of a minde more mindfull of your Lordſhips bountie, then able eyther to deſerue it, or to declare it, and as a ſmall parcell of a farre greater dutie which I

owe vnto your Lordſhip. And although in reſpect of the thing it ſelfe, the woords of the Poet may aptly be applied vnto me, Ruſticus es Corydon nec numera curat Alexis, (for I know your Lordſhip hath no neede of ſuch ſeely elements as this booke contayneth: but contrarily, both the booke and author hath neede of ſuch a noble Mecænas for their patron and protector) yet for as much as ſundrie volumes of like argument, haue often times bene dedicated (as no vnmeete preſents) to very honorable perſonages, yea, euen to princes and kings, I am a little encouraged to repell the Poet, with this ſaying of a famous learned Algebritian, to a nobleman of Germanie, vppon a like occaſion: Exhibeo numeros, numeri ſunt principe digni, for to whom may numbers be more fitly or more aptly dedicated, then to your Lordſhip, to whoſe cuſtodie and prudent ſuruey, her Maieſties reuenues and treaſurie, with knowledge of the number of her Highneſſe Cities, Townes, Fortes, Caſtles, Ships, Armies, people, and ſubiects, doth chiefely, (and that woorthely) belong and appertayne.

Humbly beſeeching your Lordſhip to accept this ſimple volume at his hands, which thinks himſelfe happie, in that it hath pleaſed God to grant him ſo much as this poore meanes, whereby to ſhewe ſome little part of his bounden dutie, and dutifull zeale towardes your honor: and truſteth alſo that by your good Lordſhips gratious patronage and fauourable protection thereof, the ſame ſhall the rather eſcape the reproch of detraction, and be the better liked and accepted amongſt her Maieſties louing ſubiects.

For me to delate vnto your Lordſhip the woorthineſſe of the Arte, or the neceſſitie thereof in humane affayres, were Sus. Mineruã, wherefore leauing and referring both that and the maner of the method, to the prudent iudgement and cenſure of your honorable wiſdome, I ceaſe longer to detayne the ſame from your more weightie affayres. And ſo, as he that is no leſſe bounden in dutie, then ready in ſeruice, ardent in affection & daily in prayers to the eternall God, for your Lordſhips moſt happy, honorable, and proſperous eſtate in his life, with life euerlaſting ſucceeding, I humbly remaine in all dutie at your Lordſhips commaundement.

Thomas Hylles.

The preface to the louing Reader.



 Entle Readers, perhaps it may ſeeme a matter vnto you importing no leſſe arrogancie, then vanitie in me, that after ſo many editions of the Art of Arithmeticke by right learned men, I a perſon of ſo ſmall literature ſhould dare or preſume to thruſt foorth my candle amongſt their bright torches, as though I would thereby greatly augment their light, or els condemne them of darknes, whereof to excuſe my ſelfe with reaſon and truthe, and in friendly maner to ſatiſfie you all, I aſſure you I neuer attempted this enterpriſe, with a minde deſirous eyther to condemne or to derogate any other mans woorkes.

But as all men are not of one minde, ſo all men delight not in one thing, yea & although ſometimes diuers men loue & like one ſelfe thing, yet generally, they do not all like it in one maner or after one faſhion, wherfore albeit that this Art hath bin already very well ſet foorth in our tongue by diuers heretofore, and eſpecially by that notable learned man (woorthy of eternal memory) *M. D. Record*, yet doth my maner of handling therof vary ſo much, both from his and all thereſt, that it may ſeeme like a diſh of the ſame meate, dreſſed after another faſhion, and ſeaſoned with a new ſtrange & extraordinarie ſence: in ſo much that peraduenture it may not onely prouoke ſome to taſt thereof which neuer intended it, but alſo allure ſome others thereunto, whoſe ſtomacks before were almoſt ſatiſfied: and therfore as ciuill gueſts at their frinds tables vſe to take their pleaſures freely of any diſh there that they like, ſetting courteouſly aſide ſuch as they like not ſo well, without eyther blaming the meate, or depraying the cooke that dreſt it: ſo good readers if you miſlike my ſimple cookery in the dreſſing and conditing heereof, my requeſt vnto you is no more, but as you are at your liberties to leaue it, and to take your choyſe of others at your pleaſure, ſo you will vouchſafe in ſcilence, to ſet it courteouſly aſide, as

a diſh

a diſh which may content ſome other, although to you it ſeeme vnſauory, and not delightfull to your appetite, and yet as I proteſt, that I obtayned that little knowledge which I haue in this ſcience by bookes onely, without other teacher, ſo (I nothing doubt) this booke without other help, ſhall be as wel able to inſtruct, any willing minde, though he haue but a meane capacitie, in the whole arte which it profeſſeth to teach: and for that purpoſe I haue enforced my ſelf to vſe the moſt playne and eaſieſt kinde of teaching that I could deuiſe, by reaſon wherof, the worke is more prolixe, and the booke growne to a greater volume, then either I ſuppoſed, or my minde was that it ſhould be, but as the large expoſitions, explications, examples, proofes, and demonſtrations of the rules & maxims, breedeth ſome tediouſnes and augmenteth the booke, ſo ouermuch breuitie neuer wanteth obſcuritie and ambignitie, wherfore he that could explicate the arte perfectly, in rules both eaſie and ſhort, might woorthely obtayne the prayſe pronounced by the ſatyricall Horatius. Poet. *Omne tulit punctum qui miſcuit vtile dulci*, which may be Engliſhed thus.

His Method doth farre all others excell,
Which teacheth pleaſantly, briefly and well.

And although for my part, I looke for nothing leſſe then that prayſe, yet may I ſay thus much, that if any miſlike the length of the demonſtrations, or if any be ſo pregnant & ſharp witted, that they can conceiue the effect of the precepts, propoſitions, Theorems, and problems of the art (which generally in this booke for breuities ſake, I call rules or maxims) without any examples, or without ſuch plentie of examples as are annexed, they (I ſay) may quietly ouerpaſſe the examples, or the more part of them, peruſing onely the rules and maxims, and ſo fewe or many of the examples as they ſhall thinke good: and to ſuch other, as finde themſelues not ſo prompt & capable, peraduenture all that is ſayd or written, ſhall ſeeme but ſufficient or little inough, wherefore as neare as I could, I haue tempred my ſtile ſo, that it may ſerue the ingenious without tediouſnes, and the

ſimpler

ſimpler ſorte without obſcuritie, for the whole arte is both truly and ſufficiently comprehended in meeters, in whoſe compoſitions (howſoeuer it hath or doth fall out in ſequel) I haue diligently endeuored, (as neere as my ſmall ſkill would ſerue, & the matter it ſelf permit) to obſerue theſe ſix points following, that is to ſay.

Firſt that the rules and maxims might containe moſt certaine & infallible truth: ſecondly that they might be proſpicious and intelligible: thirdly that euery rule might conſiſt in verſes or metres ending with like ſounds or terminations, for their better retention in memorie: fourthly that the ceaſures of the meeters might fall in apt places: fiftly that euery verſe might end with a periodus or a colon, or at the leaſt with a comma, and ſixthly that the rules and maxims might contayne as few words as were poſſible to declare their proper effect and meaning.

Wherein if I haue faled, as I am ſure I oftentimes haue in the 3. laſt points, and could not otherwiſe chuſe, I pray you to impute it rather to the harſhnes of the matter, and vnaptnes of the words and termes of arte, then to want of good will.

For this I dare aſſure you, that he which ſhall attempt the like methode in any other Mathematicall ſcience, ſhall finde it more eaſier to make a thouſand wanton amorous verſes, then ten good playne ſtaues of meeter in that ſcience, whether the cauſe be that *Venus* being (as Poets reporte) a ſtrumpet, is more eaſiely wone and obtayned, then the chaſe and learned Muſes I know not, but refer to your iudgements, and yet (be it ſpoken with modeſtie and without ſelfe-liking) I am aſſured that ſome of the rules are not contayned in fewer words, nor in clearer ſentence in any proaſe, then they are here expreſt in meeters: whereof although I confeſſe the moſt part to be ſo rude, blunt, & harſh, & ſo full of *Tautologie* (which I could not auoide) that they are not worthy to be accompted for verſes or meeters, but rather for rime doggrel, yet may they be a means to ſtir vp ſome other finer wits & better skilled hereafter to polliſh & file ſmoth that which my ſmoky forge hath here deliuered rough and raggedly

gedly: and to alter, correct, and adorne the same with more elegancie, choiser termes, and better grace: whereat I shall be no whit offended, but greatly reioyce.

But happely some will say, that this forme of teaching the Arte, is an idle vnnecessarie forme, and such as is rather ridiculous then needefull or profitable, to such I pray you let it not offend you, that I answer them with a question, asking them wherefore the Gramarians haue labored so much to reduce all the chiefest rules of their Arte into verses? why haue the logicians diuised to answer the quidditie, qualitie, and quantitie, (as they call it) of their propositions in this verse,

Quæ. Ca, Vel hyp: Qualis? Ne Vel af: V. Quanta? Par in sin.

Wherefore haue they deuised to know the reduction of an argument to due moods and figures by these verses following.

Barbara: Celarent, Darii. Ferio, Baralipton &c.

why haue our Lawiers deuised these verses insuing, as the order to be obserued for the placing of perticulers in writs of right?

Registrū orig. fol. 2.

Mes Tost. Mol, Col, Gar, Ter, Pra, Pas, Brue, Mora.
Iunca, Maris: Alne, Rus, Red, Sectare priora.

And agayne, for a man that shall wage his law.

Cur hundt in fiac.

Qui legem vadiat, Nisi lex in tempore fiat,
Mox condemnetur, Taxatio non sibi detur.

Wherefore are there so many rules of the lawes both ciuell and canon in verses? and concerning rules of Phisicke, wherfore was the whole booke called *Schola Salerni*, made in riming verses? Or why hath *M. Otto* of *Cremona* described the goodnes and choise of sundrie druggesand simples vsed in Phisick, in riming verses? and touching diuinitie, to let passe the psalmes of *Dauid*, the prouerbes of *Salomon*, the lamentations of *Ieromie*, & the Acts of the Apostles, with diuers other. Why did that learned man *Erasmus Roterdamus*, comprehend the argument of euery chapter of the new testament, in a distinction or couple of verses, prescribed to the same chapters wherevnto if they will aunswer (as they must needes, if they will aunswer truly) that these rimes and verses were purposely made for memories sake, ther-

by

by to call the matters the more readily to remembrance.

Then why may not Arithmetick vse the like aide by verses or meeters? and wherefore may not the rules, precepts, and maximes thereof reduced into Englisch meeter, serue as aptly to like purpose, and be as commodious to vs Englishmen, as it is in Latin to them which vnderstand the Latin tounge? and it is well known, that the same arte, or the most part therof, is already extant in Latin verses long agon, by a learned Mathematician our own contrimã *D. Buckle*, & is intituled *Arithmetica memoratiua*.

For to say truth, as there is no one arte more generally necessarie for all degrees, nor more commonly vsed among men, so is there none more slippery or apter to be forgotten, where it is not either cõtinually practised, or the rules fixed in a very firme and retentiue memorie, which is a gift that euery man hath not, and which for my part I confesse I neuer had, but was euer of a very weake & watrish memorie, euen from my youth. In which age, although I did not onely reade the science, but also euen thẽ (by mine own study) got such readines therin, that I thought my selfe pritty skilfull, and able to perfect, or (as the Marchants call it) to ballance any ordinary accompt or reckoning, or to answer any question dissolubl'e by the rules of vulgar Arithmeticke, yet since that time; by discontinuance and disuse of the arte, I had so quite forgotten many principall rules and operations thereof, that hauing occasion within these fewe yeares to occupy Arithmetick, about certein intricate accompts, I found my self so far to seeke, that I was constrained to go looke in my booke what I had to do: wherfore to the end I would neither be bound to be daily practising of that which I had but sildom occasion to vse, neither yet be vnfurnished therof whẽ necessity required & knowing the shortnes of mine owne wit & weaknes of my memory, I determined for the seruing of mine own priuate turne, to reduce certeine of the principall rules onely, into such a kinde of meeter, as I supposed was very apt to keep them in minde, so that if I should any more forget the practise, yet by recording or repeting the verses, I might presẽtly cal the same to

M.E.M. memorie, which when I had done, it chanced a wise & woorshipfull gent. one of her Maisties gent. Pencioners, to haue a sight of them, who being a friend to the Art, liked them so wel, that he did not onely copie them out with his owne hand, but also wished that the whole Arte were set foorth in like maner, whereunto I made no answere presently, but neuertheless I thought so much vppon his woordes, that thereby I haue at times, as it were by starts, as my leasure would serue me, waded through the whole arte of vulgar arithmeticke, not onely in meeters, but also with large expositions, examples, and demonstrations, in proase as here you may see.

The occasion of this booke

For the performance whereof, I haue perused but euen so many of the best and most famous authors and writers in this arte, as I could either buy for money or borrow for loue, gathering and collecting their best and briefest rules: for besides the order of the method & conuersion into meeter, I claime nothing herein of mine owne inuention, sauing the 17. Chapiter of the second booke, touching the golden rule artificially compoun- by Multiplication, which with the very first foundation and demonstration thereof, and (almost) al the answeres found by that rule direct, and conuerst, descending, and ascending, as well in fractions as integers, together with the merchants rule, as it is set foorth in the first Chapiter of *Musa Mercatorum*, and certayne additions to the rules of buying and selling by the 112. pound contayned in the sixt Chapiter of that booke, and the application of the compound rules to the reduction of weights and measures, as it is prescribed in the last Chapiter of that booke, with the deuice how to know certaynly by which of those foure rules, a question propounded is to be answered: I challenge wholy for mine owne, for that I neuer red or found the same in any former writer, but found it by mine owne study, and yet it may be that the same hath bene knowne also to others long before now, but that is more then hath hetherto appeared to me, and as for the rest to the end that euery man may haue his owne with due praise, I haue as occasion serued

What the anthor claymeth in this woorke.

noted

noted in the margent, the authors name whence I had it, except the questions themselues, which I esteeme as common and indifferent, as well to me, as to them.

For I see that neyther *Ramus*, nor *Vrstitius*, nor *Salignacus*, nor *Clauius*, make any doubt to call that arithmeticke absolutely theirs, whose method they inuented, although the self same questions, which they propound & answere be almost verbatim long time before propounded by *D. Tonstall*, neyther doth the same *Tonstall* forbeare to doe the like by is Arithmeticke, which neuerthelesse he himselfe confesseth to haue collected out of al the authors as wel barbarous as learned, whose toungs he vnderstood, and therefore gentle readers I trust I may be as bold to call and to accompt this arithmetick mine, for as much as the whole method and metrizing thereof (such as it is) is mine owne, and such as no man hetherto hath vsed, eyther in Englisn or Latine to my knowledge. Tonst. in profatio I homæ Moro.

And now peraduenture you looke that I should here shewe who were the first inuentors of this science, and what the commodities thereof are, of which two points I am loth to say any thing, because that by the diuersitie of opinions among writers the first is vncertayne, and by the agreement of them all, the last is infinite, for some ascribe the inuention thereof to *Abraham*, some to *Phithagoras*, some to *Phœnices*, some to the *Chaldees*, & so some to one, & some to another, but for my part I am fully perswaded that God himselfe the fountaine & author of all goodnes (from whom as saint *Iames* sayth, euery good gift, & euery perfect gift doth proceede) was the first author thereof, who by diuine inspiration infused the same first into *Adam*, among other most excellent gifts and graces, wherwith he indued him in the perfection of his creation. Iames 1. 17.

For the sacred scriptures testifie that God created *Adam* to his owne image and similitude, which cannot be vnderstoode of body and bodily shape (sith God is a spirit, but in the minde or soule, wherein if *Adam* lackt humaine knowledge, how could he comprehend the knowledge of the diuine maiestie, or in anything be more vnlike to his creator, who knoweth all things.

And therfore (no doubt) as *Adam* before his fall, resembled his Creator in righteousnes, innocencie, goodnes, holines, and immortalitie, so did he also in knoweledge of all goodnes, in so much that all the sciences which we call liberal, were not onely most perfectly known vnto him, but as histories report, & I easily credit, he also knew the natures, qualities, effects, & properties, of all herbes, plants, trees, mettals, minerals, stones, gums, birds, beastes, and other creatures and things whatsouer, and thereby did not onely giue euery lyuing creature an apt name, agreeable to the propertie thereof, but also as soone as he sawe *Heuah*, could presently say, this is now bone of my bone, and flesh of my flesh, she shall be called woman because she was taken out of man.

So that I ascribe this science to no other author, but to the immortall God, who did as verily endue man there withall in his first creatiō, as he did endue him with reason, speech, or life, which thing *Carion* in his Chronicle confirmeth, saying that it were not possible for any mortall man to attaine to the finding of so high and woonderfull things, except God himselfe did first open the knowledge thereof vnto him.

Neyther is it possible to imagine, that God would suffer *Adam* the first man, whom he appointed to be Lord of Paradice, and the originall father and roote of all mankinde, to be inferior in knowledge to *Pythagoras*, *Socrates*, *Plato*, or *Aristotle*: wherfore I conclude that God, was, and is, the first author thereof, by whose diuine power it was infused into *Adam*, as a parcell of his naturall perfection, and by him taught to his sonnes, and so successiuely deduced to their posterities, since whose fall nature being depraued, corrupted, and obscured by sinne: no man euer had any such exquisite knowledge by nature, for the great wisdom and knowledge of *Salomon*, came not to him by nature, nor yet by the teaching or instruction of any mortall man, but as the scriptures affirme, he obtained it by prayer and the speciall grace of God after he was king, which thing he himselfe witnesseth to be true, saying that God gaue vnto him the

true

true knowledge of the things that are, so that he knewe how the world was made, and the powers of the elements, the beginning, the end, and the middest of times, how they alter, and the change of their seasons, the course of the yeere, and the situation of the starres &c. Which (as I take it) is as much to say as that he had exact knowledge in the liberall sciences, and in all naturall Philosophie, for to haue the true knowledge of all things that are, doth so farre excell *Placita Philosophorum*, as *scientia certa*, excelleth *opiniones*, all which learning, for as much as he neuer obtayned by instruction from man, argueth God himselfe to be the author of all such sciences, & so consequently the very first and onely author of Arithmetick, onely the inuention of those ten numerall figures, which at this present are vsed in the vulgar Arithmeticke throughout all Europe, is generally thought to haue proceeded first from the *Arabians*, for the *Hebrewes* and *Greekes* vsed their owne letters for numbers, vvith the vvhich (as I am credibly enformed) they could eyther adde, subtract, multiplie, diuide, or execute any other operation arithmetical, both in integers and fractions, as vvel & as perfectly as we can do with the cõmon figures, vvhich thing I could neuer read nor heere, that the Latines could do vvith their letters: although it is commonly knowne that they also vsed their proper letters for numbers, as we do, yet although in a proposterous order, & contrarie to al reason: And thus much for the first author or inuentor of this Arte.

Sapience. 7.7. & 17. 18 19.

This knowledge far surpasseth the knowledge of Aristotle and all humaine Philosophers.

God is the author of good sciences.

Now touching the other poynt, vvhich is to declare what commodities the same bringeth with it, or how many wayes the vse thereof is requisite, eyther for pleasure, profit, or necessitie, in humayne affayres, would of it selfe require a large volume, for seeing the vse and commodities of number, are as infinite as number it selfe, who can by definite woordes or sentences sufficiently explicate the same, truly for my part as I confesse that I cannot, so I will not endeauour it, least whiles I should goe about so to doe, I should but greatly increase this volume, which is growne too great

great alredy and yet when I had done all that I could, I ſhould leaue much more in ſcilence, then I ſhalbe able to vtter or ſet foorth, wherefore as the adage ſayeth, *Vino vendibili ſuſpenſa hedera nihil opus*, where good wine is to be ſolde, there needeth no garland to be hangd at the doore, ſo I ſuppoſing the neceſſitie, profit, and pleaſure, of this ſcience to be ſuch, and ſo notoriouſly and palpably felt and knowne to all men that know any thing, that it needeth not the feeble floriſh of my vnfiled ſtile, to commend it or to encreaſe the credit thereof, do rather thinke it better, to haſten to ſhew the beſt & neereſt way, how to attayne the ſcience (as that without which no man can euer haue right paſſage, nor once entrie into any learning diuine or humaine, Mathematicall nor philoſophicall) then to withholde it from you vntill I haue declared all the commodities therof: of which ſcience I doubt whether you will more repent the lack, when you haue neede to vſe it & cannot, or reioyce in the habit and poſſeſſion therof, when you haue it to vſe at your pleaſures.

And yet to the entent you ſhall not be altogether ignorant of the manyfolde vſes and commodities of this arte, the verſes hereafter inſerted, by certayne friends and louers of learning will ſerue briefely to informe you in that behalfe, with the which if you be not content, (leaſt I ſhould *actũ agere, et Crambe bis coctum apponere*) I refer you to the prefaces of *M. Rob. Record* in his Ground of artes and Whetſtone of wit, & to that notable preface of *M. Iohn Dee* prefixed to *Euclid*, where you ſhal finde matter aboundant touching that argumẽt, deliuered with ſuch grace & ſweetnes of ſtile, that the very memory thereof forceth my ruſtick pen, as quite abaſhed, all amaſed & aſtoined, here ſodenly to ſtop, & abruptly to ſtay, & ſo good readers wiſhing you ſuch good ſucceſſe herein, that in reading you may vnderſtand, and by vnderſtanding may get ſuch practiſe as may increaſe both your knowledge, your profits, and your pleaſures, I cõmit you to God, and my ſelfe with this rude worke, to your curteous cenſures and good reporte,

Valete.

The

SIth Error is so naturall, and incident to man,
That it is *quasi proprium*, do he the best he can,
And stealingly doth stepp in place, where it is least suspected,
For *Barnard* seeth not all (t'is said) but oft may be corrected,
If any Error haue escapt, my authors painefull pen,
Correctors eye, or Printers hand, as some may now and thē,
Accompt it for an humaine fault which no man can eschue,
Which in respect thou art a man, amend, and so adue.

A parable of the Authors Muse.

THe precious wheate, more worth in very deed,
For humayne life, then gold or pretious stone,
Long lies halfe dead, and like a grasse or weed,
With men and beasts is oft times trod vpon,
As nothing worth, till after tract of time,
Sir *Phœbus* with his comfortable cheere,
Reuiueth it, by pleasant heat of prime,
Whereby it doth with dooble force appeare,
To spring, and spred, and couer all the fielde,
And eke ere long, doth kerne with such increase,
That ten for one, at least each corne doth yeelde,
So that although, the owner holde his peace,
And makes no boste, what gaynes his seede doth bring,
The Reapers yet, will blase abroade the thing.
Euen so my Muse,
Long time oppreſt, halfe dead and drooping lay,
Witherd with windes, of cruell inward cares,
All ouer trod, and nigh consum'de away,
By spightfull foes intrapments vnawares,
Till great Iehoue, in mercy mooued loe,
Reuiu'd her with his former louing face.
For griefe whereof, the wicked wight which so
Oppressed her, and thought her to disgrace.

To *Charon* fled, and therwithall my Muſe,
So flouriſhed aboue her wonted ſorte,
That forth ſhe brought ſuch frute as here inſues,
Which what it is, let others make reporte,
That Reape the ſame, for me it ſhall ſuffice,
If well in worth yee take and not deſpiſe,
My ſilly Muſe.

Danielis Gardiner in Thomæ Hylli Arithmeticen Carmen.

INgenium ſolers, vigilans labor, ars bene culta,
Hæc tria formarunt quod dedit Hillus opus,
Quod dedit Hillus opus, magno parit ille labore,
Aere paras paruo, quas dedit Hillus opes,
Hoc dedit Hillus opus gratum gratis, lege gratus,
Sic graue ſit gratum, quod tulit Hillus onus,
Quod cupis Hillus habet, norman facilem numerandi,
Vtere, ſis gratus, Quod cupit Hillus habet.

D. G.

Ranulphi Barlowe in Thomæ Hilli rithmos Arithmeticos Carmen.

PYthagoras olim docuit primordia rerum,
Ex Numeris fieri Simmetriſque ſuis,
Pro Mente atque Deo, Numeros poſuere vetuſti,
Sic nos priſcorum myſtica ſcripta docent,
Pondere, menſura, Numero, Deus omne locauit,
Sic nos indoctos pagina ſacra docet,
Quis dare plura vnquam Numeris preconia poſſit,
Si celebret Numeros laudibus innumeris,
Hos placidis Rithmis ἀοιδῶς finit ἀεθμῶς,
Hillus, ſit docilem te docet ille virum,

Si facili possis Numeros ediscere norma,
Et gratus venias quod cupit Hillus habet.

R. B.

Richardi Man in Arithmeticen numerosam, Tho: Hylli Carmen dialogisticum inter Montanum et musam.

DVm Numeros numeris scite brevis exprimit Hyllus, Montanus
Incertos certis: quam graue praestat opus,
Dic mihi Musa memor: quaque ars deluditur arte?
Quam tacitisque dolis ars inopina iacet?
Odit Aeduum vaga, stricta legamina vatum, Musa.
Hactenus, & renuit libera ferre iugum,
Donec Pieridum montes ascenderat Hyllus,
Edoctos sacros enumerare modos,
Queis olim rigidas cautes commouerat Orpheus,
Dum resonum digitis sollicitaret ebur,
(Tam concinnatas numeros modulamine cantans,
Possem Dulichios exuparare dolos,)
Hic epulis Musae, charites, Artesque fluentes,
Post hilares varijs discubuere iocis,
Tum bene compositis chordis ad vocis amores,
Orditur numerus Hyllus: acuta canens,
Haud secus ac Hermes soleat, vel scitus Apollo,
Dum Citharae suaui Nablia iuncta sonant,
Hunc dum suspiciunt diuinos ore canentem,
Humano modulos, Numina laeta tacent,
Aure satis facili voces simul accipit istas,
(Iuscia technarum quam?) Numerosa Dea,
Atque vt Sirenum soleant adducere nautas,
Melliflui cantus, & trahere in Scopulos:
Sic et Aeduum plane his superata camaenis,
Seu caduciferi pressa sopore iacet,
Acta paracta videns Hyllus, cito compede stringit,
Dum natas capta vinxerat Arte Dea.

Sic Numerans, Addens, Subducens, Multiplicansque,
Frangens, Progrediens, quaeque ligamen habet,
Descendit spoliis amplis, et laude triumphans,
Qua Iouis ille audax igne potitus abit,
Nec captiua viris, minus est quam libera suauis,
Fructiferæ proli, quod magis apta manet.
Montanus *Vos ergo Aonides, quibus hoc spectantibus actum est,*
Laurigerum vatis cingite fronde caput. R. M.

Another of the same R. M.

WHere hetherto Arithmetick hath often times complaind,
how that her rules are soon forgot, which labor lõg obteind
And that her lawes are harsh & hard to leaue their print behinde
Vnlesse that daily excercise, do keepe them fresh in minde.
This olde and festred sore to cure, thy care o *Hilles* hath beene,
Which by thy happy helthful hand, is found now to be seene:
For sower thou hast expeld with sweet, & sweet with profit ioind
whence wisest wits may get increase, & weakest help may find.
And wher the rough and tedious paths, did many men dismay,
Their weried wits thou doest refresh, by plasant easie way,
For why, what sot so senceless is, although he much forget,
But by the aide of riming verse, can many things repeate.
And though at first for priuate vse, this way thy Muse did finde,
that so thou mightst more perfectly, keep rules of Art in minde
Yet hauing found the larg increase, & gain the same may yeeld,
thou dost not grudge thy proper grain, to sow in commõ field.
For more respecting publique good, then any priuate gaine,
thou thoughtst it wrõg, frõ cõmon welth, thy labors to detein
Wherfore thy country vnto thee, vnnũbered thanks shal render
which thus hast taught thẽ nũbring rules by nũbers to remẽber
And with religious care shall keep, these pillers of thy prayse,
as worthy all posterities, and long induring dayes,
So that when death hath done his worst, to take thy life away,
yet shall thy name remayne with fame, that neuer shall decaye.
R. M.

Another by another.

WIth passing paines & labor great, the Ants within their Hils
do hoard vp corne as in a barne, to serue them at their wils,
The busie Bees, on buds of trees, on herbes and euery flower,
Do seeke, & search, & suck the best, to fill their hiue or bower.
The husbandmen which hope to haue their corne imbarnd in
vntil that time do seldom rest but toile beyond al reason (seasō
Not much vnlike a busie Ant, not much vnlike a Bee,
Nor much vnlike a husbandman, this author seemes to mee.
For so hath he from booke to booke, and authors many one,
with toile pact here such dainty corne as elswher there is none.
Lo heere the Hil, the hiue, the barne, wherin such store is plast,
As vnto all posterities, shall serue and neuer wast:
For why? this corne hath such a gift, that wher it is once grown
It brings increase from yere to yere, & needs no more be sown
Here is the Hils where Muses meete, forbidden to *Corœbus*,
here is the hiue of hony sweet, which feeds the sons of *Phœbus*
Here is the barne which filled is, with hony, corne, and wine,
not made by Bees, nor rept in fields, nor pressed frō the vine,
But rather sent from Helicon, by might and power diuine,
Compounded by *Apollos* arte, and all the Muses nine.
No state, no age, no man, nor child, but here may wisdome win
for nūbers teach the parts of speech, wher childrē first begin.
And number beares so great a sway, euen from the most to least
That who in numbring hath no skill, is numbred for a beast:
For what more beastly can be thoght? nay what more blockish
then man to want the onely art, which proper is to man, (than
For many creatures farre excell mankind in many things,
But neuer none could nūber yet, saue man, in whom it springs
If numbring then be (almost) al, betweene a man and beast,
Come learne o men to nūber then, which arte is here profest
If martiall man thou minde to be, or office do expect.
In court or country where thou dwelst, or if thou do elect,

In

In Phiſicke and Philoſophie, or law to ſpend thy dayes,
Aſſure thy ſelf without this arte, thou neuer canſt haue praiſe,
I ouerpaſſe Aſtronomie, and Geometrie alſo,
Coſmographie, Geographie, and many others mo.
And muſicke with her dulcet tunes, all which without this arte,
Thou neuer canſt attayne vnto, nor ſcarce to any part,
Ne canſt thou be an auditor, or make a true ſurucy,
Nor make a common reckoning, if numbers be away.
But if thou wilt a merchant be, then make this booke thy Muſe,
wher thou ſhalt find rules fit for thee, as thou cāſt wiſh or chuſe
And hauing onely handie craft, yet herein mayſt thou finde,
Such things as may oft ſerue thy turne, & much enritche thy minde
Nay, if thou but a ſhepherd be, it wil thee ſore accūber
To do thy duty as thou ſhouldſt without the helpe of number
To number all the benefits, that number brings to man,
Would be too long here to rehearſe, & more then well I can,
Wherefore to ſpeake one woorde for all, and let the reſt alone,
without this arte man is no man, but like a block or ſtone,
Auant then *Momus*, keepe thee mute, and *Zoilus* ſtand aloofe,
Depraue ye not the thing that ſerues for euery mans behoofe.
Ne carp ye at the authors worke, which none of you can mend
Whoſe paines deſerues immortall prayſe, & ſo I make an end.

Robert

Robert Hanley merchant.

THough numbers growe, and ſtractions fall,
ſo infinite without all end,
The firſt ſo great, the laſt ſo ſmall,
That no man can them comprehend.
Though numbers vſe doth playne appeare,
In towne and country euery where,
At home, abroade, both farre and neere,
At Sea, at ſhore, on earth, on ayre.
In euery arte and trade of life,
In peace, in warre in merchandiſe,
In all contracts they are ſo riſe,
They may not want in any wiſe.
Though numbers might ſurmount the Moone,
And clime aboue the ſtarry ſkye,
Yet may he make them ſtoope full ſoone,
To woorke his will obediently,
Which by theſe rules doth rule and guide,
Himſelfe and them as *Hilles* hath taught,
For beeing wiſely ſo apply'd,
They ſhewe with eaſe the matter ſought.

Robert Hanley merchant,

The names of such Arithmeticians, Mathematians, and Philosophers, as haue contributed to this present worke, or cited, or otherwise mentioned therein.

Archimedes.
Barlaam.
Bernardus Salignacus.
Boetius.
Campanus.
Christianus Vrstitius.
Christoferus Clauius.
Christoferus Ridolphus.
Cuthbertus Tonstalus.
Dionitius graius.
Diophantus.
Euclides.
Eudoxus.
Franciscus Flussas.
Gasper. peucerus.
Gemma Phrisius.
Georgius Reische.
Gulielmus Goselinus.
Gulielmus Xylander.
Henricus Billingsley.
Hieronimus Cardanus.
Humfridus Bakerus.
Iacobus Peletarius.
Ioachimus Fortius.
Iohannes Dee.
Iohannes Mellis.
Iohannis Sheubelis.
Leonardus Digges.
Lucias Pacciolus.
Mantaurens.
Nicholaus Tartalia.
Petrus de Burgo
Petrus Ramus.
Robertus Recordus.
Thomas Digges.
Valentinus Menherus.

The first booke intituled *Nomodidactus Numerorum, conteyning the first part* of the art of vulgar Arithmetike, in fourme of a Dialogue *betweene* Philomathes *and* Eumathes *interlocutors*

Of the Definition, Diuision, and Notation of Arithmeticke. The first Chapiter.

Philomathes.

1.

ARithmetickeis th'art of computation,
Teaching to number perfectly & well,
And cast all accompts in such fourme & facion
That truth shall appere vntruth to expell.
which art by this beke right sone learne he shal
That will but reade it and practise withall.

The definition of Arithmetike.

This verse or staffe conteyneth, not onely the definition of Arithmetik, shewing that it is the art of computation, or numbring, whereby is taught to number perfectly and well, and to cast all accompts, or reconings so truly, that no error shal appeare or be found therein, but also it sheweth you, that he that is willing to learne the sayd art, may by this booke onely (without other scholemayster or teacher,) learne & apprehend the same easily, if he do but onely reade it, and practise the thing which he readeth.

Euma. Sir as I do vnderstand the meaning of the verse very wel, and the better for your explanation thereof, so doubt you not, but I will read the booke diligently, & practise it as I read it continually, till I feele & finde ỳ habilitie in my selfe which the verse promiseth.

Phi. So shall yo do well, and not fayle to obtayne your desire, forseeing that you proceed orderly, not medling with that which is behinde, till you be perfect in that which is before.

Euma. Very good, you haue geuen me a good warning which I will not forget to obserue.

Phi. Then will I now shew you what number is according to the opinion and definition of Euclide & other auncient Arithmeticians, and Mathemeticians, who define it thus.

The definition of number. Euclyde 1. and 2. Definicion of the. 7. one nomber.

2. And number is a quantity discret,
Or multitude of vnits seperate,
Each by it selfe being entire and complet,
And in one summe together aggregate.

So that wher is not two vnits at leſt,
There is no number I ſay and proteſt,
For vnits, integers, or things entire,
Are either but one, where there is any,
Or els they are mo, wherby they require.
The name of number becauſe they are many,
But any one thing propoſed or aſſignde,
Is not a number of that proper kinde.

Hereby you ſee how number, beeing ỹ matter wherof this Arte conſiſteth, is 2 defined to be a Mathemetical quantity diſcreet, or a multitude of vnits, integers or things entire ſeperated one from another, but yet collected agregated or aſſembled together into one ſumme, as it were in a heape.

Euma. What meane you by the woords vnits, and integers?

Phi. By an vnit or an integer (which ſometimes I alſo cal an Ace) is vnderſtode any one whole & entire thing of whatſoeuer kind it be perfect in quantity continuall, vnbroken & vndeuided whether you conceiue it only in minde, mathematically as abſtract & ſeperated from matter ſubſtance or ſence (as pure mathemetical vnits are) or whether you apply it & contract it, to one man, one beaſt, one penny, one pēũd, one el, one inch, or any other one entire thing, as you may doe to one graine of muſtardſeed, or one graine of ſand, if you will.

Now many of ſuch vnits, integers or aces, agregated or aſſembled together, into one ſum, make a nũber, becauſe that by their multitude or pluralitie, they become aquãtity diſcreet or ſeperate, & ſo the agregatiõ aſſembling & collection of ſuch ſeueral diſtinct, vnits, ingers, or ones, into one ſum is the forme wherby they are made a nũber. The action or operation wherof is done by the minde, & vttered by the tongue when we in numbring ſay, one, two, three, foure &c.

For if we ſhould ſay one, one, one, one, &c. a thouſand times ſuch ſaying were no agregation, aſſembling or gathering together of vnits into one ſum, but a meere leauing of them ſtill ſingle diſperſed and loſe, and therefore in ſaying ſo, we neyther number any thing, nor make any nũber, for want of the due fourme thereof, wherby it is euident ỹ no earthly crature is capaple of nũbring, but onely man.

Numbers numbering Number numberred Number by which we number.

For as number is principally diuided into a threefold ſtate to wit, Number numbring, Number numbred, and Number by which we number. So number numbring, conſiſteth onely in the mind or reaſonable ſoule of man. Who by the organes of the body hath power to number, Number nũbred, is the things that are numbred as men, beaſts, cattell, money, &c. And all things ſubiect to number weight or meaſure, the numbers by which we number are one, two, three, foure,

foure, fiue, &c. which are those which are called Mathamaticall.

And for asmuch, as no single vnit or integer, of whatsoeuer kinde it be, is or can be discreet or seperate from it selfe, but is in it selfe truly & naturally, a quantitie continuall indiuisible (for if it be once diuided, then is it no longer one) & also sith there cannot (by reason) be any collection or agregation, where nothing is discreet or seperate a sunder, therefore the second verse here present, affirmeth truly, that where there are not two vnits at the least, there is no number, because there cannot be an assembling or putting together of fewer things then two. vnitie is indiuisible by nature

And by the same reason and this definition, If I shall speake properly and exactly, I must say that one is no number, but rather, the beginning of number, yet is it a part (and that the least part) of euery true number. And is consequently two is the least number. An vnitie a part of euery true number vz. parts, constituēs siue componens.

For as the third bearse here present, saith, that where any integers are proposed, there is either but one, or els there are mo, so if they be mo then one, then are they rightly to be called a nūber, because they are many (for mo then one is many) although they be but two, and euery true number conteyneth many vnits, or integers, but any one thing only proposed, assigned or set footh, is not a nūber of ꝑ kinde.

Euma. I vnderstand you thus, that one man, is not properly to be sayd a number of men, nor one horse, a number of horses. And likewise ꝑ the sun or moone, being but one, cannot truly be called a number of Sunnes, or Moones, but if there were mo then one man, or mo horses then one, or mo Suns or Moones then one, then were they a number, and so do require to be counted and called, though there were but two of each kinde.

Philo I perceiue you vnderstand my meaning sufficiently.

4 Wherefore note well that an vnit or ace,
Of all things entire is fewest and least,
But not of numbers, for why in that case,
Two is the smalest that can be expresst.
As for the greatest no tongne can recite,
Sith numbers encrease doth rise infinite,

By this rule you are to note that an vnit, an ace, an integer, or one, is the fewest or least quantity 2. the least number.

of all whole things that can be expresst, whereof no man hauing reason can doubt, for he that hath not one Angel in his purse, hath none at all there, no although he haue two crownes, or tenne shillings.

Euma No hath? Why is not one Angell and two crowns, or one Angell and ten shillings all one?

Philo. No verely, for the matter of the Angell (as I meane it) is

gold, the fourme whereof is the print and superscription, and the weight or quantitie therof geueth it the valure, which quantitie, is continuall and therfore is a magnitude, and not a multitude or number, but although the matter of the two crownes be all one with the Angell, yet are they a quantitie discreete and seperate eche from other into two vnits, and so are the ten shillings likewise into ten vnits, wherefore they are rightly called a number of crownes, and a number of shillings, but not a number of Angels.

Euma. Yet men comonly call ten shillings an Angell, and an Angell ten shilllinges.

Philo. But that is abusiuely and onely in respect of their valures being equall, and yet the things themselues are no more one the other, then a peny loafe is esteemed to be a peny, because they are both of equall valure. wherefore the effect of this verse is, that as one is the fewest or least of all integers, so two is the least of all numbers, but as for the greatest it cannot be recited or rehearsed by any mans tongue which is as much to say as that the least number is certainely knowne to be two, but the greatest is vncertaine and vnknowne, and therefore cannot be named with tongne, nor yet described with pen or otherwise, sith number of it selfe is infinite in encreasing.

Euma. Why sir if number be infinite, how can there be any Art thereof: for I haue heard learned men say, that of things infinite, there is no Art?

Philo. That saying is very true, for of things infinite, there can no rules nor precepts, be collected or composed, neyther can any mans wit comprehend them, wherefore we refuse such infinitnes and entermeddle onely but with numbers which are certaine finite, and determinate. For,

Numbers giuen is infinite, the greatst and smallest fractions are namcles.

5. The vnits of any whole number assignde
Are not infinite nor innumerable,
But yet the greatest of numbers to finde,
Then which to giue greater, no man is able,
Or of all fractions to describe the lest,
Is an impossible Idle request.

So that this verse assureth you þ they are not infinite or innumerable vnits, in any number geuen or assigned, for although numbers in power and increase, are so endles, that the greatest cannot be named or expresst. Yet no number giuen, assigned, or expresst, That is to say, no number propounded, or which is in art already knowen and certainly limited named and determinate (as all the numbers whereof we intreat are) can contayne infinite or innumerable vnits, because euery number

that

that can be named propounded or expreſt, hath a certeine known & determinate number of vnits in it, as forty which conteyneth iuſt fortie vnits, and ſo forty thouſand conteyneth iuſt forty thouſand vnits, and neyther mo. nor fewer, and ſo forth of euery other number, wherefore this verſe concludeth that whoſoeuer requireth to knowe what number is greateſt, or what fraction is leaſt, aſketh a vayne Idle requeſt and ſuch as is impoſſible to be anſwered.

And thus haue I ſhewed you the definition of number according to Euclide, and other auncient writers, but the latter writers, as namely Ramus, and ſuch as haue written ſince his time, affirme not onely that an vnite or one, is a number, but alſo that euery fraction or part of an vnite, is a number. And they ſhewe many reaſons, and bring diuers arguments, for the confirmacion of their opinions, which as they are to long here to rehearſe, ſo I know they are as yet, to hard for your vnderſtāding. Yet one of their reaſons I wil repeate which is this, that one is reckoned among alkinde of numbers as the firſt Originall of euery kinde, as namely it is holden and accompted for the firſt digit number the firſt odde number, the firſt Triangle number, the firſt Square number, the firſt cube, and ſo forth of others. And in the ſtrength and power, it is truly ſo as they ſay. Rama. pag. 1. Saligna rus. pa. 6. Vrſticius Siſteuens. pag. 1. Boetius, 2. Cap 8. Arith.

But the reaſon of this priuiledge of vnitie, Boetius very well declareth, where he ſayth for aſmuch as vnitie is the originall mother of all numbers, it muſt needes contayne in it ſelfe (by a certayne naturall power) whatſoeuer is found in theſe numbers which are conceiued of it, or proceed from it. And Pelletarius a learned Mathemetician ſayth that an vnite or vnitie is no number, but onely the beginning of number, and that more is, it is the beginning of it ſelfe. For ſayth he it is the product of it ſelfe by multiplication, the quotient of it ſelfe by diuiſion, the roote, ſquare, and cube of it ſelfe, and to be ſhort it hath the proprietie of all regular numbers, And being by nature indiuiſible immutable immaterial & without poſition, hath in it ſelfe no imperfection, wherefore it is compared to the diuine nature, as aproching more nigh thereto, then any other thing.

And the moſt famous Philoſopher, of this our country and of this our age, affirmeth vnitie alſo to be no number. Yet ſayth he number hath a certayne originall ſeede (as it were) of an vncredible propertie, and of man neuer able fully to be declared, by which ſeede he vnderſtādeth vnitie, for ſaith he, though number be not produced of an

Unit, yet doth it consiste of vnits as of the very materiall cause.

But although vnitie, by speciall proprietie & priuiledge in some respects hath the power of diuers numbers, yet actually or essentially no vnit can represent the fourme of Triangle square, cube, or other figure geometricall as true figurall nūbers do, but is resembled to the indiuisible pricke in geometry onely, as you shall better perceyue, when we shall come to that kind of Arithmetike wich is called figurall.

1. accompted a number of euery kind, potencially onelye.

Euma. Do you then esteme or accepte vnitie for a number or not?

Philo. I do accompt it after a sorte for the first or least number of euery of the said kinds that is potentially and as it were in power and possibilitie onely and not already actually or essentially euen as an egg, with in power possibilitie containeth a bird though really and actually it is none, other wise I take two, for the lest number in in arte really.

And likewise I esteeme, one halfe, two third parts, three quarters and such like (which are called fractions) to be numbers, yet not of vnities or integers but of the parts of Integers, for if such fractions should be contracted to any sensible thing, as namely to an apple, could you say that one half apple, or three quarters of an apple, were any number of apples.

Euma. No in deed in my iudgement, but me thinks they might rather be called numbers of the peeces of an apple.

Phi, you saie wel, but being cōsidered mathematicallye that is rather by Imaginations, or conceipt in minde, then by outward senses, they are repu'ed for numbers, and to saye truth all such fractions, are but meere conceipts & fictions of the mind, for if they should be applied to matter subiect to the sences, how can one inch. or any other one thing be deuided into a thousand, or an hundreth thousand parts, so þ they shalbe al iustly equall, & none more nor lesse then other, but onely by Imaginations, in so much that Stifelius is bold to cal them feigned numbers, and yet do such fractions bring great profit commodities & precise truth to this arte, And thus much for the definition of numbers and the parts thereof.

Euma. very good, but now I pray you geue me leaue to aske you one question by the way, whilest it is in my mynd, and it is this, why do you in the tittle of this booke call this Arithmetike vulgar.

4. kynds of Arithtike.

Philo. I call it so in respect of other things of the arte which are not so commonly in vse of the vulgar people, for besids this vulgar arithmetike, their is also arithmetike Circuler arithmetick figural,

and Arithmetike Radical. But to proceede with this vulgar kinde first, I come now to the diuision thereof which is thus.

Arithmetieke is diuided into 2. generall parts whereof the first conteyneth the notation and operrations.

6 Arithmetike hath two parts as fundations,
whereof the firſt, which firſt muſt be taught.
Shewes the notation and operations,
Of numbers and fractions in fourme as it ought,
Of integers firſt, and the next of fractions,
But each in due order to ſtoppe all obiections.
7. The ſecond part ſhewes the workes and effectes,
Of numbers relatiue or being compard,
In quantitie firſt, and qualitie next,
As in the ſecond booke ſhall be declard.

Hereby you are to learne that this art hath two generall partes whereon it ſtandeth as on two fundacions, or pillers, whereof the firſt part ſheweth the Notation, and opcracions, aſwell of integers or whole numbers, as of fractions, but yet in their due order, his firſt the notation or deſcription of the integers, or whole numbers, and then of fractions, and ſo agayne after that, firſt the operation of integers, and then of fractions. As for the ſecond part, as it belongeth to the ſecond booke, ſo it ſhall be there declared at large.

Euma Very good I cannot miſlike of this good order.

Philo. Then ſhall you vnderſtand that the notation is thus defined.

the Notation defined.

8. To write and to reade all maner of ſummes,
Expreſt with figures is called the notation
which thing ten figures alone ouercums,
And yet the tenth hath no ſinification,
But ſerueth onely to fill vp a place,
It ſelfe not being ſo much as an Ace.

So that notation ſheweth you how to fourme or faction the numerall figures, as they muſt be written, and alſo howe to pronounce them as they are to be read, with their valures and ſignificacions, which they haue aſwell by their propper fourmes or ſhapes, as by their ſeuerall places or degrees. And this by the moſt part of writers is comonly called Numeration, which name ſith it is ſo well knowen & ſo much vſed, you may alſo vſe if you wil, for it forceth not ſo much for the name of the thing, as for the true vnderſtãding what is meant by it. Neuertheleſſe, to ſpeake exactly, the name of Notation is more apt and more agreeable to the nature of the thing, then is the name of Numeration, how-be-it as I ſayd before, vſe you which of them you liſt.

Now the ten figures or numerall letters wherewith all numbers be they neuer so great may be expresst are these following, each of the which signifieth by his proper shape onely so many in number, as you see written directly ouer it.

the forme or shape of the Numerall figures

one, two, three, fower, fiue, sixe, seuen, eight, nine, nought,
1. 2. 3. 4. 5. 6. 7. 8. 9. 0.

Euma. I will imprint both the shape of these figures and also their seuerall values so well in my minde (godwilling that I trust to be able to describe them with my pen and to pronounce them with my tongue, whensoeuer any occasion shall serue hereafter.

Philo. So must you doe, and then marke what followeth for that which is sayd yet, extendeth to the value of euery figure but onely by his proper shape. But now you shall here each of them to haue other values by reasons of their places or degrees of situation except the last which signifieth nothing of it selfe, in what place soeuer it stand, yet proueth it to augment the significacion of any of the other by filling a place as this rule present teacheth.

the value of the figure by their shape & by thier places.

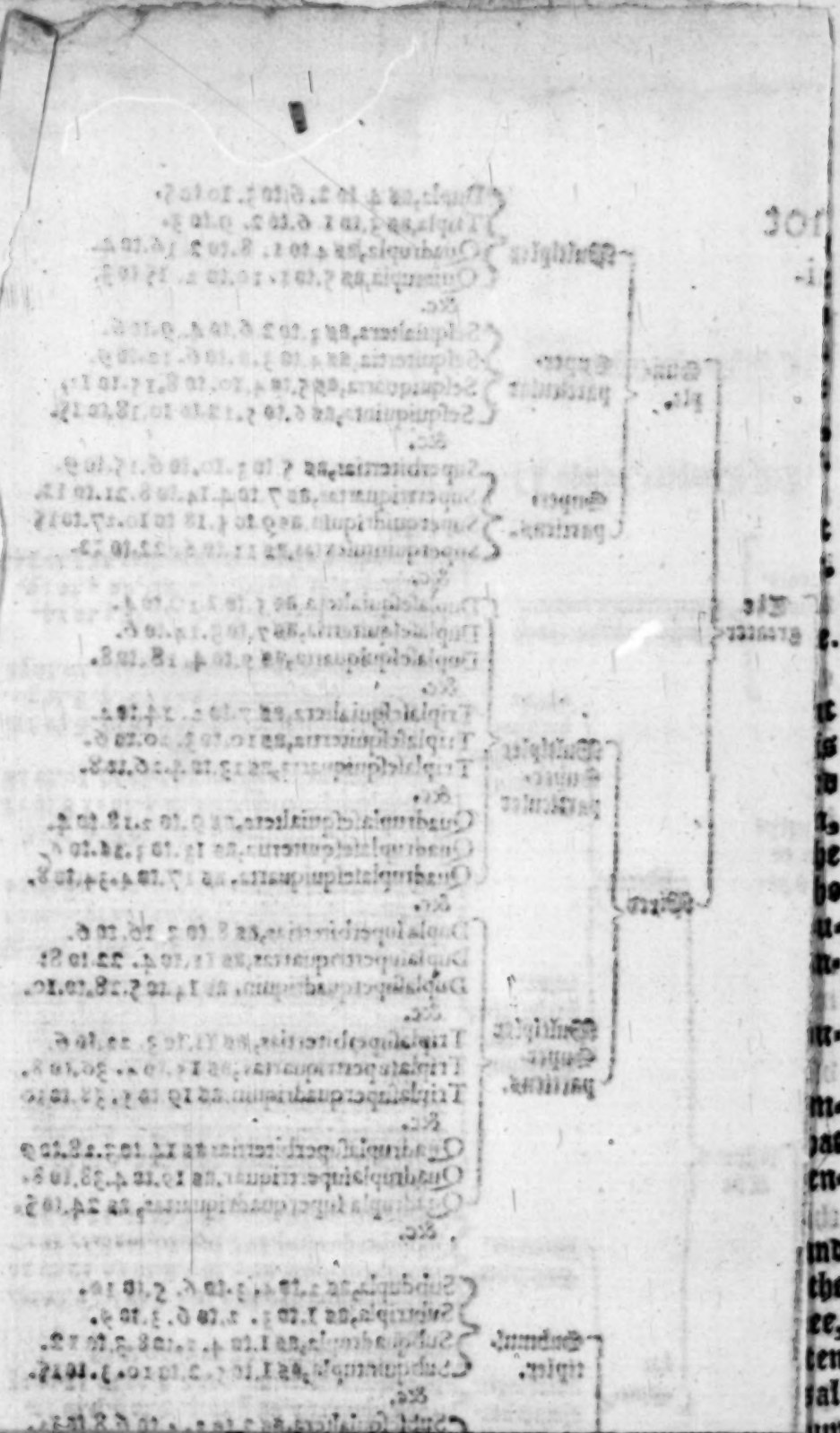
9. And as the shape of each Figure doth showe,
In first place his value but once and no more
So doth the second place make it to growe,
ten times asmuch as the place next before.
The third place augments the second asmuch,
And so in all other their nature is such.
10. And note that the place next to the right hande,
is called the first place be they neuer so many,
Wherefore the figure which so fast doth stand,
Or els the Cipher if there be any,
Is properly sayd to hold the first place,
Next whom is the second, and so foorth the race.

And so this rule telleth you that as euery figure in the first place is his owne value but once onely, so in the second place it is tenne times asmuch (by reason of that second place) as it signifieth by his proper shape, and in the third place it is tenne times asmuch as it was in the second, by reason of that third place, and in the fourth place, it is (by reason of that place) ten times asmuch as it was in the thyrd, and it telleth you further that the first place is that which is next your right hand, and that the place is next to the first, is the second, and the place next following after that is the third & so foorth continually, as for example I set these 4. Letters A. B. C. D.

Left hand. Right hand



Now



wayes euen tens, And that in the third place,

the forme or shape of the Numerall figures

the value of the figure by their shape & by their places.

Left hand. Right hand

next your right hand, and that the place is kept to the left, to the second, and the place next following after that is the third & so forth continually, as for example I set these 4. Letters A. B. C. D.

Now

Now would I know of you in which place euery of them standeth,

Euma. I know not what you meane by a place, but if I shall tel you which is first, second, third, or fourth, Then by your doctrine I must say. D. is the first, C. the second B. the third and A. the fourth.

Philo. you haue said truly, and you might aswell haue sayd, that D. standeth in the first place, C. in the second, B. in the third and A. in the fourth, for a place is nothing els but the very roome which a figure standeth in and occupieth, And looke how many figurs, be in any one number, so many places hath that number, and neyther mo nor lesse, and as you rekoned the places of those Letters. A.B.C.D. so must you reken the places of your figurs alwayes in this arte

Euma well then I trust I vnderstand welinough, what a place is, and which way places are to be accounted, touching their order.

Philo. Then tell me what number this is, 1111.

Eu. This number hath but foure places, for you sayd euen now that how many figurs were in a number, so many places hath ỳ number.

Philo. I sayd so in deede and it is true.

Eu. And these figurs by their proper shapes are but each of them one.

Philo. all that is true.

Euma wherfore the first is but one, because of his first place, But the second 1. because of his second place, is ten times asmuch as his former fellow, so it is ten, And the third 1. bycause he is in the third place, is ten times asmuch as the second, but the second 1. is ten, wherefore this third 1. is ten times ten, that is an hundreth, and the fourth 1. because of his fourth place being ten times asmuch as the 1. in the third place, must needes be ten hundred, that is a thousand, and so the whole sum or number is, one thousand one hundred, Ten and one.

Philo. you might aswel & better haue sayd, it had ben one thousand, one hundred and eleuen, for is not ten and one eleuen?

Eum. That is true, but if there be no easier way to read these numbers, me thinks it wil aske a long time to pronounce a number, that hath a dosen or twentie places, because the reconning of their tenfold values from place to place in this order will tedious.

Phi. No, not a whit, for after a little vse, you shall both quickly and easily pronounce any nũber you wil, for do you not perceyue that in the first place, 1. is alwayes but one, and 2. but two, and 3. but three, and so of all the rest, And that in the second place, 1. is alwayes ten 2 is twenty, 3. thirtie, 4. fortie, and so fourth, so that this place is alwayes euen tens, And that in the third place, 1. is alwayes one hun-

dred 2. two hundred, 3. three hundred, and so forth, so that this place goeth alwayes by euen hundreds. And that in the fourth place, 1. is alwayes one thousand, 2. is two thousands. 3. three thousands, and so fourth, so that the fourth place is euermore iust thousands.

Euma. In truth now I consider your number prescribed with this your woordes, I thinke the tenfold encrease of the figures by their places (in that number) to be such as you say.

Philo. Very well, then if you can remember but the values of these foure places, that is to say, First, ones Second, tennes, Third hundreds: And fourth, thousands, you shall neuer need to troble your selfe, with any further accomptes, as for example: I set this number, 2345. to be redde. Now I knowing the fourth place to be all thousands, the third all hundreds, the second all tennes, and the first but ones, I reade the same imediately thus, two thousand, three hundred, forty, and fiue. And this number 3864. I reade thus, three thousand, eight hundred, sixty and foure.

Euma. This seemeth not very hard to do, for his wit is to short that cannot remember foure places, but what if a number haue many mo places? I see not hereby how I shall read or pronounce all the rest.

Philo. That will another rule teach you anone, but in the meane season I would heare how you can reade a number or two, and first how reade you this. 6891.

Euma. I reade it thus, sixe thousand, eight hundred, ninety and one.

Philo. And how these. 5408. 2200. 1000.

Euma. The first I reade thus, fiue thousand, foure hundred, and eight, the second thus, two thousand, two hundred, and the third thus one thousand.

Philo. That is wel done, & in these three last numbers you see the vse of the Cipher (for so the figure 0. is peculiarly named, although it be generally called & accompted as a figure) which being nought or nothing it selfe, yet by the filling of his second place, in the first number of those 3. It causeth the 4. & 5. following to be one in the third place, and the other in the fourth, and thereby to be fiue thousand, foure hundred, which could not be so much if the Cipher filled not the second place, for if the Cipher were absent, the same number would be thus, 548. which is but fiue hundred forty eight.

Eu. That I perceiue very well, And further in the second number I see, that though the two Ciphers there signifie nothing of them selues, yet do they thrust the two figures following into the third

and

and fourth places, whereby the same figures signifie two thousand, two hundred, which without those Ciphers, would signify but two & twenty: And for the last number which is one thousand, I see that if one of the Ciphers were taken away, it would be but one hundred, and if two of them were taken away, it would be but ten, and if all three Ciphers were taken away, it would be but one,

And thus I perceiue clearely that the vse of a Cipher is by fylling a place, to augment the valure of other figures in that number, but if a Cipher happen in the last place of any number as thus, 0342. I cannot see how it encreaseth the value of any of the figures.

Philo. Neyther can I see to what purpose any cipher should occupie such last place, for Ciphers serue onely to fill a place so, that by meanes of them, some significant figure, may be preferred to a place of greater valure then it had before. But in this number 0342 there is no significant figure preferred, or thrust into any place of greater valure by that cipher, but the cipher it selfe standeth in the place of greatest valure, & yet is nought stil, as it would be in any other place neither is the number either the more or the lesse for his being there, wherfore such ciphers ought neuer to be suffered in the last place of any number.

A Cipher ought not to stand in any last place.

Fuma You haue fully satisfied me for this matter, wherefore now I pray you shew me how I shall expresse any number consisting of mo places then foure.

Phi. Then marke wel the rules following which teach ỹ same fully.

11. Wherefore when you would describe any number,
You ought to begin first at the right hand,
Placing your figures all like farre a sunder,
To left handward bending as right as a wand.
Which to that order is contrarie quite,
That commonly men do vse when they write.

12. That done to pronounce and read that you wrote,
First prick the fourth place, and ouerskip twaine,
The figure the next with prick also note,
And leauing two mo, prick next place agayne.
Proceeding foorth so whilest figures afford,
Ommitting still twaine, and pricking the third.

13. Then with the last prick at left hand begin,
As if it were but the first place of all,
And if any figures stand yet beyond him,
Count the one the second, the other third call.
Then reade them as though there were but those three,

The figurs alwayes ought to stand distant like a sunder.

How numbers are vttered and pronounced.

Pronouncing thousand forceth pricke you see,
14. And so to the next prickt figure proceede,
Accounting that now to be the first place,
VVhom with his fellowes in former sorte reade,
Pronouncing thousand agayne in like case.
For euery prick which rightward doth stand,
Beginning still with the prick last in hand.

As for example here is a number consisting of fifteene places, to wit, 324356985420794, all whose figures I haue set right and distant like as you see, in whose fourth place being (as you know) ỹ place of thousands, I finde a Cipher, which I prick, by setting a prick ouer the heade of it, and then ouerskipping or omitting two places, I likewise pricke the third, so proceeding and continuing as long as the figures will afford places, whereby when I come to the thirteenth place, and haue prickt it, I finde that the figures will not yeeld places to prick any further, and to leaue two vnprict, as the rule present requireth, wherefore there I say and the number standeth thus, 32435698̇5420̇794. Hauing 4. prickes onely.

Now for the reading of it, I begin (as the 13. verse biddeth me) at the prick last made (which in reading I must accompt the first) where I finde 4. which 4. also I accompt to be in the first place, and because 2. standeth next beyond him, I esteeme that 2. to be in the second place, and sith there standeth 3. yet beyond that 2. I repute that 3. to be in the third place, and so taking those 3. figures 324. belonging to the first pricks, as though they weare no more but they three onely, I reade them according to their values in respect of those places, pronouncing withall foure times Thousand, because there are foure pricks from the place were I began, saying thus, three hundreth twenty foure thousand, thousand, thousand, thousand then come I to the next prickt figure, which is 6. stāding in the tenth place who with his fellowes standing beyond him maketh 356. which I read in the selfe same maner as before, saue that now I pronounce but thrise thousand withall, because there is but three pricks from the place where I now began, vnto the first place saying, three hundreth fifty sixe thousand, thousand, thousand, from whence I come to the third pricked figure (standing in the seuenth place) which is 5. whom with his fellowes, taken after the former order, I finde to make 985. to whom I adioyne twise thousand, because of the two pricks appearing betweene the place where I left before, and the first place, saying, Nine hundreth eighty fyue thousand,

thousand

thousand. Finally comming to the last pricke (I say the last in reading but indeede the first in making) I see the figures belonging to that pricke make 420. whereto for that one prickes sake, I adde once thousand, calling it foure hundred and twentie thousand, and as for the rest hauing no prick I read them as they are, saying, seuen hundred nintie and foure, which number if I should reade roundly by my selfe without example of teaching, I would reade thus, Three hundred twenty foure Thousand, Thousand, Thousand, Thousand, three hundred fifty fiue thousand, thousand, thousand, nine hundred eightie fiue thousand, thousand, foure hundred and twenty thousand, seauen hundred ninety and foure.

Euma. Truly this round reading I like well, & me thinks by this playne example I could do the like with any number, but the beginning at the right hand, and to write the figures toward the left hand (as it were backwardly) troubleth me much.

Philo. As for that matter let it not trouble you, for you may begin to write you number at which hand you list, so that you be once sure of your places, as if you would expresse ten thousand two hundred & fortie, you may begin first (euen as it is spoken) with ten thus, 10. setting such a prick ouer his head, because it must be thousands, and then next the figure 2. for two hundred thus 102. and lastly the figure of 4. for forty, with a 0. after him to fil vp the first place thus, 10240. And so we commonly doe in Numeration, or the Notation, but in al operations of this art, you can not doe so, but must describe your figures in such order as the worke doth require.

Euma. Well then touching this Notation, I pray you giue me leaue to set downe but one number my selfe, and see me prick it, and here me reade it, & I will desire no more helpe in this matter.

Philo. Go to in Gods name.

Eu. This is the number with his pricks 459008000260730.

Philo. You haue pricked it rightly.

Euma. Well then beginning at the prick next the left hande, I read the three figures 459. belonging to that prick thus, foure hundred fifty nine thousand, thousand thousand, thousand, then cõming to the second prick (as I now accompt it) I find it hath no mo figures belonging to it, but that 8 which is directly vnder the prick, for the Ciphers are nothing but serued only to augment the valure of the other figures which I red before, wherfore I take that 8. as it wee in the first place, but call him for the three pricks sake yet to

come eight thousand thousand thousand: But now comming to the third pricke, which is in the seuenth place, I finde nothing belonging to that pricke but cyphers: So that I wot not what to do with them, except I should omit them quite, as I thinke I must, for once I am sure they are nothing.

Phi. you say truth, and therefore you must accept them for nought, but must omit them quite, and their pricke also, and so go thence to the next pricke.

Euma. Then in so doing I find. 2 6 0. which hauing no more but one pricke I reade thus: Two hundred and sixtie thousand, and finally I reade the figures remayning without any prick euen as they are thus, Seuen hundred and thirtie.

Philo. you haue done well, wherefore sith you seme perfecte inough in this notation, reade it all ouer once againe roundly, and then we will proceede to other matters,

Euma. That will I do quickly now I doubt not, wherefore new writing the number agayne as here you se it. 459008000260730. I saye thus, Foure hundred fiftie nine thousand, thousand, thousand, thousand, eight thousand, thousand, thousand, Two hundred and Sixtie thousand, seuen hundred and thirtie.

Phi. Thus as you haue described & pronounced this number, so you, may by the same meanes vtter any other, how greate soeuer it be.

Euma. I haue no distrust thereof, for the matter now semeth very plaine and easie.

Philo. Then wil I shew you how Integers, or whole Numbers are first distinguished which is thus.

Integers are diuided into digits, articles and mixt of both.

15. And note that of numbers be. 3. sundry sorts,
First DIGITS, which neuer amount aboue nine
The ARTICLES next which alwayes imports,
Numbers iust tennes, sone known by their signe.
VVhich is a Cypher in first place to gide them,
And euermore ten, will iustly deuide them. } 1
16. The third sort are numbers MIXT or compound,
Of digits and articles ioynd both together
VVhich vnder 2. figures can neuer be found,
VVhereof the first being a digit euer.
The rest must of force be articles all,
VVhich therefore by right MIXT numbers we cal.

Euma. Although the sence of this rule seme very playne to be vnderstood, yet before you proceede any further I pray you here how

I

I conceyue it, least by misvnderstanding thereof I may hereafter incurre erro?.

Philo. You cannot lighly mislike it, but say on.

Euma. I conceiue the meaning of this verse thus, that any number which can be written with one figure onely, is named a Digit, and therefore 1.2.3.4.5.6.7.8.9. are onely digits, and all the digits that are but 10. and all numbers aboue 10. cannot be written vnder two figures at the least, whereof as many as haue a Cipher in the first place, are iust tennes. and therefore are called Articles, such are, 20.30.40.50.100.110.120 and such like, but all other numbers being aboue 10. & hauing no Cipher in the first place (though they haue Ciphers in any other place) yet are they not Articles but Mixt numbers, mixed & compounded of Digits. and Articles put together, by reason wherof they obtayne that name, such I take these to be insuing, 11.12.15.18.22.36.103.2004. withall others like. Mixt numbers primarie.

Philo. You say very truly, and vnderstand the meaning of this Maxim in all things as rightly, wherefore here I will ende the Notation or Numeration of Integers, or whole numbers, and now we will go in hand with the Notation of fractions.

Of the definition of fractions, and fractions of fractions, and their Notation. } Chap. 2.

17. A true fraction is some one part or mo,
Of an Integer or thing that was entyre
VVhich some diuision hath seuered so,
That vnto the whole it cannot aspire.
And is writiten as here may be seene,
In two rankes of termes with a line betweene.

A fraction defined.

This Maxim conteyneth the definition of a fraction, shewing ỳ it is some part or partes of an vnit or whole thing, which beeing so seperated or diuided, ỳ the parts remayning, suffice not to make vp the whole vnit agayne, is therfore called a fractiõ or broken number: as if you hauing an Apple, diuide it equall into three partes, and giue away two of the same parts, then haue you but one part left, which is called one third part, & if you had geuen away but one of those parts and kept the other 2. Then had you not had one part onely, but parts of that Apple that is to say two third parts, in both which causes as the part & parts which you haue left after such giuing, cannot be equal to the whole apple, to make it vp againe, because there wanteth either two, or at the least one of those parts, so therfore the part or parts so remayning is called a fraction, that is as you would say a fragment, or a thing broken in peeces: And the Notation, writen

The Notations of fractions with their values.

writing or representatiõ of such fractions, is alwaies don in 2 ranks of termes with a lyne betwene them thus $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{4}$. where the first fraction expresseth one halfe parte, the second, one third part, and the third, one fourth part, which we most commonly call a quarter, Also these fractions $\frac{2}{3}\ \frac{3}{4}\ \frac{4}{5}$. signifie ech of them not one single part, but parts of some vnit or whole thing, whereof the first betokeneth two third partes or thirds, the second, three quarters or fourths, and the third fower fift parts or fift, And so the are red and called.

Eu. All this I vnderstand indifferently well, But why do you call the. 2 digitts or numbers which make a fraction, 2 rankes of terme I understand not, for I remember not that I haue hard you vse that word (termes) before.

what the termes or the bounds of a number are

18. the Termes or bounds of a number are those,
by whose Caracters that number is expreste,
which termes whole numbers in one Rank dispose,
but fractions require. 2 Ranks at the leste
And sometimes whole numbers vpon vrgent cause,
vse also 2 rankes without breach of lawes } 11

Lo, now you are taught, what is ment by the termes of numbers, to wit, that they are nothing else but the very Characters or figurs whereby any nũber or fraction is expreste, as here I set downe these three whole nũbers 8. 12. 18. Now the same three stãd al in one rank or row as you see, & are as well & as vsually called & knowne by the name of three termes, as of three numbers, But if I set downe these three seuerall fractions, $\frac{1}{2}\ \frac{1}{3}\ \frac{1}{8}$ you se they stand not in one rank as the Integers do, but occupie two ranks, for in the vpper rank you see are, 1. 3. 5. and in the lower ranke are. 2. 4. 6. And the figures 1. and 2. are the termes of the first fraction. And so 3. and 4. are the termes of the second. And likwise. 5. and 6. are the termes of the third, And here you are further taught that sometimes vpon iust occasion whole numbers may be and are described in. 2 rankes also after the order of fractions, with out breach of Arithmeticall lawes, or Rules, as thus $\frac{1}{1}\ \frac{2}{1}\ \frac{3}{1}\ \frac{4}{1}$. which is asmuch to say, as 1 Ace. or 1 vnit 2. vnits, 3 vnits, 4 vnits, for when we in numbering saye. 1. 2. 3. 4. and so fourth, we alwayes meane indeed so many vnits, Aces, or ones,

Eum. I do not greatly doubt, that in our common numbring, when we say. 1. 2. 3. 4. 5. and so forth, but we meane so many ones, or vnits But to write such integers or whole numbers at any time in two ranks like fractiõs, I se neither necessity nor reasõ, for it is but more labor then needeth, to wryte 2. figurs wher one would serue, & the nũbers (in my mind) are rather more easie and better to be vnderstoode

when

when they are expressed in one ranke, playnely after the common fraction, then in such double rankes.

Philo. I sayd not that whole numbers are alwaies written, but onely sometimes, and that vpon iust occasion, or rather as the verse it selfe sayth vpon vrgent cause, which therefore is neyther without necessity nor reason, for both neede and reason sometimes vrge the same to be so done, as you your selfe will confesse hereafter, when you shall come to the experience thereof, but because you cannot as yet vnderstand the reason of the matter, I suppresse it till time serue more fit for the purpose. And in the meane season, I will shewe you a rule teaching you by what names the termes of fractions are named, and called, and what their seuerall offices are which is thus.

19. The Termes of fractions haue seuerall names,
which ought to settle each in his owne place,
One vnder another with interlind frames,
The one for the toppe the other for Base,
Which Topterme by name is called Numerator:
And likewise the base the denominator.
20. The Numerator alwayes expresseth,
how many partes a fraction conteyneth,
The Denominator for his part confesseth,
The names of those partes and truly complayneth,
In what kind of parts the whole is deuided,
And so by them both the thing is decided.

12

The top-terme, the toppe and the numerator, are one,
The Base and the Denominator are one.

Now as euery fraction consisteth of two termes at the least, so this rule present indeauoreth first to distinguish the same termes, aswel by seueral places, as seuerall names, shewing ỳ the vpper terme, which standeth aboue the line is commonly called the Numerator, which also for more shortnesse is sometimes called the Topterme or top onely: and that the lower terme is vsually called the Denominator or Base.

Secondly the same sheweth you that the office of the Numerator, is to expresse how many parts of an vnit, a fraction conteyneth, and that the Denominator serueth to declare the name and kind of the parts whereinto the whole vnit is diuided: as whether it be diuided into halfes, third parts, quarters, fifth parts, sixes, or such like: That so by the Numerator, and Denominator, the certaynty and truth of the parts represented may appeare, as for example, to make it yet more manifest: Let this fraction signifie some parts of an entire shilling, now how much of the shilling is represented by this

fraction, the termes thereof declare in this wise.

That is to say: the Basse or Denominator beeing 4. sheweth that the whole shilling is diuided into foure partes or quarters, and not into halues, nor thirds, nor yet into any other kind of partes, but onely quarters: And the Top or Numerator beeing 3. sheweth how many of those quarters, the whole fraction containeth: that is to say, neyther 1. nor 2. nor yet all 4. but onely 3. of those quarters, & so betweene them both a certayne portion of the whole shilling is truly expressed, which in this example is onely $\frac{3}{4}$. and amounteth iust to 9. pence, euery quarter being 3. pence.

Eume. I thanke you much, for this playne example, for I hope it hath made me perfect for euer, touching the representation of al maner of fractions, and also concerning the offices & properties of their termes whether they be Numerators or Denominators.

Philo. Then because you hard before that true Integers may be and oftimes are, written and described in 2. rankes of termes, like vnto fractions, I will now shew you how you shall know a true fraction, from such Integers, and the rule thereof is thus.

How a true fraction is known from an improper fraction or from true Integers.

21. The Numerator of euery true fraction,
Is lesser then the Denominator,
For if they be equal both termes by collection,
Returne to an vnit their former creator:
But if the Denominator be lest,
Then more then an vnit is thereby expresst. } 13

By which rule you may soone discerne true fractions from Integers, described fractionlike: For it assureth you that the Numerator of euery true fraction, is lesser then the Denominator, as in these $\frac{2}{3}.\frac{3}{7}.\frac{3}{4}.\frac{4}{5}$, which are true and proper fractions: For (sayth this maxim) if the Numerator and Denominator be equall, then they returne to an vnit, from whom as they first came, into such partes created by the diuision therof, so by the collection together of al the parts wherinto any thing is deuided, that which amounteth shalbe equall to the whole, such are these, $\frac{1}{1}.\frac{2}{2}.\frac{3}{3}.\frac{8}{8}.\frac{10}{10}$. and such other like, which though they be written fractionlike and stand in the shape of fractions, yet are they not truly fractions of vnits, but each of them is one entire vnit exactly. Agayne if the Denominator be the least of the 2. termes then this verse telleth you, þ they expresse more then an vnit or integer, so that in þ case also, they are no true fractions: such are these $\frac{8}{5}.\frac{15}{3}.\frac{20}{4}$. wherof the first containeth 4. vnits or Integers exactly, & the second & third containe each of them 5. Integers exactly,

wherefore

wherefore eyther sort of these 2. kindes are rightly called improper fractions, because they are truly Integers, and not fractions of Integers indeed, although they appeare in the shape or vesture of fractions, as it were men disguised in woemens apparrell: who being deuested or stripped of their improper garments, foorthwith appeare in their owne proper shape, and shew what they are. And agayne many times in the same case, where the Denominator is lesser then the Numerator, the fraction so expressed, may containe not one Integer or many exactly: but also some fraction besids, as in these exãples, $\frac{3}{2}$. $\frac{7}{3}$. $\frac{15}{4}$. where you may perceiue the first to containe one whole vnit, and an halfe: For three halfes of any thing you know is asmuch as one and an halfe. And the second by that reason contayneth two whole vnits, and one third, and the third contayneth three entire vnits and three quarters: These therefore and such like are not absolutely to be named Integers, nor fractions, but they are most aptly and naturally called Second Mixt Numbers, because as your former Mixt numbers mentioned in the 16. verse consisted of Digits and Article together: So these consist both of Integers and fractions, or of whole numbers & broken: and being seperated asunder as they may be, to wit the vnits by themselues, & the fractions by themselues, they wil apeare each in his proper likenesse thus, $1\ \frac{1}{2}.\ 2.\ \frac{1}{3}.\ 3\ \frac{3}{4}$. Secondary mixt numbers defined.

Euma. I suppose I vnderstand all that you haue sayd, & it seemeth to me very reasonable: when whole numbers and fractions hapen together (which you cal secondarie Mixt Numbers,) that then they should be both written, red and pronounced as they are, as these $1.\frac{1}{2}.\ 2.\frac{1}{3}.3.\frac{3}{4}$. which you described last, who being so orderly distincted in their owne fourmes: I reade & pronounce thus, one and an halfe, two and one third, three and three quarters.

But it seemeth a thing very strange and in mine opinion vtterly vnreasonable, that Integers and fractions being hudled togeather confusedly, should beare the shape of one fraction: and much more that pure Integers should be so much transformed, or (as you cal it) disguised from their naturall shape, that they should seeme fractions, contrarie to that which they are.

And truly (with your fauour be it spoken) I cannot yet deuise any reason, nor cause why foure whole vnits or Integers, should at any time be expresst in halfes, and called $\frac{8}{2}$. Or why fiue Integers should be represented thus $\frac{10}{2}$. or that $2.\frac{1}{3}$. should be called $\frac{7}{3}$. Or that $3.\frac{3}{4}$. should be called fifteene quarters: For if you should aske me, how many yardes of cloth made my gowne, and I should

answere you ½ of a yard: I suppose you would thinke it a strange Phrase, and no vsuall kinde of answre: And that it were better to say. 3. yardes and ¼. or rather 3 ¼. yardes: and yet I remember, that in the glosse of the 18 verse, you sayd that both reason and necessitie did sometimes vrge true Integers to be so described.

Philo. I sayd so indeed: and it is true, not onely touching Integers, but also touching these second Mixt Numbers, which consist of one, or mo Integers and a fracion: both which kinde of numbers, declare themselues, & their owne values most significantly, when they are expresst distinctly each in his owne proper forme, in such order as you say, & would haue them: and therefore in that case I allowe of your latter answere, as a better phrase and more vsuall then the first: but in Arithmeticall operations, the Arithmetitian or worker with such numbers, shal of necessity be oftimes constrayned to chang the shape both of Integers & Mixt numbers, into the forme of such improper fractions, and likewise of such improper fractions into true Integers, or mixt nũbers, as occasion serueth, or otherwise he cannot proceede in his operation, which transformations both wayes is done by that part of this arte which is called Reduction, as you shall learne when we come thereto: and in the meane season, I will shew you how you shal knowe the greater fraction from the lesser, the rule whereof is thus.

1. cõmon sentence added by campane and Flussates, after the 13. definition of 7. booke of Euclid.

22. That fraction is greatest whose Denominator,
Is of all numbers the least and most small,
So that an vnit be the Numerator,
For then is there no greater fraction at all. 14
For no kind of fraction of one part alone,
Exceedeth ½. the least is vnknowne.

Fuma. This maxim seameth so playne of it selfe, that I thinke I vnderstand the meaning thereof already, for it affirmeth that fraction to be the greatest, whose Denominator being the least Number, hath an vnit or one, for his Numerator: whereby it approueth that which you sayd before, in the glosse of the 5. verse, where you saide that ½. was the greatest fraction of one single part, which I see must needes be so by reason, for no whole thing can be broken or diuided into fewer equall partes then two, which parts must needes be halfes, because they are both equall.

Philo You say truth, and here if you marke it, the Denominator being two, argueth the same to be the least number, according to the fourth verse: For if 1. be taken for a number, it followeth that

2.

2. is not the leaſt: But if one be made both Numerator & Denominator thus, $\frac{1}{1}$. Then by the very laſt verſe foregoing, the ſame is become no fraction, but a meere Integer or vnit: And agayne as the leaſt number, and greateſt ſingle fraction are knowne, and may be certainly geuen, expreſt, and deſcribed: ſo the nature of Numbers is ſuch, that the ſelfe ſame tearmes which expreſſeth the one, expreſſeth the other, without any diuerſitie, ſauing onely chaunge of place or ſituation, In ſo much that as $\frac{1}{2}$ is the greateſt ſingle part or fraction, ſo $\frac{2}{1}$ is the leaſt true number.

Euma. Truly this reaſon perſwadeth me fully now that 2. is the leaſt Number indeede.

Philo. Or els this rule preſent cannot be true, but this rule is moſt true infallibly: For who knoweth not that a Teſtren, being $\frac{1}{3}$ ſhilling, is a greater part of the ſhilling then $\frac{1}{3}$ which is but a groat: Or $\frac{1}{4}$ which is but 3. pence, or $\frac{1}{8}$ which is but 2. pence, or $\frac{1}{12}$ which is but a penny. Wherefore for my parte, whereſoeuer I ſpeake preciſely of true whole numbers, I alwayes eſteeme 2. to be leaſt: And as for 1. I accompt it for no number, but rayther as it were the ſeede or beginning of number: Yet neuertheleſſe ſo, that with Boetius I confeſſe it in ſtrength and power to contayne the nature of euery kinde of number, which is an vnſpeakable ſecrete, belonging peculierly to vnitie: and thus hauing done with the Notation of fractions, of vnits, or Integers, I come to fractions of fractions, and their Notation thus.

The definition of a fraction of fractions.

23. Fractions of fractions are defined to be,
The parts of a fraction ſubdiuided newly,
For as ſome part of an vnit you ſee,
Maketh a fraction euen ſo truly.
A fraction new parted in parcels agayne, 15
Is called the fraction of a fraction playne. 16

The Notion of a fraction of fractions.

24. which fractions of fractions are alwayes deſcribed,
By ſetting there termes one vnder another,
Like fractions of vnits, but are not diuided,
with any ſuch line leaſt one time or other.
The ſhewing alike and both of one facion,
Might be miſtaken in their computation.

This rule ſheweth you what is vnderſtood by a fraction of a fraction: For it ſayth, when ſome part or partes of a fraction or broken vnit, is eftſones newly broken or diuided into other parcels: then are thoſe laſt parcels ſo made by ſuch ſubdiuiſion called the fraction

of a fraction. And they are described and written, by setting their Numerators ouer their Denominators, euen like as fractions of vnits are, sauing that for a difference, to knowe the one from the other, The fractions of fractions haue no such lyne betwene their termes, as other fractions haue, but stand thus $\frac{2}{3}$ $\frac{1}{2}$. least being both like without any difference, erroꝛ might growe by mistaking them, because the valures of the one are farre different from the other, And to make all things more euident by examples mark wel this figure following, Whereof the vppermost. 12. spaces betoken one whole shilling diuided into 12. parts or 12 pence,

The shilling diuided into pence.

The first partision.	1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12	1. s.
The second.	3 | 6 | 9 | 1 | $\frac{1}{4}$ of	
The third.	3 | 6 | $\frac{2}{3}$ of	
The fourth.	3 | $\frac{1}{2}$ of	

Now of these 12. partes, you see the second partision (hauing 3. spaces) cutteth of $\frac{1}{4}$ which is 9. pence as you may apparantly perceiue by the strikes running vp & downe: And from that 9. pence, diuided into 3 parts, you see the third partisiō (hauing but 2 spaces cutteth of $\frac{2}{3}$ thereof, which is 6 pence as also appereth euidently, And finally from that. 6. pence, you may perceiue the last and lowest partision contayning one onely space to cut of $\frac{1}{2}$ thereof, which is 3 peuce, Now therfore this $\frac{1}{2}$ which remayneth last, is not $\frac{1}{2}$ of the whole shilling (for it is but 3 pence as I sayd, & as you may easily iudge, by this occularie demonstrations) but it is $\frac{1}{2}$ $\frac{2}{3}$ $\frac{3}{4}$. of a shilling, that is by due pronuntiatiō, one halfe of two thirds, of three quarters of a shilling, and is al one or equall with $\frac{1}{12}$ of a shilling, And here you see the first fractions to wit $\frac{1}{2}$ being a true fraction, written with his lyne as it ought to be, and the other two that is to say $\frac{2}{3}$ and $\frac{3}{4}$. to be written without any lyne as their vse and orderis, And alwayes in pronouncing them. you must put the word (of) betweene them as you see me to haue done.

Ema. very well, I will not forget that, And now touching this rule, you haue so cleared it by exposition and examples, that I see no doubt, nor cause to moue any further question about it, sauing that vnder your correction, me thinks the fraction proposed by you for an example, was not onely a fraction of a fraction (as you sayd) But rather the fraction of a fraction of fractions

for

for if it had ben but thus $\frac{1}{3}\frac{2}{3}$. that is one halfe of two thirds, I suppose that it had ben a fraction of a fraction, But your example is. $\frac{1}{2}\cdot\frac{2}{3}\cdot\frac{3}{4}$, that is one halfe of two thirds of three quarters,

Phi. You seeme to haue reason in that you speake, But notwithstanding the matter is all one, as touching the names of these fractions, for if they be broken and subdiuided, 20. tymes one after another, yet are they all comprehended vnder the name of fractions of fractions, (or as some men call them particles, that is as you would say parcels of parts.) eyther of which names sufficeth to them all, els should there neuer be an end of names for them, sith (as I told you before) there is no end of their decrease.

Fractions of fractions are also calle particles.

And now, although I haue sayd inough already in the glosse of the 21. verse, touching the often necessitie of the Notation or decription of Secondary Mixt numbers, in the forme of improper fractions, yet I thinke it not a misse (Because I see you scarcely satisfied therein,) to adioyne the rule it selfe which comprehendeth the reason thereof more fully, before I end the Notation, which Rule is thus,

25. Second mixt number like as they consist,
of Integers mixt with fractions together
So do in some kind of workes what you list,
yet shall you find them vntractable euer:
Till both be made shew in shape of a fraction,
Or of an whole number at your owne election. 17

Secondary mixt numbers consist of integers and fractions

So that as Secondary Mixt numbers consist partly of true Integers & partly of fraccions, so this verse telleth you plainly, That do you what you list or what you can, yet shall you neuer finde them tractable to some kinds of opperations, Till they both (that is to say) aswell the Integers as the fraction, be made shewe eyther in the shape of one simple fraction, or of an whole number at their one election, That is not in what shape you would haue, or will appoynt them: But in such fourme and shape as they themselues will be apt to admit or agree to take, by which meanes they are made subiect to all Arithmeticall opperations, which otherwise were not possible to be regularly brought to passe, For which cause also, True integeres are sometimes noted and expressed in two rankes of termes, according to the 18. verse and the glosse thereof.

Eu. Although both this verse & your expositiō therof be very plaine yet do I not conceyue it so perfectly as I should do, if I sawe

the experience thereof, by example of some such operation as you mention.

Philo. That shall you doe hereafter when time serueth: but first you must learne what an operation is: wherefore hauing thus past ouer and finished the Notation aswell of fractions as of Integers, like as now by order of Science. The operation of Integers, ought to follow next: So will I first tell you what an operation Arithmeticall is, as it is generally taken, and what rules and orders are generally to be obserued in euery such opeation: And then will I proceed to the speciall practise of euery operation perticulerly.

Of Operation Arithmeticall generally, viz. What it is, what is the end thereof, and what rules and orders are therein to be obserued. Chap. 3.

The definition of an operatiõ Arithmeticall.

26. An operation is that in Arithmeticke,
VVhich by two numbers first named or assignde,
Finds orderly out a third which we seeke,
Except it chance so by meere course of kinde,
That reason deny to finde any third,
For then to finde any were very absurd. } 18

This rule is very playne, shewing you how that it is called an operation in Arithmetick, when by two numbers first named or assigned, we finde out orderly, a third number sought: Of which kind of operations there are principally foure, to wit, Additiõ, Substraction, Multiplication, & Diuision: Nowalthough there be aboue two numbers so named or assigned, as oft times there is, yet is the rule true, for where there are mo then two numbers, there are also two numbers, howbeit in all the sayd operations, there is neuer vsed aboue two numbers at once towardes the finding of the third.

And therefore if these numbers, 3.4.5. and 6. were geuen me to adde together, I ought first, to take but the first coople of them onely, saying, 3. and 4. make 7. and so by these two geuen numbers, 3. and 4. is found a third number viz. 7. Then secondly I ought also to take the same 7. & adde it to the next number, saying, 7. and 5. make 12. and so by these two numbers, another third number (viz. 12.) is likewise found.

And lastly I must adde the same 12. to 6. the last of the numbers assignd, saying 12. and 6. make 18. And so haue I by those 2. last numbers found yet one other third number (viz. 18.) which is the whole summe of the numbers, 3.4.5.6. But here you may consi-

der

ver by the very words of the rule, that if reason deny to grant any such third number, Then can there none such be found, And yet is the opperation true notwithstanding, for if two equall numbers be geuen to be subtracted the one from the other, as, 3. from 3. or 4. from 4. there cannot by reason be any third number found by such opperation,

Fuma. That is very true, and therefore if any third number should be found in that case, it weare a mere absurditie. Philo.

27 And when the 2. numbers assignd are expresst,
Each with one figure alone and no mo,
The third number then is soonest and best,
Found wholely at once, but if it chance so,
That they haue mo figures, then to escape crymes
The worke must be wrought, by parcells at times. 19

This Parim sheweth you, that when in any opperation, the 2. numbers geuen are each of them expresst with one only figure, then the third number sought for, is best and soonest found, wholy at once, as when we say that 3. & 4. make 7. or that 6. abated from 9. leaueth 3. or that 5. times 6. is 30. where you may see that 7. 3. & 30. the seuerall third numbers of euery such opperation, are found wholy at once, But if the geuen numbers haue mo figurs then one, then is not the operation to be wrought wholy at once, but by parcels or at diuers times, as when 36. is to be added to 52. here 6. must be first added to 2. & then 3. to 5. And likewise in multiplying 36. by 4. first by the same 4. we multiplie 6, & afterward the 3. For it is al one thing to worke either by the whole number, or by the parcels or digits therof, But the working by the digits is more easie, because small numbers are easlier rekoned then greate.

28 And whilst you are working you must haue regard,
To think and perswade your selfe for that space,
That all the digits both vpward and downeward,
Are either alone, or in the first place,
For thought many figures at each side may lurck,
Yet count you that place, still first, where you worke. 20

The meaning of this rule is no more, but that whilst you are working in any operation, you must alwayes imagine & perswade your selfe that all the digits as they stand in a ranke, vpward or downeward (for that time) are either alone as it were without any mo ranks, Or els that they are in the first place onely, which is all one thing as you shall see anone, For it is a generall order in al opera-

Orders generall to be obserued in all operations

tions (before the worke begin) to place the figures of the geuen nũbers directly the one vnder the other (though not in al opperations alike) yet euermore so that therby their figurs must of consequence fal in ranks alwel vpward & downward, as sidelong: as if I would adde 24. & 35. together, first before I begin I must place the same geuen numbers, one ouer the other thus | 24 / 35 | or thus | 35 / 24 |

And so you see they make 2. rankes of figures in both numbers, whether you behold them sidelong, or vpward and downeward: Now in adding them together, 35. must not be added to 24. nor 24. to 35. wholy at once, but first I must adde together the first ranke of figures, as they stand vpward and downeward, that is in this example, 4. and 5. saying 4. & 5. make 9. & then cõming to the next rank, although indeed they standing in the 2. place, represent 20. & 30. yet I esteeming them, as though they were there onely alone, or but in the first place, (as I did the other) adde them togeather thus, saying 2. and 3 make 5. And then the very place where the same 5, setteth by nature of the operation, maketh it 50. whereby the whole summe of this Addition, appeareth to be 59. And as I haue done in this example of 2. rankes, so should I proceede still if there were mo such rankes of digits or figures.

Eu. I vnderstand you in al this hetherto very well, and see that this kind of imagination is very commodious and bringeth a great ease to the operation.

Philo. Then shall you haue one rule more, which is no lesse generally to obserued in all operations then the others proceeding, and so an end of the generall rules, and it is thus.

29. And last if in working the third number found,
Consist of two figures it craueth by kinde,
Two places to hold it whereby you are bound,
To keepe the latter of both in your minde.
From ranke to ranke still, till you come to the last,
Where then such 2. figures may both be wel plast. 21

As if 56. and 68. should be added togeather, After I haue set the numbers, in order, one vnder the other thus, 56 / 68 — I say 6. and 8. make 14. but now because this 14. the third number found of this first ranke, consisteth of two figures which require to occupie two places: and therfore cannot be both placed vnder 6. and 8. which make the first ranke, therefore I say

say I must onely write downe 4. there, the first figure of 14. and keepe the latter (which is 1.) in my minde till I come to the next ranke, which done, and comming to the next ranke, I adde the parcels, digits or figures thereof, saying thus, 1. in my minde and 5. make 6. and that with the other 6. make 12. which 12. although it consist of 2. figures, yet seeing it commeth of the figures of the last ranke (for there are no mo rankes nor figures after that) I may now by this rule write downe wholy beyond my former 4. thus, 124. But if there had beene yet mo rankes of figures, then should I haue written downe onely 2. of the 12. and haue kept the latter 1. in minde agayne till the next ranke, and so should neuer subscribe both figures, till I had done with the very last rank, the like order is to be obserued in euery other operation: And thus much for the operation in generall. Now to euery operation perticulerly, and first touching Addition.

Of Addition the first operation Arithmeticall. Chap. 4.

The office and effect of Addition.

30. Addition vniteth two numbers or more,
To th'end to make them all but one totall,
VVhich were dispersed in parcels before,
VVherein the grosse sum so found must be equall,
To all parcels els is the worke nought,
And the totall found not such as it ought.

Euma. This rule sheweth the office of Addition very playnely and to what purpose and effect it serueth, but I pray you, what meane you by the totall?

What nũbers are called the parcells & which the totall.

Philo. The totall is the grosse sum resulting or amounting by the adding together of diuers lesser parcels or particuler sums: as if you would adde, 2. s. 3. s. and 5. s. together, then you might say the totall thereof were 10. s. and that the same 2. s. 3. s. 5. s. are the parcels or particulers, which make that totall.

Eum. This I vnderstand well, wherefore I pray you proceed to shewe me the practise worke of Addition.

Philo. With a good will wherefore marke well these rules ensuing, which teach the same very playnely.

31 Addition of numbers in this wise is wrought,
First write each number right vnder other.
Figure to figure iust as can be thought,
Beginning at right hand so passing to th'other:

The form of practise of Addition.

VVhich done drawe a line all along vnder,
Thereby to decerne the totall asunder.
32. Then to the first ranke at right hand resort,
Each figure there adding to the next beneath it,
Whose totall if it but a Digit import,
Right vnder that ranke beneath that line leaue it,
From whence proceeding past to the next row,
VVhose totall collected straight likewise left tow. 23
33 But if the totall of any ranke found,
Yeeld you 2. figures, then play you this pranke,
Subscribe but the first for the other is bound,
To stay and be added vnto the next ranke,
VVhich in the meane while must rest in your minde,
And in this due place be first that is ioynde.

Although you may in a manner sufficiently vnderstand the whole practise of Addition already, partly by this rule, and partly by the examples thereof geuen before in the glosses of the 28. and 29. verses: yet because I will not leaue you doubtfull in any thing, I will lighten this Maxim also with examples, whereof let this be the first.

1. A question of sheepe. There are in one shire 4. great sheepe maisters, the first hath 8620. sheepe: The second hath 6845. The third 4600. And the fourth 3030. now my desire is to know how many sheep they 4. haue in al.

Wherefore as this rule biddeth me, I first write downe,
The seuerall numbers of euery mans sheepe, each right vnder other, and draw a line vnder them, as here you see.

	8620
	6845
	4600
	3030

Then resorting to the first ranke of figures at my right hand (I meane as they stand vpwards and downewards) I beginne with the vppermost figure of that ranke, which I finde to be 0. or a figure of nought: Wherefore I let it passe for nought, and descending downeward to the rest, I finde onely but 5. in all that ranke (for the rest being ciphers, need not so much as once to be mentioned) because they are nothing as you know: and now for asmuch as 5. being the totall of this ranke, is but a digit, I therefore subscribe the same wholy right vnder the same ranke beneath the line thus.

	8620
	6845
	4600
	3030
	5

This done I passe to the next ranke beginning with 2. the vppermost figure thereof, which I adde to the 4. beneth it, saying 2. and 4. make 6. which with the 3. beneath the 0. make 9. (for I omit to name the ciphers as before.

And

And so for asmuch as this total 9. is but a digit also I subscribe the same wholy right vnder the same second ranke beneath the line, thus as here you see.

And now proceeding to the third ranke I adde all the figures thereof togeather, saying thus, 6. and 8. is 14. and 6. is 20. which totall because it containeth 2. figures I must (both by this rule, and by the 29. verse) subscribe but the first of them, which is the 0. and keepe the other which is the 2. in my minde to be added vnto, the next ranke: Wherefore keeping the same 2. in mind accordingly I subscribe onely the 0. vnder the third ranke beneth the line thus.

And now comming to adde the parcels or figures of the fourth and last rank, I must by this rule first ioyne or adde the 2. in my minde vnto them (so shall I be sure not to forget them nor yet to charge my memorie to long in that behalfe) therefore to performe the rule, I say 2. in my mind and 8. is 10. and 6. is 16. and 4. is 20. and 3. is 23. And now although the total of this ranke consisteth of 2. figures, yet sith it is the last rank, I may yea and must now of duty, by the prescript of the sayd 29. verse, subscribe them both: But yet so as each of them may possesse his propper place, to wit, I must set 3 the first figure right vnder the same last rank: and then the 2. one place further towards the left hand as here I haue done, and so haue I ended the addition & the whole fabricke of the worke standeth thus: whereby it appeareth that the totall number of all their sheepe is, 23095.

8620
6845
4600
3030
95
8620
6845
4600
3030
095
8620
6845
4600
3030
23095

Eu. I pray you giue me some like numbers to adde, whereby you shall see how well I haue marked the order of this worke, and your proceeding therin, for I hope I will not misse much in the working of any like example presently.

Phi. Then adde togeather these 3. numbers 8054630. 9080220. and 7080240.

Eu First I write them downe one directly vnder the other, figure to figure, as the rule willeth me and make a line vnder them thus.

8054630
9080220
7080240

Then in the first ranke I find nothing but ciphers which are nothing, wherefore I let them passe, & go to the 2. place.

Phi. But though the ciphers of them selues be nothing yet you know they stand not there for nothing, but for some proofe.

Eu. Yea that is true, I know they stand there to fill vp a place

for the augmentation of the other digits folowing towards the left hand, But they make nothing in Addition.

Philo. And yet they fill a place also in addition, wherfore you must euer in such cases fill the place also derectly vnder them with a cypher, saying 0. 0. & 0. make 0. and so subscribe that 0. vnder the first rank, or els the figurs of the next rank, wilbe but in the first place, which in truth are, and ought to be in the second,

Euma. Indeede I perceiue reason willeth so, for els I see I should loose one of the places of my numbers, wherefore I say, nought, nought and nought maketh nought, and so subscribe 0. the figure of nought, accordingly thus,

8054630	
9080220	
7080240	
	0

But the cause why I was imboldned to ouerpasse the cyphers, was because that in your former example, I saw you did not only ouerpasse the cyphers of the first rank, but also you sayd, they nede not so much as once to be mentioned or named, but I confesse I did not consider ẏ ther was both in that rank & euery other rank of that example some one other figure (at the left) with whose Addition you filled that place, But I will take better heede hereafter, And now to proced with my worke, I come to the second ranke or place, whose digits I add thus saying 3. and 2. is 5. and 4. is 9. which 9. being a digit or number of one figure, I do subscribe beneth the lyne derectly vnder his own rank, And so the fabrick sheweth thus,

8054630	
9080220	
7080240	
	90

Then come I to the third ranke or place and add all the digits there of, saying 6. and 2. is 8. and 2. is 10. But 10. being an article or number of 2 figures, I subscribe the, 0. onely and keepe the 1. in mynd, to giue to the next ranke. And so the addition standeth thus.

8054630	
9080220	
7080210	
	090

This done I come to the fourth ranke: saying 1. in mind and 4. is 5. wherefore (omitting the ciphers, because they are nothing, and I haue a digit to fill that place) I subscribe that digit 5. vnderneth the lyne derectly in his owne place thus,

8054630	
9080220	
7080240	
	5090

Then add I all the digits of the fift place likewise: saying 5. and 8 are 13. and 8. makes 21. which being a number of 2. figures, I subscribe the first thereof which is 1. and cary the second viz. 2. in mind to be added to the next place and so the fabrik stands thus And now making accesse to the

8054630	
9080220	
7080240	
	15090

first

first place I find nothing there but cyphers. wherefore by your former doctrine, I must subscribe a 0. but what shall I doe the whilst with the 2 in my minde.

Philo. Your selfe sayd but euen now, that you must subscribe the first figure and carry 2. the second in mind to be added to the next. & yet now you go about to subscribe a Cipher there, & aske what you shall do with the 2. in mynd. Do you not consider that the place where the Cyphers are, is the next place: In which though you find nothing, yet when you haue added the 2. in your mind, Then haue you the same 2. to subscribe in that place, and not a Cypher.

Euma. I perceiue my fault, wherefore I say 2. in mynd and 0.0.0. is 2. which being a digit, I subscribe directly vnder the ciphers beneth the lyne thus Then finally comming to the seuenth & last place or ranke, I adde the digits thereof saying 8. and 9. is 17. and 7. is 24. whereof I subscribe the 4. directly vnder the ranke whereof it came, and the 2. one place further towards the left hand, according to the 19. verse, because it is the last place, & so the fabrick sheweth as here aboue appeareth, and the whole totall of the three numbers geuen is, 24215090.

Philo. Well sayd, it is so in deede, wherefore now I, Will set you but one example more of Addition, and so we will go forward, and it is thus I would haue these seuen numbers, brought into one Totall,

Euma. That will I endeuor to performe god willing, wherefore first I collect all the digits of the first place, and I find them to make 61. whereof I subscribe 1. the first figure vnder the lyne directly which the first rank as here you may see, and keepe the second viz. 6. in mynd,

Then add I the 6. in my mind to the digits of the second ranke, and so they make in all 67. whereof I subscribe the 7. in his due place, and keepe 6. in mynd agayne, to be added to the figures of the third place. But comming to the third ranke, I finde nothing but Ciphers, wherefore (as you taught me euen nowe) if I adde the 6. in my mynde vnto that place, then that

8054630	
9080220	
7080240	
215090	
8054630	
9080220	
7080240	
24215090	
9099	
8098	
9079	
8099	
9099	
98	
99	
1	
9099	
8098	
9079	
8099	
9099	
98	
99	
671	
6	

6. wilbe to be subscribed there which therefore I subscribe under the same ciphers beneath the lyne, accordingly as here appeareth, And so now haue I nothing in mind to charge my memory: Finally comming to the last place, I collect all the figures there, and they make. 43. Both the figures of which 43. (because they come of the addition of the last rank, I subscribe presently under the line according to the 19. verse, that is to say first 3. the first figure directly under the ranke whereof it came, and then 4. the second figure, one place further towards my left hand, and so haue I done, And the totall as appeareth is. 43671.

Philo. Well I think this sufficent for the practise of Addition in whole numbers, But now for the proofe of your worke take this rule.

9099
8098
9079
8099
9999
98
90
43671

The proofe or Triall of Addition.

34. Addition is tried by casting out nine,
First from the parcells and next from the totall,
For nothing their rests at th'end of a line,
The worke is well wrought if both appeare equall.

As for example the 3. numbers which I first added, with the totall of them amounting, were such as here you see,

74235
1321
2142
776,8.

Now to trye whether the totall be true or not, I take all the figures of the first parcel (not valuing them by their places, but by their proper shapes onely, wherefore I may begin either at the right hand or at the left: as I like best) & they make. 21. from which I cast twise 9 and so remayneth 3. which I ad to the digits of the second parcel & they make in all. 10. from which I cast 9. & so haue I but 1. remayning, Then ad I that 1. to ỳ figures of the third parcel & they make also 10. from whom I cast 9 & so resteth 1. onely, which I set at the end of a lyne thus 1 ——, Then come I to the totall saying 7 and 7. make 14, and 6. makes 20. and 8. make 28. (for as for the 9 I let him quite passe) & so casting 9. thrice out of 28 there will also remayne 1 or els accompting 28. as they stand thus, 28. but for 2. and 8. then make they 10. from whence casting 9. the remnant will also be 1. which therefore I set at the other end of the lyne thus. 1 —— 1. And now because I see these rests or remnants are both equall, Therefore I say that my worke was done well and truely,

Euma. This I like well, and therefore I will assay how I can make the like triall of my last worke, whose parcells and totall

tall was as here it stands. 9099
Wherefore beginning with the first parcel, I finde 8098
nothing but Nines and 0. all which therefore I ouer- 9079
passe, but in the second parcel (besids a Cipher & 9. which 8099
I also ouerpasse) I finde twise 8. which make 16. from 9099
whom I cast 9. and there resteth 7. to which I put the 98
7. of the third parcell (omitting the Nines there also) & 99
they make 14. from which abating 9. there resteth 5. 43671
that 5. I adde to the 8. of the fourth parcell and they make 13.
whereof reiecting 9. their remayneth 4. which I ioyne to the 8. of
the sixt parcell (omitting cleane both the fift and seuenth parcels,
because they be all nines.) and so haue I 12. from which casting 9.
there resteth lastly 3. wherefore I set that 3. at the end of a line
thus as you did 3.—And then comming to the total, I finde all
the digits therof to make 21. and out of that casting 9. twise, there
wil also rest 3. which I set at the other end of the line thus 3 —— 3
and now sith these restes be both equal you say, the addition is wel
wrought and the totall true.

Phi. It is so, and by this meanes you may examine your worke
alwayes when you stand doubtfull thereof. For if the 2. numbers
remayning after all the nines be so reiected be not equal, then you
may be sure that you haue missed some where: but now harken
what belongeth further to Addition.

The badg or signe of addition is +

35. Also there belongeth vnto Addition,
One speciall signe peculier therevnto,
VVhich is a playne crosse and by exposition,
Signifieth more as here you see it doe,
For here 3 + 4. doth signifie seauen,
And so 5 + 6 betoken eleuen. 25

Euma. The sence of this rule is manifest inough, but I see no
neede of such kinde of crosse, for me thinkes I might as well say,
3. and 4 are 7. or 10. & 10. are 20. as to say 3. more 4 are 7. or 10.
more 10. are 20. for so I perceiue you wil me to pronounce the +.

Philo. You must do so, and the effect is all one, but the + is some
made, and serueth more aptly for the purpose, as your selfe will
confesse hereafter, when you shall know the vse thereof better, and
in the meane season, I do but declare the signification of that + vn-
to you, against you shall come to it, and now I will shew you cer-
tayne properties which Addition hath, and so conclude it.

The properties of Addition.

36. Addition hath these properties in store,

Eucl. 2. & 5. comon sentence 1 boke.

That if equall numbers be ioynd in accompt,
VVith numbers which were coequal before:
Then both their totalls shall equall amount,
But equal numbers ioynd with vnequals,
Euermore causeth vnequall totalls. 26

As here is 8. and 8. being equal numbers, now if to each of them I adde 4. & 4. which are also equal nũbers, then their totals wilbe 12 & 12. which are likewise equal together, but if I adde 4. & 4. being equals to 6. and 8. being vnequals, their totals wilbe 10. and 12. which are also vnequals.

Eu. These properties, are made very manifest, by your examples.

Philo. Then here concluding Addition, I will now proceede to the next operation, which is Subtraction.

Of subtraction the second operation Arithmeticall. Chap. 6.

The office of Subtraction.

37. Subtraction abateth one summe from another,
Greater then it selfe or equall at the left,
So leauing a third as part of th'other,
VVhich part is called the remaynder or rest,
VVhereto if you adde the number abated,
The former totall returneth created. 27

This sufficeth for the Argument, to declare the nature of subtraction: the office thereof and wherefore it serueth.

Now marke the practise which is thus.

The practice.

38. Subtraction requires to haue the sums placd,
Each vnder other as in Addition,
The greater sum first, and lesser sum last,
Figure to figure right in position,
VVhich done let a line be drawne vnder all,
To part the remayner from the Totall,
39. And then beginning first at the right hand, 28
Bate euery figure from that right aboue it,
Forcing the remnant at each time to stand,
Right with his owne ranke for so all workes couet,
But where there remaines not so much as and ace,
There set a Cipher to fill vp that place.

This rule sheweth you that subtraction requireth to haue the sums or nũbers right & orderly placed, euen as you did in addition, with a line vnder them, in the selfe same maner, which line as it serued in Addition, to seperate the totall from the parcels, so here it serueth to seperate the remayner left, after that the one number

is

is abated or subtracted from the other.

And it willeth you to place the greater sum first, that is to say the Total, from whence the numbee lesser, or equall, must be subtracted: for you know it is impossible in reason, to abate the greater from the lesser, but a lesse number may be subtracted from any one which is greater, or equall at the least: As you may take 3. the lesser, from 4. the greater, and there will remayne 1. Also you may take 4. from 4. both being equals, and the remainer wilbe 0. but you cannot abate 4. from 3.

Euma. All this is but reasonable, and I vnderstand it very well, and doubt not but to vnderstand all the rest of the rule by an example of the practike worke, wherefore I pray you spend no more time about the explication of the rule, but shew me an example.

Philo. That will I do with a good will, by a question which I will propound and also aunswere my selfe, and it is this.

2. A question of a Subsidie.

A certayne prince hath graunted to him by his subiects a subsidie of 235970. pounds, deducting therof for the wages of diuers garrisons of souldiers, remayning in certayne townes within his dominion, 15850. poundes: Now the question is, what sum of money came cleare to the princes treasurie after such deduction.

For answere whereto first 235970. being the Total or greater sum must be first placed, and then the lesser right vnder it, which therfore I set downe thus, w a line vnder them as here appeareth:

And then beginning first at the right hand I abate the figure beneath, from that right aboue it saying thus Nought out of nought, leaueth nought, And so put a Cipher, or figure of nought, vnder the line, directly with that ranke as in this example.

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235970
 15850
-------
      0
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Then come I to the second place of the lower number where I finde 5. which I subtract from the 7. right aboue it saying, 5 out of 7. there remayneth 2. that 2. I set vnder the line, directly with that second ranke thus.

```
235970
 15850
-------
     20
```

This done I go to the third place, saying, 8. out of 9. leaueth 1. Wherefore I note that 1. beneath the line, right vnder that third ranke, as here appeareth thus.

```
235970
 15850
-------
    120
```

And so comming to the fourth place, I find 5. in both numbers. Wherefore I say 5 out of 5. there remaineth nought and therefore, as the rule biddeth me, I set a Cipher onely vnder the line, to fill vp that place, thus.

```
235970
 15850
-------
   0120
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And so I go to the fift place, where I abate the 1. beneth out of the 3. aboue it, saying, 1. out of 3. leaueth 2. remayning: which 2. I subscribe vnder the line directly with the same place as here you see it. And because there yet remayneth 2. in a further place of the vpper nũber from which I find nothing in ẏ lower number to be subtracted, therfore I subscribe that 2. wholy as it is, vnder the line, in the correspondent place: as ẏ which hath nothing abated from it. For, seeing that nũber, which by my worke is setled vnder ẏ line, is the portion remaining of the Totall or vpper number, when the lesser is substracted from it, & that the lesser doth not extend so farre, as to subtract any part of the 2. in the last place of the vpper number or Totall, it is reason that the same 2. be placed vnder the line, with the remayner, as a parcell thereof, and so haue I done, and the worke standeth thus.

235970	
15850	
220120	

Whereby I finde (as it appeareth) that when the, 15850. pounds were deducted from the whole subsidie of 235970 pounds, yet there came cleare to the ẏinces treasurie, 220120 pounds, as the remayner, after such deduction.

Eum. This rule seemeth easie inough to vnderstand, and the worke is easie to doe, wherefore I pray you let me trie what I can do in it.

Philo. Then take this number, 6340002565. for the totall, and substract from it, 40974630. and tell me what the remayner is.

6340002565	
40974630	

Eum. first I place the numbers orderly, with their lyne vnder them: thus.

Then beginning at the first place of the lower number, I find ther a Cypher or figure of nought. Wherefore if I abate that nought from 5. the vpper number there, then the same 5 (by reason) must needs remaine 5. still: which I subscribe therefore vnder the lyne in the first place, thus:

6340002565	
40974630	
5	

Then, coming to the second place, I say 3. out of 6. leaueth 3. remayning: wherefore I set that 3. vnder the lyne, directly with that second rank, thus:

6340002565	
40974630	
35	

And so going to the third place, I find 6. to be taken from 5. which I cannot do by reason. And I remember that both by the 27. rule, and by your owne notes vpon the 28. rule, it is impossible to abate a greater number from a lesse.

Philo. Why, weether think you to be greater: of the totall or of the

the abatement:

Euma. Which call you the abatement:

Philo. The number to be ſubtracted or abated, is alwayes called the abatement or Deduction. For, to abate, ſubtracte, deduct or take away, ſignifie all one thing in effect. What this word (abatement) ſignifieth.

Euma. Then I muſt confeſſe, that the Totall ſemeth to be greater in ſight by 6 2 0 0 0 0 0 0 0 0. at the leaſt: becauſe it exceedeth the abatement clearely, by the ynth and tenth place.

Philo. Then aſſure your ſelfe, this ſubſtraction is not impoſſible ſith you cannot deny, but that it is poſſible to ſubtract a leſſer number, from a greater. But, if the Totall and Abatement had both like number of place: then, though all the figures of the Abatement were leſſe then thoſe of the Totall ſauing the very laſt, yet if that laſt figure of the abatement do exceede the laſt of the Totall, then were it impoſſible to be ſubtracted. As for example, here in theſe numbers.

3456
4000

Where though all the figures of the Abatement, ſauing the laſt, are but Cyphers as you ſee: Yet is not that lower number poſſible to be ſubtracted from the vpper number: And that onely, becauſe 4 (being the fourth & laſt figure of the Abatement) is greater then 3. being the laſt of the Totall, and like in degree or place.

Eu. That appereth reaſonable, becauſe 4000. is greater then 3456

Phi. But it is not ſo, in your numbers. For if I ſhould require you to abate 19. from 20 would you thinke it impoſſible?

Euma. No, indeede.

20
19

Philo. And yet, ſet the numbers, as they ſhould ſtand thus. And you ſee, you cannot ſubtract 9. out of 0.

Euma. That is certayne. Wherefore if there be any ſpeciall rule how to do it, I pray you, ſhew it me.

Philo. Then, marke it wel. For, the rule which ſerueth in this behalfe, is generall to all manner of vulgar ſubtractions, and is thus.

40. And when the digit to be abated
is greater then that of the vppermoſt row
Let theſame vpper digit be rated,
for ten mo in number, then it doth ſhow:
From both which conioynd, the lower then take,
and vnder the lyne, the remnant downe ſtake.
41. But note that as oft as any ſuch digit,
or Cypher aboue, doth borrow ten ſo,
So oft the next digit, in lower ranke ſet,

Must borrow an ace, to make it one mo,
VVhich digit so rated, 1. more then it is,
Must so be abated, remember well this.
4?. And if that case you happen to find,
the place next beneath, quite emtie and void,
Yet must you repay 1. there in your mind?
For loane of the ten, which last you enioyd?
Subtracting that 1. from numbers aloft,
VVhich for the more suretie is told you so oft.

The former rule did teach you, by a very easie example, as your selfe confesse, the cause wherof (as you may now perceiue) was, for that no one figure of the Abatement, was greater then the figure of the Total right ouer it. But now this rule telleth you, that when the digit to be abated, is greater then the other ouer it (from which it should be subtracted) then you must recon that vpper digit, for 10. mo then it is indeede: And so ioyning that 10. and such digit togeather, you must from them both subtract your vnder figure, and so subscribe the rest, in the due place.

And further it teacheth you, that for euery ten that any digit or Cipher of the vpper number doth so borrow, to make it selfe shew greater then it is indeed: So often the next digit or cypher of the batment or lower number, must borrow an act to make it more then it is in deede. And the same lower figure being so reconed and accompted, for 1. more then it is by his shape, must be so wholy subtracted, from the vpper digit of the Totall.

And therefore, now to proceed with your former subtraction, which you left thus, doe as this rule willeth you. Put ten more to 5. the vpper digit, and how many will it be then?

6340002565
40974620
35

Euma. It wilbe 15.

Phi. Well then, from that whole 15. now abate 6. your vnder figure be subtracted, and there will remayne 9. Which 9. set vnderneath it, as the remayner thereof, thus.

6340002565
40974620
935

And, because you did now (by borrowing of 10. make 5. (the last figure of the Totall) to be 10. more then it was before, therefore you must by the same rule) repaye 1. to the next place of the Abatement, or lower number which is 4. Saying 4. and 1. that I owe, is 5. And so rebate that 5. wholy, from the figure of the Total aboue it.

Euma. Now I thinke, I vnderstand the matter better then I did. For it seemeth to me, that the 1. which I repaye in the

Abate-

Abatement, is as much in that place, where it is to be repaid, as the ten (that is borrowed of the Totall before) is, in that place where it is vsed. For, sith (that by the 20. rule) the place where I am partly working, must alwayes be accompted to be the first place, then ℔10. (which was lately put to the figure of 5 in the totall, to make it 15.) was but an vnit, in that place whence it was borrowed, which was from the second in the fourth place. But in comparison of the third place, (where it was ocupied) it was 10.

And therefore, when I repay 1. to the 4. in the fourth place of the Abatement, the same 1. is asmuch there, as the 10. which was borrowed before, to encrease the 5. of the Totall, in the third place, and maketh iust satisfaction for it.

Philo. You haue found out the very truth. And therefore, if you vnderstand your owne wordes, and remember the rule well, you may go through with the subtraction begun, or with any other that you shall take in hand. Howbeit for your better instruction, I will finish your worke begun, to the end it may be a generall example to you, for all others. And therefore marke well the order thereof. And I wil begin, where you left where the numbers stode thus.

6340002565	
40974630	
	935

And here first, because 5 (the third figure of the Totall) was euen now encreased, and made 15. by borrowing 10. from the fourth place (as you sayd) therefore I repay 1. to the 4. in the next place of the Abatement, saying as before, 4 and 1. that I owe, is 5. which I would take out of 2. the ouer figure, but cannot: Wherefore adding 10 to that also, it maketh 12. from whence I abate 5. & so resteth 7. to be set vnder the line thus.

6340002565	
40974630	
	7935

And here, sith the same 2. was made 12. comming now to 7. (the next digit of the Abatement) I accompt it for 1. saying 7. and 1. that I owe or repaye, make 8. which I should take out of 0. the vpper figure directly, but cannot: Wherefore I accompt that 0. for 10. And so abating 8. from that 10. there remayneth 2. which I set directly vnder the same place, beneath the lyne, thus.

6340002565	
40974630	
	27935

And so proceding to the first place of the Abatement or lower number, I finde 9. vnto which I forthwith repay 1. for loane of my last 10. borrowed, and so it is 10. which I should subtract from 0. the figure directly aboue it, but, sith it cannot be, I make that 0. ten, in like sort as before, and then

say, 10. substracted from 10. leaueth 0. which 0. I therefore subscribe vnder the line, in his proper place, thus.

6340002565
40974630
027935

This being done, and being now come to the seauenth place of the Abatement, I finde there a Cipher, vnto which I must repayre 1. the vsurie of my ten last borrowed aboue, and so it is 1. which I should abate from the 0. aboue it, but cannot. Wherefore I yet borrow 10. more of the next place of the Totall, and put it to the same 0, and so it is 10: From which I substract that 1. and there resteth 9. which I subscribe accordingly, thus.

6340002565
40974630
9027935

And now comming to the last figure of the Abatement or deduction, which is 4. I first repaire thereto, the 1. due for my last ten borrowed by the totall, saying 4. and 1. that I owe is 5. which I should take from 4. the digit aboue it, but cannot, wherefore I accompt that 4. for 14. (so is there ten put to it) And then say 5. out of 14. leaueth 9. which I set vnder the line thus.

6340002565
40974630
99027935

And because the sayd vppermost 4. was so made 14. as you see, therefore I must repayre 1. to the next figure of the lower row as I did before, for although the Abatement haue but 8. figures, and therefore consequently but 8 places in all, yet must I in minde imagin a ninth place there, vnto which I must repayre the one due for the loane of my last borrowed 10. And so the rule willeth saying thus.

But if in case you happen to finde,
The place next beneath quite emptie and voyde,
Yet must you repay 1. there in your minde,
For loane of that 10. which last you enioyde.

Substracting it from the numbers aloft, &c.

And therefore I say 1. that I owe, abated from 3. leaueth 2. which I subscribe therefore, directly there vnder thus.

6340002565
40974630
2299027935

Finally (saith 6. the last figure of the Totall, hath nothing vnder it to be abated from it, neyther doe I owe any vnit to the place there, I say nought out of 6. leaueth 6. still, and so subscribe the same lastly, vnder the line thus, as you see.

6340002565
40974630
6299027935

And thus you see that 40974630. being substracted from 6340202565. there will yet rest, 62990 27935.

Euma.

Eu. Now I pray you let me worke one example of subtraction that you may therby parceyue what heede I haue taken to your worke: And you shall see I will not be long about it, nor vse much repetition neyther of wordes nor figures, til I come to the end, for I perceyue you haue vsed many more words and circumstances in your worke then needed, only of purpose to make the same more playner to me.

Philo. I haue done so in deed. And sith you are so desirous to proue your cunning: go to, subtract 64980575. from 10460013064.

Euma. To do this after that I haue placed the numbers ouerly with their lyne vnder them thus, I begyn at 5. without vsing more wordes, but say 5. out of 14 there remaineth 9. which I subscribe directly keping 1. in mind to repaye in due place. Secondly I say 7. & 1. in mind are 8 which taken from 13 leaue 8. That 8. I forthwith subscribe directly, keeping 1. agayne in mind Thirdly I say 5. and 1. in mind is 6 which subtracted from 10. there remayneth 4. That 4. I do forthwith subscribe directly keeping yet another 1. in mynd. Forthly I say 0. and 1. in my mind is 1. which I abate from 3. & so resteth 2. That 2. I therfore subscribe directly (without keeping any thing now in mind, because the last figure of the total borrowed no ten as his fellowes before did.)

Fiftly I take 8 from 11. and so resteth 3 which I subscribe directly keping 1. now againe in mind to repaye in due place:

Sixtly I say 9. and 1. in mind make 10. which deducted from 10. leaue 0. therfore I subscribe ỳ 0. directly, & keepe 1. in mind againe.

Seuenthly I say 4. & 1. in minde are 5. which taken from 10. cause 5 to remaine, therfore I subscribe that 5. directly & keep 1 in mind.

Eightly I say 6. and 1. in my mind are 7. which I abate from 16. And so remaineth 9. That 9. therfore I subscribe directly not forgetting 1. in my mind, which I repay in the supposed ninth place of ỳ abatement, vnder 4. the ninth figure of the total, saying 1. out of 4. leaueth 3 which 3. I subscribe directly vnder the same 4. And so haue I done sauing that there yet remayneth 2. figurs of the total, which I thinke I may either subscribe as I find them, or else for faction sake I may say nought out of 0. leaueth 0. and so subscribe it directly. And then say likewise 0. out of 1. there remayneth 1. & so subscribe that 1. also directly. And so both wayes being of one effect the numbers will stand thus: And the remainer or third number found by this opperation, will be as here it is vnder the lyne.

10460013064	
64980575	
10395032489	

Philo. You haue done very wel, and so well, as if you forget not the order here by you obserued, I iudge you sufficiently instructed in subtraction: But yet, to the end that you your selfe may know when your worke is well and truly done, I will giue you a rule for the triall thereof, which is this.

The triall of Subtraction.

43. Try your subtraction by casting out nine.
Both from the abatement, and from the remayner,
And note what doth rest at th'end of a line,
So shall you find it appering the playner,
Then casting all nines likwise from the totall,
The work is well wrought if both rests be equall. 38

Eu. This rule is like the trial of addition, wherfore I will assay to examine my last worke by it: and therfore first casting out all the nines out of the Abatement, I find the remainer to be 8. Secondly casting likwise al ỹ nines from the remainer, I find ỹ there resteth 8. also, which put together make 16. From whence I take 9. and there resteth 7. which I note downe at a lines end thus. 7 ——
Then casting all the nines of the Totall number I find also 7. to rest, which therfore I set at the other end of the line thus 7 ——7
And so the qualitie of these 2. digits argue my worke to be true.

Philo. It doth so, but the best and surest proofe of subtraction, is Addition: And likewise the best proofe of Addition, is subtraction, For when you haue added 2. numbers together, by subtracting of the one of them agayne from the Totall, the other will returne and be the remainer, if the addition were truly wrought: And so when you haue subtracted one number from another, if you adde the abatement and remayner together agayne, the first Totall will returne, if the subtraction were truly wrought before, or els not as appeareth by the 30. and 37. verses.

Addition and subtraction are operations by equalitie

And therfore some Arithmeticians accompt and name Addition and subtraction, to be operations by equalitie, because that as in addition, the Total found by that operation, is and ought alwayes to be equall to all the parcels: So in this operation of subtraction the remayner thereby found, with the parcels subtracted, is and ought alwayes to be equall to the first Totall. And so equalitie, doth as it were, prescribe the very rules, lawes, and trials of these 2. first operations. And now to goe forward with that which resteth, yet touching subtraction I say thus.

44. Subtraction aswell as Addition
Doth challenge a signe vnto it pertayning,

Which is a right line receiued by tradition,
To signifie lesse where it is remaining
Whereby we say 10 — 3, is but seuen,
And 15 — 4 maketh only eleuen } 31

The badge or Signe of Subtraction

Eu. So that I perceiue a + signifie addition, and this — here, signifieth subtraction or abatement. And the one is pronounced more, and the other lesse: and therefore as 10 —— 3. is but seuen, so 7 + 3. is 10. And likewise as 15 —— 4. is but 11. so 11. + 4. is 15.

Philo. You conceiue it rightly, and euen by that you haue said you may perceiue that Addition and subtraction be each of them a proofe of the others worke. But now you shal here the properties of subtraction. And so an end thereof.

Euclid 3. common sentence of the first booke

45. Subtraction doth clay me these properties here
That if equall numbers be bated from equalls.
Then both their remainders shall playnly appere,
As equall together, as were the first totalls } 32

Euclid 4. common sentence first book

46. But equall numbers from vnequall bated,
Or vnequall taken from equalls before
Leaues their remainders vnequally rated,
Vnto their first totalls by Euclide his lore.

The properties of Subtraction.

As for example from 8. and 8. being equall numbers, abate 5. & 5. being also equal together, and the remayners wilbe 3. and 3. which are as equall still together, as were 8. and 8. the first totals. But if 5. and 5. being equall numbers, be abated from 8. and 9. which are vnequall numbers, the remayners wilbe 3. and 4. which are also vnequall. Likewise if from 8. and 8. being equall numbers you subtract 4. and 3. which are vnequall numbers the remainers wilbe 4. and 5. which are also vnequall together.

Euma. All this is easie inough to be perceiued.

Philo. Then may we proceed to Multiplication which is thus.

Of Multiplication the third operation Arithmeticall. Chap. 6.

Euc. 16. d of the 7, booke

47. Multiplication bends all his deuotion,
By folding together 2. numbers assignde
To bring forth a third in such like proportion.
To one of those 2. as the other behinde
Is vnto an ace, which number we call,
The product or ofcum, where euer it fall. } 33

The office and end of Multiplication

This verse sheweth you the definition of Multiplication as namely, that it is the folding together of 2. numbers appointed, each in

other, the to intent thereby to produce and bring forth a third number which may be in such rate and proportion to any one of the 2. numbers multiplied, as the other is to an ace or vnit

As if I were bidden to multiplie 6 by 4. here the same 6. and 4. are the 2. numbers assignd, which when I multiplie together, I do nothing else but seeke for such a third nũber, as may be in like rate, behauior, & proportion, to 4. as 6. is to 1. which number so sought for, being once found, is properly named the product or diccome as the rule sheweth. But, 6 times 4. (or the product of 6. by 4.) you knowe is 24. wherefore by this rule I affirme that the same, 24. is to 6. as 4. is to 1. For if you first note downe the product & the next the 2. geuen numbers, and lastly an ace, or 1. thus 24. 6. 4. 1. you may easilie see, that as 24. is 4. tymes 6, so likewise 4. is 4. times 1. or if you set the same termes thus 24. 4. 6. 1. yet you see them still keeping a like rate, for as 24. is 6 times 4. so 6 is likewise 6. tymes 1. Agayne if you place them thus, 1. 4. 6. 24. or otherwise howsoeuer (So that the product and 1. be alwayes made either the 2. extremes, or else the 2. meanes) yet will each couple be still like in rate & proportion together: For now as 1. is $\frac{1}{4}$ of 4. so is 6 also $\frac{1}{4}$ of 24. which thing followeth allwayes, as an accident inseperable cleauing to the product in all multiplications whatsoeuer.

And therefore in multiplication, euermore the product or third number found, with the 2. numbers geuen to be multiplied, and 1. are generall accounted and truly named 4. numbers proportionalls.

Eu. Truly I confesse I vnderstand not perfectly what you meane by proportion nor proportionalls, nor yet by the 2. extreames or meanes of a proportion.

Philo. I cannot blame you for to say truth, as you haue not heard of those words heretofore, so their explication belongeth not to this first booke, but is to be reserued vnto the second, wherefore I will speake no more of them here, then I must needes for your present Instruction, which is thus.

When any 4. termes or numbers assignd are in such rate, behauior, & proportion together, that the first couple are rated like the latter couple, as you see the 4. former termes are. Then such 4. termes for their likenes and agreement in such rate and proportion are called proportionalls. This is (as you would say) nũbers of one proportion or proportionable together, which name with ỳ cause & reason therof I wish you to imprint wel in mind. For although I shal here

Euclid. 7. defi. 5.

here after in due place, intreate more largly thereof. Yet in the meane season, you shal not be able to vnderstād, neither the rule present (nor many other following) so well as you shalbe, if you know and remember what numbers are properly called proportionalls. And of euery such 4. termes the greatest and the least are most properly called the extermes, because that as the one exceedeth all the rest in greatnes, so the other exceedeth all the rest in smalnes, whereby they represent a certayne kind of extremity, as here 6. 3. 4. 2. where 6. being the greatest is called the greater extreme, and 2. being the least is named the lesser extreame. But 3. and 4. being neyther of them so great as 6. nor so little as 2. are therefore called the meanes, as those that conteyne a certayne meane or mediocritie betwene the 2. extremes. And the nūbers so placed are truly called extremes and meanes as well in respect of place as quantitie. Howbeit many times and that very generally those 2. numbers which are placed first & last of the 4. are called the extremes, & the ople which are placed in the middle betwene them are called the meanes. as if the selfe same termes stood thus, 3. 6. 2. 4. Now is 3. and 4. accompted the extremes. and 6. and 2. is accompted for the meanes, but these numbers are so called only in respect of their places and not of their quantities.

Extremes & meanes both by place and quantitie.

Extremes & meanes only by place & not by quantitie.

Eu. So I perceiue that numbers are somtimes called extremes and meanes in respect of their quantities onely, and somtimes in respect of their places onely. And sometimes in respect of both. And as for numbers proportionall I hope I know them so well, now that I will both remember them & the cause of their name also Phi. Then know you further that the 2. numbers geuen or assigned in euery multiplication, haue each of them a peculier name, for the greater is called the multiplicand and lesser is named the Multiplier. And with all you shall note that Multiplication forsaketh in stead of many additions, for it doth asmuch at one operation as addition doth at many.

Multiplicand

Multiplier

As if I should aske you how many is 30. times 40. And for knowledge thereof you would go about to add 40. together 30. tymes, it would be a tedious worke, wherefore by the inuention of Multiplication, the same is despatched at one operation.

Eu. Then both by the rule and your speach, here I learne first what Multiplication is, and whereto it serueth, and secondly that there is 3. numbers in euery Multiplycation, whereof 2. are geuen, and the third is sought for, each of the which haue their peculier

 names,

names, for the greater of the 2. geuen numbers you say is called the multiplicand, the lesser is named the multiplyer, and the third which is sought for and amounteth of the Multiplication, you say is called the product or ofcum: and lastly you say that the same 3. seuerall numbers taking an ace to them to supply the roome of a fourth, make 4. proportionalls, which I must not forget, but what if I would multiplie together 2. numbers equall together as 20. and 20.

Philo. Then may you make which of them ỹ ou will the Multiplicand: and so may you do in all other numbers also if you will, though not so aptly for working as when the greater is applied thereunto.

And now would I presently shew you the order of the working, were it not for that Multiplication of great numbers, cannot be learned but by the Multiplication of digits. Wherefore I wish you first to be perfect in the Multiplication of euery digit, each by other, which is a thing must needes be learned by hart & comitted to memorie, before you can proceed to ỹ multiplication of Mixed numbers.

Pythagoras his table.

1	2	3	4	5	6	7	8	9
2	4	6	8	10	12	14	16	18
3	6	9	12	15	18	21	24	27
4	8	12	16	20	24	28	32	36
5	10	15	20	25	30	35	40	45
6	12	18	24	30	36	42	48	54
7	14	21	28	35	42	49	56	63
8	16	24	32	40	48	56	64	72
9	18	27	36	45	54	63	72	81

And for your furtherance therein behold this table, whereof some affirme Pythagoras to be the first author.

The vse of this table is thus, when you would know the product amounting by any 2. digits, loke for one of the same digits first at the left side of the table, and then loke for the other in the heade or vpper ranke of the same, and hauing found them, goe directly from the digit found at the left side, towardes the other side of the table, till you come iust vnder your digit found in the head or vppermost ranke: and looke what number you finde enclosed in the little square of their meeting, the same is the product of those 2. digits, as for example if I would know the product of 6. times 9. First I finde 6. at the left side of the table, and then 9. at the top or head thereof, and going right forth from 6. towardes the right hand, til I come directly vnder 9. I find in the square of their meeting 54. which is the product of 6. multiplyed by 9.

Euma. This is one learned, and for a triall, I will assay to find the product of 8. times 8. Wherefore hauing found 8. at the left side of the table, and also at the top, I keepe my finger euen with the 8. at the left hand, going forth towardes the right side of the table

table till I come vnder the other 8, at the top or head where I find 64. as the product of their multiplication. And this table I doubt not but to learne so perfectly, and haue in minde so readily, that you shall finde no cause of stop or stay in me, but that you may proceede to shew me the worke of Multiplication in compound or mixt numbers, when it pleaseth you.

Philo. Well then pursuing that you eyther haue, or shortly wil haue the product of euery digit by other, at your fingers end, as perfectly as the table it selfe, I wil instruct you in the Multiplication of mixt numbers, both by rules and examples, but the rules ought to proceed, the examples whereof this is the first.

48. Multiplicacion first maketh request,
To haue each number in order so placed,
As in subtraction before, was exprest,
Figure to figure with line vnder placed
That is to witt, first the multiplicand,
And then multiplier beneath it to stand

By this verse or rule, you are taught the placing of the numbers in order, agreeable to the nature of ỹ worke, that is when you haue 2. nũbers geuen or apoynted, to be multiplied together, you must first write downe the Multiplicand, which is best to be the greater of the two, as I told you euen now: and then directly vnder it, figure to figure, must you write the Multiplier which done you must draw a line vnder both as you did in Subtraction.

Emma. Then if I would multiplie 432. and 24. together I must first write downe the 432. and then the 24. directly vnder it, and then drawe a line vnder them both thus.

432
24

And so 432. being the greater number standeth now as Multiplicand & 24. being the lesser standeth for Multiplier as this verse & your instructions importe: And so I perceiue that al this is but a preperation to the worke, neyther doth this verse extend to any further doctrine.

Phi. It is euen so, but now the next rule teacheth the worke thus. The practise of Multiplication.

49. Then cause the digit in first place belowe,
To multiplie euery digit aboue,
Orderly placing the products that grow,
Vnder the line as art doth approue,
VVhereof if any a digit surmount,
Subscribe the digit, the rest in minde count,

50. To be transeported vnto the next place,
As by former rules, you may vnderstand,
And when this first digit hath run his full race,
And multiplied all the multiplicand
The second digit must lead the like daunce,
And so each other as falls to his chance
51. In placing whose sums, you must not forget,
That euery digit of the Multiplier,
Haue the first figure of his product set,
Right vnder it selfe as straight as a wyer,
And all other figures that thereof accrue,
To lefthandward thence, each in his place due,

As the last former rule taught the preparation and orderly placing of your numbers fit and readie to the working of Multiplication, so in this rule is fully contayned all the order of the whole operation.

As for example I will multiplie together the numbers which your selfe late proposed, which were these 432 and 24. set- 432
led thus, wherefore here as the verses bid me cause the di- 24
git in first place below, to multiplie euery digit aboue, euen
so I cause 4. (which is the digit in the first place below,) first
to Multiplie the 2. directly aboue it, saying 4 times 2 is 8. which
8. I subscribe straight as a wyer, directly vnder the same 4- 432
whereof it was produced thus. 24

Then cause I the same 4. to Multiplie the next figure a- 8
boue, which is 3. saying 4. times 3. is 12. And because his
product yeldeth 2. figures and so surmounteth a digit, I subscribe
onely the first thereof viz. 2. counting and keeping the 8-
ther in my mind to be transported to the next place, as this 432
rule willeth, and as the 21. rule, and last verse of the 23. 24
rule before taught. And so the worke standeth thus. 28

This done, I cause the same 4. to Multiplie the third &
last figure of the Multiplicand, which is also 4. saying thus 4. times
4. is 16. vnto which I must now put the 1. in my minde and that
maketh 17. wherefore (if there were any mo figures in the vpper
row now to be multiplied) I should subscribe the digit 7. and keep
1. the second figure of this product in my minde to be transported
to the next place as I did but euen now.

But sith this is the last digit, which 4. hath to multiplie I must
now by the 21. rule, subscribe both the figures of this product to
wit,

wit, the whole 17. setting 7 in the next place directly vnder 4. and the article 1. one place further toward the left hand in this maner.

And now this first digit hauing run his ful race and multiplied all the Multiplicand, the second digit must lead the like daunce &c. that is to say 2. the second digit of the Multiplier, must likewise multiplie, euery figure or digit of the multiplicand, for performance whereof according to this rule I say thus, 2. times 2. is 4. Which 4. being but a digit, I set downe straight as a wier vnder 2. the same figure of the Multiplier wherof it is the product, as here appeareth

Then by the same 2. do I multiplie 3. the next digit aboue saying, 2. times 3. is 6. which 6 being but a digit also I place in the next place towardes my left hand vnder the line beneath the former 7. thus.

Finally I cause the same 2. to multiplie 4. the last figure of the Multiplicand or vpper row, saying 2. times 4. is 8. which forasmuch as it is also but a digit, I subscribe it forthwith in the next place towardes the left hand, which is vnder the 1. and so haue I done, for both the digits of my Multiplier haue orderly Multiplyed euery digit of the Multiplicand, and so the worke is thus.

	432
	24
	1728
	432
	24
	1728
	4
	432
	24
	1728
	64
	432
	24
	1728
	864

Euma. I haue taken so good heede to you in the working of this example, that I thinke I do not onely vnderstand the order thereof. But also could neere hand do the like agayne in any other, but yet I do not perceiue what the whole sum or product of this multiplication is, for here are 2. sums, whereof the first is 1728. and the second is 864.

Phi. There are two numbers yet, indeed whereof the first is 1728. as you haue sayd, but the second is 8640. & not 860 as you supose, for you must alway reken your places from the first figure of your numbers as you do in Addition, so that as 8. standeth in the first place beneath the line, so in the second place standeth 2. and 4. and in the third place are 7. and 6., and likewise in the fourth place, stand 1. and 8. But to assure you what the product of this Multiplication is, harken what the next rule sayth.

52. And when you haue finisht Multiplication,
By enery digit of the Multiplier,
Draw a new line that next operation,
May shew the product in one sum entire,

36

Which thing by addition is easily wrought,
So haue you the ofcum or product you sought.

So that by this verse you se the multiplication is perfectly finished already, But the totall thereof, which as it appeareth by this rule and also by the 33. rule is properly called ỳ product or ofcum, must be found by another operation, ỳ is to saye by addition, which I doubt not therfore, but ỳ you your selfe can perfectly performe.

Eu. That shall you sone see if it please you to byd me do it.

Philo. I pray you do it.

Eu. Then as the verse willeth me, (the numbers standing thus) I draw a new lyne vnder them as here you may see, and adding onely the figures betweene the two lines I say first 8. is 8. which I subscribe directly vnder the same 8 beneath the last lyne, Then comming to the second place I say 2. & 4. is 6. which I subscribe in the second place. Thirdly I say 7. and 6 make 13. whereof I subscribe the 3. in third place and kept the 1. in my mynd. And finally comming to the fourth place: I say 1. in my mynd and 1. is 2. and 8. is 10. Wherof I subscribe the Cipher in the fourth place, And the article 1. in the fifte place thus And so the whole product of 432. multiplied by 24. is 10368.

432	
24	
1728	
864	
432	
24	
1728	
864	
10368	

Philo. It is euen so. And where as you said euen now that because there were two sums appearing after the Multiplication ended, you could not perceiue which of them should be the product, now you see that both those parcels or numbers added together make the product, and if the Multiplier had bene a number of 3. or 4. figures or more, there would haue risen so many parcels, or perticuler sums in the multiplication, All which must haue bene added together to make the product as you shall see anon.

Euma. So I perceiue that Addition is necessarily taught, first because other operations following cannot be accomplished without it, But now I pray you giue me 2. numbers to multiplie together, that I may proue my cunning therein.

Philo. That will I doe with a good will, and because I would make one example to serue as a sufficient instruction for you in all other, I would haue you practise your cunning vpon these 2. numbers to wit 4520480. and 150620. wherein if you shall doubt of any thing, I will be at hand to resolue all your doubts, and to helpe you.

Euma

Euma. I thanke you for it. And vppon hope thereof I will set the numbers in order, as the 3 4. rule requireth and your former example taught me which is thus.

But I am at a stope all ready, for in the first place al- 150620
well of the multiplicand as of the multiplier, I find a
cypher or figure of nought, which multiplied together can make
but nought still, as it was before, wherefore except I should sub-
scribe a cypher or figure of nought in the first place, vnder the line
I wot not what I should do,

Philo. You consider it well, and you must not onely (as you say)
subscribe a cypher in the first place vnder the lyne, but also in euery
place throughout that lyne or rowe.

For as in the first place you say nought times nought maketh
nought and so subscribe 0. So likewise comming to 8. in the se-
cond place aboue, you must say nought times 8. is nought also,
For whether you say nought times 8. or 8. times nought, al is one,
& both wayes nought encreaseth thereof at any time,
and so doeing your numbers will stand thus.

Or els if you will, in all places where 0. is the mul- 00000000
tiplier, after that you haue set 0. once vnder the
line as the first product thereof, you may passe it quite ouer, and go
in hand with the next figure of the multiplier. Alwayes prouided,
that the first digit or 0. of any product made by any figure of the
Multiplier be placed right as a wyfe directly vnder that figure of
the Multiplier of whome it was produced, And the rest forwards
towards the left hand as the 3 4. verse requireth.

Eu. I pray you worke out a figure or two of this example, for
my better vnderstanding of your wordes.

Philo. I will worke out the 4 first figures of the Multiplier,
and leaue the rest for you to do. And such you see the fourme of the
worke as I left it before. I will begyn it agayne, working it now
after the order last rehearsed, That so you may perceyue my meaning
the better, wherefore newly placeing the
numbers with their lyne in order thus as you see.
First I say nought tyme nought, is nought,
which product being 0. I subscribe beneath the
lyne directly vnder 0. the first figure of the Multiplier
thus.

And now for asmuch as this first product is placed, And that

if I should proceede to Multiplie all the rest of the vpper figures by the same multiplier 0. the whole line would be but ciphers standing Idle as they did before, therfore I now omit so to doe, ouerpassing to make any further Multiplication by the same first 0. And going to worke with 2. the next figure of my multiplier, I say first 2. times 0 is 0. which 0, I subscribe directly vnder the same 2. being my Multiplier thus.

Then continuing The Multiplication of euery figure of the Multiplicand or vpper row, by this second Multiplier 2, I next say 2 times 8 is 16. wherof I subscribe 6. in the third place, and keepe the article 1. in my minde, to be carried to the next place, and so the worke standeth thus.

Thirdly I Multiplie 0. the third figure aboue, by the same 2, saying 2. times 0. is 0 & 1. in my mind maketh 1. which therefore I subscribe in the fourth place beneath the line thus.

Fourthly, I say 2. times 4. is 8. which 8. I subscribe in the fift place thus.

Fiftly I multiplie 0. the fift figure of the vpper rowe by the same 2, saying, 2. times nought is nought & so subscribe 0. the product therof in the fift place thus.

Sixtly I enfold 2. the first figure aboue by 2. my present multiplier and it maketh 4 which 4. I subscribe vnder the line in the seuenth place thus.

Seuenthly I fold 6. the seuenth figure of the Multiplicand in the same 2. saying, 2 times 6. is 12. wherof I subscribe the digit 2. in the eight place, and keepe the Article 1. in minde, to be transported to the next place thus.

Eighthly, I Multiplie 4. the last figure of the vpper row by this Multiplier 2. saying 2. times 4. is 8. which with one in my minde make 9. That 9. I therfore subscribe in the 9. place vnder the line, & so haue I wrought ouer the 2. first figures of the Multiplier and the worke is such.

Wherefore now comming to vse 8. the third figure of the Multiplier, I doe by it multiplie 0. the first figure aboue and it produceth 0. which being the first product of this Multiplier 6. must therefore

therefore (by the 51. verse) stand right vnder the same 6. in the third place as here it doth. Secondly by the same 6. I multiplie 8. the second figure aboue saying 6. times 8. is 48. Therefore I subscribe 8. in the next place toward the left hand keeping the Articles 4. in my minde and so the worke will appeare thus.

Thirdly I say 6. times 0. is 0. whereto putting the 4. kept in my minde, there is the same 4. onely to be placed in the next left hand place, which I doe accordingly thus.

Fourthly I make 6. to multiplie the fourth figure of the multiplicand, saying 6. times 4. is 24. whereof I subscribe the digit 4. in the next left hand place vnder the 0. keeping the articles 2. in my minde, and so the worke will shew thus.

Fifly I say 6. times 0. is 0. which with 2. in my minde is 2. wherefore I subscribe that 2. in the seuenth place thus, as here you may see.

Sixthly I multiplie 2. in the vpper number by my Multiplier 6. saying, 6. times 2. is 12. whereof I subscribe the digit 2. and keeping the Article 1. in my minde, I am ready to go to the next place and so the worke is thus.

Now in the next place aboue I finde 6. which multiplied by 6. my Multiplier maketh 36. and the 1. in my minde maketh 37. Wherefore I subscribe 7. in the ninth place, and keepe the 3. in minde for the next place, and so the worke is thus.

Finally I multiplie 4. the last digit of the multiplicand by this Multiplier 6. saying, 6. times 4. is 24. which with 3. in my minde make 27. Whereof (as before) I should subscribe onely the digit 7. & keepe the articles in minde, if there were any more figures of the multiplicand to be multiplied. But for asmuch as there are no mo, but that this 4. last multiplied is the very last figure thereof. Therefore now I subscribe both the digit and articles thus, as here you see.

And thus haue the 3. first figures of the multiplier done their office, and paſſed their duties in this worke. Wherefore now I will worke out the fourth also

46204080	
150620	
924081600	
46204080	
150620	
924031600	
80	
46204080	
150620	
924081600	
4480	
46204080	
150620	
924081600	
24480	
46204080	
150620	
924081600	
224480	
46204080	
150620	
924081600	
7224480	
46204080	
150620	
924081600	
27224480	

which is a 0. and so commit the rest to you.

Eu. May I pray you let me worke out the other three figures remaynyng? my selfe, for me thinkes now I can not mysse the order, you haue shewed me so playne an example.

Philo. Well if you will needs do it, go to then.

Eu. Now I remember you sayd before, that when bis the multiplier, I should not needs to multiply euery figure of the multiplicand thereby, but onely to note vnder the first product of his multiplication directly vnder it selfe. And so to passe it ouer afterward. Wherefore multiplying the first figure of the vpper number onely, which is also bis my multiplier, I say nought times nought maketh 0. which I subscribe in a new rowe of lynes directly vnder my present multiplier 0. in the fourth place thus.

462040	30
	1500 20
	9240816 00
	277224480

And so this first product of my Multiplier being placed duely I consider as you sayd that if I should multiplie all the rest of the figures of the multiplicand by the same 0. there would arouns nothing thereof but cyphers still. Which would fill an whole lyne to no purpose. But by this other way, I see that the worke is not onely abridged of those cyphers in sight, But also of some superfluous labour.

Wherefore according to your doctrine I now ought to prosecute the multiplication of all the other figures of the vpper rowe, by the same 0. For I perceiue this 0. already subscribed (as the first product therof) serueth but to fill a place, and to driue the produce of the next figure (the more orderly) one place further toward the lefthand.

Phi. It doth serue to no other vse as you say, but to retayne order for the better placing of euery particuled product in his owne place, Wherefore after that you are a little perfect in this operation you may quite ouerpasse such mylde cyphers altogether, so that you obserue the due order in placing the rest of the produces. And so may you likewise ouerpasse the first cyphers also. But with this condition, that after your multiplication is ended you alwayes in that case restore so many cyphers as were in the whole product, as you omitted both in the multiplier & Multiplicand, but do you proceed.

Euma. Well then hauing set downe the first product of my multiplier, 0. directly vnder it selfe. I passe it now quite ouer as if it had run ouer all the multiplicand. And coming to the next multi-

tiplier, I do by it multiplie euery figure of the multiplicand successiuely, beginning first with the first figure thereof which is 0. saying 5. times 0. is 0. which being the first product of this new multipiler, I set directly vnder it selfe thus.

46204080
150620
92408 1600
27722 4480
00

Secondly multiplie 8 the second figure aboue by my multiplier 5. saying 5. times 8. is 40. whereof I subscribe the 0. in the next lefthand place, and keepe the articles 4. in mynd: And so the worke standeth thus.

46204080
150620
924081600
27722 4480

Thirdly I say 5. times 0. is 0. which with 4. in my mynd maketh 4. those 4. I subscribe therefore in the next lefthand place (which is the seuenth place) And so the worke is as here you see it.

46204080
150620
924081600
277224480
4000

Fourthly I fold the fourth figure aboue (which is 4.) in 5. my multiplier and they produce 20. whereof I subscribe the 0. keeping 2. in memory thus.

46204080
150620
92408 1600 0
27722 4480
04000

Fifthly I say 5. times nought is nought, which with 2. in my mynd make 2. and so I subscribe the same 2. in the ninth place thus.

46204080
150620
924081600
277224480
204000

Sixthly I infold the 2. of the vpper nũber in 5. my multiplier, saying 3. times 2. make 10. whereof I subscribe the 0. in the next lefthand place and keepe the 1. in memory thus.

46204080
150620
924081600
277224480
204000

Seuenthly I infold the 6. of the multiplicand in 5. my multiplier, saying 5. times 6. is 30. & 1 in memory makes 31. whereof I subscribe the 1. in the next lefthand place and keepe 3. in mind and so the worke now appeareth thus.

46204080
150620
924081600
277224480
10204000

Lastly I say 5. times 4. is 20. and 3. in my mynd make 23. And because my multiplier 5. hath now fully done his office & hath no mo figures to multiplie, I subscribe this product wholly, to wit first the digit 3. And then the articles 2. one place beyond that towards my lefthand as reason willeth me, both by your example and also by the doctrine, which I learned in Addition, and Subtraction: And so the

whole worke standeth in this order

Philo. This is well done of you hetherto, but now (by the way) learne this for a principle that 1. doth neither multiplie nor diuide any nomber for if you would multiplie 3. by 1. It would be but 3. still, as it was before, for 1 time 3. is but 3. and 1. time 4. is but 4. and so of euery other number, wherefore now sith your last multiplier is 1 you shal not neede to say 1. time 0. is 0. nor 1 time 8. is 8. and so forth. But you may subscribe the whole Multiplicand as the true product of you last Multiplier, forseeing alwayes that you misplace it not, but set euery figure thereof in his due place, as I told you before.

Eu. Then if I vnderstand you so well as I thinke I doe, I must set the first 0. thereof right vnder 1. my last present Multiplier, and so euery other figure one place further towardes my left hand as here I haue done thus.

Phi. You vnderstād me very well & haue placed all the figures truly, wherefore now adde the parcels together & so make them but one product.

Euma. To do that, I draw a new line vnder all, as here you see, and comming to the first and second places, I finde nothing but ciphers, which I subscribe vnder the line in their due places, in the third place I finde onely 6. which I therefore subscribe in his proper place. And in the fourth place I find 1. & 8. which make 9. wherefore I subscribe the same 9. accordingly: and so passing still from place to place, ouer all the places, the whole product is as appereth vnder the last line, 695925852960o.

46204080
150620
9240816oo
277224480
2310204000
695925852960o

Philo. If you remember this that you haue learned, I hold you sufficiently instructed in the worke of multiplication.

Euma. Yes I doubt not thereof, but if I happen to forget anything I will forthwith repayre to your prescribed rules, and now here I remember one thing that you told me, which I cannot see appeare in this last multiplication. Which was that how many figures were in the multiplier, so many parcels or perticuler sums should rise of that multiplication. But here, although there were 6. figures in the multiplier yet is there but 4. parcels risen of the Multiplication.

Philo. The reason thereof in this Multiplication is for that I caused you to auoyd 2. lines of Ciphers in abridgement of the worke

worke, which els would haue made vp 6. parcels according to my saying, for if the worke had bene cõtinued as I began it at the first, (and as men most comonly vse it) the fourme of your Multiplication, and situation of your numbers, would haue bene such as here you see.

Euma. you haue satisfied me well: And I perceyue that both wayes bring forth one product, but the other way semeth to me both more compendious and more commendable,

Phi. Now you knowe both wayes, you may vse which you like best: Also you shall vnderstand, that there are besides these sundrie other waies of Multiplication, aswell with squars as without which if you list to learne I referre you to M. Records ground of artes, where you may finde plentie of varietie: But I minde not to trouble your memorie with no rules then I must of necessitie: Only this I haue thought good not to pretermit that if your Multiplier at any time be a number consisting of one single Article or vnit, and the rest being Ciphers, as 10. 100. 1000. 10000. and such like are, you shall neede but to adde so many Ciphers to your Multiplicand, as your Multiplier hath: And so the whole multiplication is ended, as if you would multiplie 24. by 10. because 10. hath but one 0. put you one 0. to 24. thus 240. and it is done, likewise if you would multiplie the same 24. by 100; sith 100. hath 2. Ciphers, put you 2. Ciphers to it thus, 2400: and so haue you the product thereof, And so the lyke is of all other.

Anabridgment of Multiplication.

Eu. I thanke you much for this, for it may be a great ease to me many times, besides the shortning of the worke, but I pray you what triall is there of this operation, whereby I may knowe whether my worke be well and truly done or not.

Phi. As you had a rule for the triall of Addition and Subtraction by reiecting all the nines, So is there a rule for triall of this operation much after the same sort: but as I told you then, that the best and surest triall of Addition is by Subtraction, & of Subtraction, by Addition: So I say now that the most perfect triall of Multiplication, is by Diuision, and of Diuision by Multiplication, but till you haue learned Diuision, you may vse the other rule whereof I speake which is thus.

	46204080
	150620
	00000000
	92408160
	277224480
	00000000
	231020400
	46204080
	6959258529600

The triall of Multiplication.

53. Multiplication by this meanes is tried,
First cast out all Nines, from the Multiplicand,
Setting the rest at such a crosses side,
VVhich done next take your Multiplier in hand,
From whom when all nines be cast out likewise,
Repose the rest at the other side the
Then folding both sides, from that which doth rise,
All Nines likewise, taken the remnant engrosse,
At toppe of the crosse, and there let it rest,
Till lastly all Nines cast out in like sorte,
From the whole product the remnant be prest,
Quite vnder the crosse the same to supporte,
This done if the top be now like the tayle,
The worke is wel wrought without any fayle,

37

Eu. If this be all, I doubt not but I can do it alreadie, and the the soner by that which I learned in the 24. and 30 rules, wherefore with your fauour, I will trie the worke of both the Multiplications, which you gaue me before for examples.

Philo. Go to let me see you do it.

Euma: The first multiplicand was 432. the Multiplier 24. and the whole product 10368. Now therefore begynning first with the multiplicand as the verses bid me, I find it iust 9. for 2. 3. and 4. make iust 9. wherfore the rest or remainder being nothing I set a cipher as the proper and peculier note of nothing at thone side of such a byas crosse as the verses shew me thus.

which done, next I examine the multiplier, wherin I find but onely 6. wherfore sith it amounts not to much as once 9. I take it for the remainder, to be set at the other side of the crosse thus.



And now the verses bid me fold both sides togeather, whereby I vnderstand, that I must multiplie or folde the numbers of the sides together, that is 0. by 6. wherefore 6. 9 times 0. is 0. Now from this product, I should cast all the Nines, also if there were any, but sith there is none to be taken thence, I engrosse or place the same 0. at the top of the crosse, as the verse will me thus.



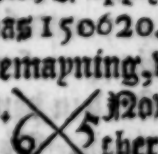
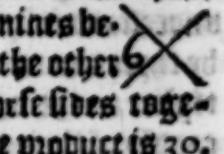
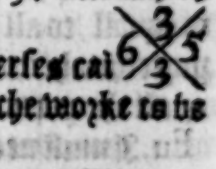
Finally I must cast out al the nines from the whole product also, which I may do twise iust, & there wil also remayne 0. which 0. I presse therefore vnder the crosse as the the rule willeth me thus.



And now the verses say, if the top be like to the taile, the worke is good without any fayle, whereby I take it is ment, that if the figure at the top or head of the crosse be like to that below, beneath it

it(which is as it were the taile of the crosse) Then the worke is wel wrought and otherwise not, wherefore sith the figures both aboue and beneath the crosse in this triall be like & all one (that is both 0) I suppose the first multiplication to be good and truly wrought.

Philo. It is so, and you conceiue all the wordes of the verses very wel & according to their true sence and meaning, wherefore now proceede to the examination of the last worke

Eu. That will I quickly do godwilling, first the Multiplicand was 46204080. Whereof all Nines being reiected I find the remaynder to be 6: which I apply to the one side of a crosse thus. And the Multiplier was 150620. whereof al the nines being abated, I finde 5. remayning, which I place at the other side of my crosse thus.  Now multiplying these sides together, to wit 6. by 5 the product is 30. from which casting out the Nines the remaynder is 3. which I set at the head or top of my crosse thus.  Finally the product was 695925852960o. From which casting all the Nines, I finde the remaynder to be 3. also which being subscribed vnder the crosse thus.  It is apparant that the head and foote (or as the verses cal it the top and tayle) are like, whereby I conclude the worke to be well wrought, and the product thereof to be true.

Philo. I cannot, but commend your promptnes and dilligence for you haue done very wel herein, In so much that I think you so perfect both in multiplication, and also in this kinde of trial thereof that we might enter into Diuision, were it not that this worke of Multiplication, (aswel as Addition and Subtraction) hath certaine speciall properties peculier thereunto, which would require not to be ouerslipt, the first whereof is thus.

The properties of Multiplication. Euclid. 16. propo. 7.

54. Two numbers multiplied which waies you will,
Their products both wayes shalbe equall still,
For 5. times 20. if you consider,
And 20. times 5. are equall together,

Eu. The effect of this verse is so manifest, by an example contcined in it selfe, that it needeth no glosse or exposition, for I can easily coniecture by the example therein proposed, that any 2. numbers multiplied together, forwardly or backwardly, bring forth all one product: and hereby I perceiue that as 4. times 0. is 0. So no times 4. is 0. whereas before, (although I sayd nothing) I doubted that no times 4. might yet be 3. 2. or 1. at the least, because neyther 1. 2. nor 3. make once or one time 4.

Philo. You vnderstand the verses rightly, and your doubt hath

some shew of reason, but now you are quite of it, I wil proceed to reherse another propriety belonging to multiplicatiō which is this

Euclid 2. booke Barlaam 1. prop. Ibidem Campa. prop. 4. in Eucl. fol. 220. b.

55. If any number be a Multiplier,
Of diuers numbers in percels proposed,
And eke of their totall, in one sum entire,
The product so by that totall disclosed
Shall countervaill all the products before,
Made by the percells though they weare a score. } 39

Euma. I promise you, I vnderstand not this verse.

Phi. It may be so, & yet I doubt not, but by an example to make you vnderstand it by and by, For let 4. 5. and 6. (whose totall is 15.) be three diuers numbers or percells proposed, And let 3. be multiplier to them all, first as they stand in percells seperated and secondly of their totall made by addition, Then you know that 3. times 4. is 12, 3. times 5. is 15. and 3. times 6. is 18. all which products conioyned make 45. Then secondly let 3. multiply the totall also of 4. 5. and 6. (that is 15.) and you shall find 3. tymes 15. to be also 45. And so the products of their totall countervales (or is equall) to all the products made of the percells as the verse imporeth: which is asmuch to say, as to multiply any whole number, or all the parts thereof by one Multiplier, is both one thing.

Eu. I must needes confesse now, that the matter is clere innought to vnderstand, wherefore I pray you go on with the rest.

Philo. Another propriete is thus.

Flussas in a corollary to the 17 prop. 7. Euclid.

56. If the 2. products of any 2. fractions,
Produced by byas Multiplication,
Be equall together, then so are those fractions,
Both equall, and both of like estimation. } 40

This verse sheweth a certayne propriete of Multiplication which is incident aswel to Integers (in some respect) as to fractions, as you shal learne hereafter, when you come to proportionalls, wherefore I thought good, not to ouerpasse it: But now to explaine the verse by an example, I propose these 2. fractions with a crosse betweene them vz. $\frac{2}{4} \times \frac{3}{6}$ And now if I multiply their termes biasly that is to say 2. by 6. and 4. by 3. as the byas crosse poynteth the products willbe 12 and 12 wherefore seeing their products or ocomes are equall I affirme the same fractions to be equal, and both of like valure or estimation.

Eu. This is not difficult to vnderstand.

Philo. Then I proceede thus.

57. Any

Salignac. lib. 1. ca. 4. verse. 6.

57. Any 3. numbers vnequall assignd,
Causeth the product produced of their meane,
By totall of both the odds which you find,
Betweene the same meane and eyther extreame,
To counteruaile both the products proceeding,
Of each extreame by other ods breeding. } 41

Euma. This verse passeth all the rest in obscuritie, wherefore I pray you expound it by some example, for otherwise I doubt I shall neuer finde the meaning of it.

Philo. I cannot greatly blame you, for this verse is misticall, and of more importance then you perhaps presently would thinke, and yet easie inough to be vnderstood, and conceyued as by an example will appeare, the sence therefore of it is thus.

If any 3. vnequall numbers be proposed, they haue this propertie, that the product of their meane number, by the total of both the ods or differences, whereby the extreames differ from the same meane, counteruayles both the products made of each extreame by his fellowes differance or ods: As for example let 10. 15. and 16. be their vnequall numbers: Now the ods or differance betweene 10. (the lesser extreame) and 15. the meane or mid number is 5. which ods I set vnder 16. the greater extreame, then the ods or difference betweene the same 16. the greater extreame, and 15 the meane number is but 1. which one I set vnder 10. (the lesser extreame) And then the totall of the differences or ods of both the extreames added together is 5+1. that is 6. which I set vnder the meane or middle number it selfe and so the 3. numbers with their ods or differences stand in this fourme.

10	15	16
1	6	5

Although here 15. stands in the mids, yet is it not accounted to be the meane or midnumber therefore: But in respect of quantity because it is more then 10. and lesser then 16.

Now I say that the product of 15. by 6. is equall to both the products made of 10. by 1. and of 16. by 5. for demonstration wherof, first let 6. multiplie 15. & the product shalbe 90. Then secondly let 1. multiplie 10. & the product (by the glosse of the 52. verse) shallbe but 10. still: Finally, let 5. multiplie 16: and the product shalbe 80. Now put this 80. produced of 16. the greater extreame by 5. the others ods, to 10. the product of the lesser extreame by his fellowes ods, and they wil make iust 90. also, equall to the product made of 6. (the totall of both the ods, by 15. the meane number as the verse importeth and here appeareth.

10	15	16
1	6	5
10.	90	80
	90	

Here 10. and 16. are Extreames by quantity, and not by place: although here they stad in the extreme places.

And although the weight of this verse cannot

as yet appeare vnto you, yet I assure you it is the chiefe, or rather the onely ground of the Rule of Alligation, for the inuention whereof Archimedes receyueth (and that worthely) so great fame and comendation.

Eu. Very well I doubt not now, but I vnderstand the meaning of the verse, but the full fruite thereof you say I cannot as yet reache vnto, neuertheless trusting that I shall attayne it before it be long I will in the meane season liue in hope.

Philo. Another propertie of Multiplication is thus.

Euclid. 19 prop. 7. Boetius lib. 2. ca. 44.

58. If 4 numbers make the ofcum of their meanes,
Iustly coequall to the ofcum growne,
Betweene the 2. other called the extremes,
Then are they 4. true proportionalls knowen

This verse geueth you a very sure token howe to knowe numbers proportionall, for example whereof let, 9. 3. 6. 2, be fower numbers whose meanes or myddle numbers are 3. and 6. and the extreames are 9 and 2. Now it is manifest that the product of 3. by 6. is 18. and so is the product also of 9. by 2, wherefore these 2. products being equall, I affirme thereuppon by this rule that the geuen numbers 9. 3. 6. 2. are 4. true proportionals. And therefore as 9. is to 3. so is 6. to 2. that is to saye, as 9. is thrice 3. so 6. is thrice 2.

Euma: That appeareth also to be so, by the glosse of the 33 rule where you willed me to remember not onely the name of proportionalles but also the cause of their name, howbeit I remember not that you gaue so playne and easie a rule any where as this, whereby to knowe such numbers proportionalls

Philo I commend your carefullnes in remembring the names as I did bid you: But to geue any rule in this behalfe before now, was not needefull, And yet if you vnderstood the 40. rule so well as there you seemed to do (where you confessed it was not difficult to vnderstand: And could apply it so farre, as it may be well extended, you might make as easie a triall of proportionalls thereby, as by this rule present, which thing the glosse there doth partly mention.

For if you hauing any 4. numbers to write them by cooples one vnder another, as the termes of the fractions in that glosse are described: And multiplie the byas numbers together, as I there did multiplie the 4. termes of the 2. fractions: Then if their 2. products be equall, it is as good a triall that suche 4. numbers are proportionalls, as it was there, that those 2. fractions were equall.

And therefore for your more easie knowledge of proportionalls, you shall haue this present rule once a gaine repeted I say the selfe same

rule

rule in effect, though changed in fourme of wordes, and it is thus.

59. If any 2. coople of numbers be found,
Placed directly each vnder other,
VVhose products byasly made do rebound,
Equall together the one to the other.
Then be well assured and haue no mistrust,
Such 4. numbers are proportionals iust. } 40

A collorary to the same 19. propo. 7. added by Flussates.

Eu. I pray you let me trie the former proportionals by this rule.

Philo. With a good will, I am content.

Eu. Then to obserue the wordes of this rule, I place the numbers by coples, one vnder the other like fractions, & then make a byas crosse betweene them as you did betweene the fractions thus: and now I go no soner about to multiplie ỹ byas nũbers, viz. 9. by 2. and 6. by 3. But I foresee presently that both the products wilbe 18. & so are equall as appeareth in the fabrique, wherefore these numbers are doubly confirmed to be proportionals.

$\frac{9}{3} \times \frac{6}{2}$ $\frac{}{18}$ $\frac{}{18}$

Philo. You haue done well inough, sauing in one thing (whereof I should haue told you,) & that is this. For asmuch as these numbers represent proportionals, and not fractions, you should eyther haue made no line at all betweene them, or els you should haue made twaine thus.

$\frac{9}{3} \times \frac{6}{2}$

That so they may not onely differ from fractions, but also be certainly knowne to signifie proportions & proportionals.

And so doing, you may see now, that the first terme or number aboue, and the last beneath, euen as they are the first and last in making, so are they the 2. extreames, and the other 2. are the middle tearmes or 2. meanes, euen as they were before.

And because I esteeme this representation of proportions and proportionals, to be the aptest fourme of their notation, and description, therefore hereafter I purpose most generally to vse it, wherefore these numbers $\frac{9}{3}$ and $\frac{6}{2}$ being thus expressed, are thus to to be pronounced 9. to 3. and 6. to 2. and so the like of others.

Eu. Very good all this I trust henceforth will I remember.

Phi. Then let vs proceed to ỹ nexte propertie of Multiplication

60. If any one number do multiplie twaine, (which is thus.
The products insuing such Multiplication,
Shall in proportion alwayes remayne,
Like to the first coople without alteration,
VVherefore of fiue force those 4 numbers must,
By consequence be proportionals iust. } 41

As for example, let this number 3. multiplie these 2. nũbers 4. & 5.

And let the products of such multiplication, be 12. and 15. As here you see they are.

Now this rule sayth, that the same 12. and 15. are in like proportion and rate together, as 4. is to 5. without any deference or alteration, wherby it foloweth that the same 4. 5. 12. and 15. are 4. true proportionalls: For trial wherof, you may vse the practise of the last rule before.

4	5
3	3
12	15

Eu. That wil I quickly do, for letting the numbers stand euen as they do, I will but drawe a byas crosse betweene them, and so multiply together the byas nũbers which the same crosse coupleth that is 4. by 15. and 5. by 12. And both wayes the product is 60. As here appereth, Wherefore I conclude the same 4. numbers to be truely proportionalles, And that as $\frac{4}{5}$ so is $\frac{12}{15}$. in all respects of proportion.

4		5
3		3
12		15
60		60

Philo. These proportionalles are well expressed by you, And now although there be many other properties, rising by multiplication very necessarie to be known, Yet because they depend aswell vppon Diuision as multiplication, And so growe as it were by the mixture of both operations, therfore I will deferre them, till after I haue taught you diuision, And so closing vp Multiplication, with that which hath bene sayd, we will now presently go in hand with Diuision,

Of Diuision the fourth opperation Arithmeticall. Chapter 7.

The effice and dutie of diuision.

61. Diuision doth search how oft the diuisor,
In Diuidend may be quoted or found
Whereof the quotient is the decidor,
VVhich shewing how oft the same doth redound
Is to the Diuidend ratable euen,
As 1. is to the diuisor so geuen. } 15

Hereby you may gather that the office of Diuision, is to search how often, one number (which in this worke is named the Diuisor) is contayned in another which is called the Diuidend, As if you should aske me how often, or how many times 4. is in 8. I would answere you twise or 2. times.

Now here 8. is the diuidend (that is the number diuided) And 4. is the diuisor or number parting and diuiding the same 8. And 2. which sheweth the number of tymes, is called the quotient, which quotient by this verse, must allwayes be in such rate & proportion to the diuidend, as 1. is to thee geuen diuisor, Or else it is no true quo-

tient, whereof it followeth that when we diuide one number, by another, we do therby onely seeke such a third nũber, as may be in like rate and proportion to the diuidend, as 1. is to the diuisor, which when we haue found we call it the quotient.

The intent and effect of Diuision

As for example in the late diuision, when 8. was diuided by 4. the quotient found was 2. wherefore now I say, that as 1. is to the diuisor 4. So is the same quotient 2. to the diuidend 8. for as 1. is ¼ of 4. so is 2. likewise ¼ of 8.

The quotient qd.

Eu. That is very true, wherefore by the 42. and 43. rules before the same 4. termes 1. 4. 2. and 8. are 4. proportionals.

Philo. You haue sayd truth: And therefore like as in multiplication you sawe, that the product, with the multiplicand, multiplier & 1. made alwayes 4. proportionals as there was told you, so likewise here you see in diuision that the diuidend, diuisor, quotient and 1. make euermore 4. proportionalls also: For which causes some men name Multiplication and Diuision, operations by proportion, as workes, whose rules, lawes, and proofes, are grounded & consist, in a certaine proportionalitie.

Euma. So then I perceiue there are 3. notable numbers in euery diuision, whereof the first is named the diuidend, which is the nũber to be diuided, and therefore seemeth alwayes to be the greater, the second is named the diuisor, which is it whereby the others is diuided, and therefore seemeth alwayes to be the lesser: and these 2. numbers be geuen, & the third which issueth by the working of diuision is called the quotient, which is it that answereth to the question, how many times, and is alwaies sought for or required.

And in this operation you say, that the 2. geuen numbers, with the quotient found, and 1. must make 4. proportionals or els the quotient found is not true.

Phi. That is most certaine: and touching the names of the nũbers the diuidend (as you haue sayd) seemeth generally to be the greater number, because it cannot in reason (to your understanding) be demaunded how often the greater, is contayned in the lesser: but yet you shal vnderstand hereafter, that the greater number may diuide the lesse, aswell as the lesse the greater: Marry then the quotient of euery such Diuision will alwayes be a fraction and no true number, but let vs growe towardes the practise.

62. Diuision begins first at the left hand,
Contrarie to all the former workes wrought,
Wherefore on that side both numbers must stand,
Agreeing directly or els they stand nought,

The ordering of the numbers in Diuision.

Saue when the diuisor exceedes that aboue it,
For then a place forward you ought to remoue it.
63. Diuision doth place the Diuident first,
And then the Diuisor right vnderneath it,
Thirdly the quotient (the thing which we thirst)
Issueth by worke ing as you shall perceyue it,
Which done the remayner, if any thing rest,
Must helpe make a fraction, and so be expresst. } 46

This rule sheweth you how the working of diuision beginneth at the lefthand, & so proceedeth towards the right, contrarie to the working of Addition, Subtraction, and Multiplication, and therfore, that the giuen numbers in this worke, must stand, or be set agreable directly at the left side, except it chance so that when the diuisor is so placed it doth exceede the number of the diuidend directly aboue it, for then the diuisor must be set one place more forwards towards the right hand, (which (in this worke) I wil from henceforth cal the second place: For sith al the working hereof beginneth at the lefthand, and so groweth towards the right, I will (for your more easie vnderstanding of that which shalbe said hereafter) cal ẏ place the first, & that figure first, which is next the lefthand, & that next to it, the second, & so each other in that order, although indeed ẏ places be quite cõtrary by nature, & the value of them farre vnlike but notwithstanding whilst you are working in any place, you know by ẏ rule that you must repute that to be the first place for ẏ time. Moreouer this maxim sheweth you that the diuidend, must be placed first and vppermost, & the diuisor right vnderneath it, and so the quotient being the number for which we seek, and the thing which we do as it were thirst to know, issueth by the operation, & finally ẏ when diuisiõ is finished, if any part of ẏ diuidend remain, then ẏ same must help to make a fractiõ, & so must be expressed in fractiõ forme.

And although al things in the rule are so manifestly explained ẏ you cannot lightly misunderstand any thing, yet for your better instruction touching ẏ placing of ẏ numbers according to the meaning of the Rule, I propound 568. to be diuided by 4. which according to this rule must be placed thus $\frac{568}{\ }$. But if I would diuide the same 563. by 6. then must I set the 4. numbers thus. $\frac{563}{\ }$. because if I should set them thus. $\frac{563}{\ }$. I could not subtract 6. from 5. ẏ number aboue it. But in setting the figurs as before, I may subtract 6. from 56. which number is there adiudged to be ouer 5. for like as in ẏ notation or Numeration when you were taught to pronounce anũber by pricking it, you tooke not the bare digit only vnder the

pricke

pricke to belong to that pricke, but also all suche other figures as were beyond it towards the lefthand, so in this worke, those figures of the diuidend, which stand aboue the deuison beyond it towards the lefthand, are aswell accompted to be aboue the deuisor, as the rest that stand directly plumbe ouer it, but so are not those that stand beyond it towards the right hand.

Also, if I would diuide the same 568. by 58 I must place the numbers thus. 568 / 58

For if I should set them thus. 58 / 568 the diuisor as you see will exceed the numbers 56. aboue it, contrarie to the perscript of this rule.

Eu. All this is exceeding plaine, but this one thing, maketh me think my selfe decepued before, for that it seemeth now you make 4. numbers belonging to Diuison, whereas before I supposed there were but 3. For besides the diuidend Diuisor and quotient, both the rule and you also talke of a Remayner. Remayner

Philo. You were not deceiued before. but rather now, for the remayner is no fourth number, nor any other thing, but onely a perell of the Diuidend, lesse then the deuisor, (least after the deuision is ended) for which cause it being diuided by ỹ diuisor. cannot bring fourth a quotient in whole number (as I told you in the glosse of the former rule) neuertheless it may not therefore be omitted or neglected, but must be expres in forme of a fraction, and annexed to the quotient as a part thereof, which maketh the same perfect and complete, but alwayes when the diuisor diuideth the diuidend iustly as it did in the example which I propounded of 8. diuided by 4. then is there no remayner at all.

Eu. Then if you should aske me, how many times 3. is in 16. I could not saye 6. tymes for 6. tymes 3. is 18. And to say but 5. tymes absolutely, seemeth too little, because it conteyneth 1. ouer and besides the 5. tymes wherefore I thinke I must say 5. tymes besides 1. remayning (for you say the remayner must not be omitted) And so here the Diuidend is 16. the diuisor. 3. The quotient 5. and the remayner 1.

Philo. You haue declared your selfe to vnderstand all my wordes very well, and now this 1. remayning (as the rule importeth) & as I told you before must helpe to make a fraction, and so be adioyned to the quotient to make it perfect, which thing how you shall do, I will shew you hereafter.

And here (by the way) you may easily see. that if you had taken, but 15. or els 18. to your diuidend. as you tooke 16. then had there been no remayner at all, because 3. would haue diuided either of them

both iustly and euenly, that nothing would haue left remayning

Eu. That is very true, But now I thinke it long, till I know to what vse Diuision serueth, and also to se the practise of it.

Phi. Diuision serueth in stead of many subtractions, for euery diuision is but the subtracting of the diuisor from the diuidend, as often as it is conteyned therin, which is ỳ cause ỳ when ỳ diuisor exceedeth the figures of the diuidend aboue it, so that it cannot be subtracted from the same: then it must be set forward one place as you heard euen now, whereby it may then be subtracted, not from such figures of the diuidend as stand perpendicularly ouer it onely, but from them, with the other lefthand figures ioyntly.

The practise of Diuision.

Also Diuision serueth to many other needefull vses as your selfe will confesse hereafter in the meane season the rules of the practise of it are these.

64. The diuidend and diuisor once plac'd,
Account the first place where first you begin,
But neuertheless begin with the last,
Of all the diuisor thus dealing therein,
Aske how many the same is depending,
In th'ouer number directly ascending.

65. And if it do serue to answere you twice,
Or mo times or lesse, as hapneth to rise,
The number of times set downe at a trice,
Inclosd in a crooke, for so is the guise,
To place the quotient which loueth to lurke,
In such a crooke at right hand of the worke.

66. Then multiplie next the quotient thus found,
By your diuisor and looke what amountes,
Subtract it out of the diuident round,
Defacing th'old figures but still keepe accounts,
That the remayner may rightly be plac'd,
Aboue the figures which earst were defac'd.

67. Now if the diuisor consist in this case,
But of one diget with Ciphers or none,
It may be remou'd forth forward one place.
After each quotient once quoted and knowne,
At each time asking (how many times newly,)
And doing agayne all former workes duly.

47

Here you see that in this operation of Diuision, when the numbers be duly placed, you ought to begin at the last place of the diuisor

uisor, & yet esteeming it as if it were the first (for the time) to aske how many times the figure there standing is contayned, in the ouer number directly ascending.

And as it serueth to answere you, be it twice, thrice or more or lesse, so to set downe the number of times, in the quotient line or crooke, at the right side of the worke: As for example I will take the numbers before propounded, and will deuide 568. by 4. which therefore I place working to the 46. rule which is thus.

568
4

Now beginning with my diuisor 4. which (sith it consisteth but of one digit) is both the first and last figure of my diuisor I aske how many times 4. in 5. the ouer number, and it answereth me but once, wherefore I set downe 1. (which is the number of times) at the right side of my worke within the quotient line or crooke thus.

568 (1
4

Now the 66. verse of this rule biddeth me multiplie this quotient found, by my Diuisor, and to subtract the product thereof, from out of the deuidend. And then to deface thold figurs that is to saye, aswell subtracted as from whome subtraction was made, & so to place the remayner, aboue the figures defaced in their due places wherfore now to performe the same verse I say 1. tyme 4. is 4. which I subtract from 5. (ỳ figure ouer it) and the remayner is 1. Therefore I deface both 5. and 4. and place the 1. remayning ouer it as ỳ verse willeth me thus.

1
568 (1
4

And thus my diuisor hauing done his office in one place (sith it consisteth but of one digit) may by the last verse of this rule, & must by order of worke be remoued forward to ỳ next place, after the finding and quoting of this quotient so after euery other, wherefore to doe accordingly I remoue it vnder 6. the next figure of the diuidend thus.

1
568 (1
44

And now asking how many times newly, I say how many times 4. (not in 6. but) in 16. and it answereth me 4. times,

1
568 (14
44

wherefore now doing all former works duly, I write 4. the number of times) within the quotient line or crooke before the 1. there thus.

And next multiplying this quotient by my diuisor, I say 4. times 4. is 16. which taken out of the 16. aboue it leaueth 0.

10
568 (14
44

remayning, wherefore I defaced both the 16. aboue, & the 4. beneath superscribing the remainer 0. thus.

Agayne imediately after this second quotient, is found and placed, and the multiplication and subtraction thereto belonging per-

fayned I remoue my diuisor as I ought yet one place further forward: that is to say, vnder 8. the last figure of my diuidend (as here I accompt it) and so the worke standeth thus.

568 (14
444

And now newly asking the question agayne, how many times 4. in 8. it answereth 2. times, wherfore I write 2. in the quotient with his fellow figures there, and then do all things agayne as before, that is to say, I multiplie the quotient 2. last found by 4. my diuisor, and subtract the product from the diuidend, saying, two times 4 is 8. which I take out of the 8. ouer it, and there remayneth 0. wherefore I deface both the olde Cipher, and the 8 and 4. setting this last 0. now remayning ouer the 8 as I ought, and so the diuision being fully ended standeth thus.

568 (142
444

And so the whole quotient of this diuision is 142 whereby I know and affirme that 4. is contayned 142. times in 568.

Exception

Eu. Now I pray you set me an example to worke out my selfe.

Phi. May soft this operation, hath another exception annexed therto, then you haue yet heard: for if you marke it, this last rule teacheth the Diuision of numbers but by one digit or figure onely, whether it haue no Ciphers as these 1.2.3.4.5.6.7.8.9. or haue any Ciphers as these 10.20.30.400.500. and such like. But if the diuisor contayne 2. digits or mo, whether it be an article number, or a mixt nũber, this rule wil not serue nor hold in that point, wherefore you were best to here the rule in that behalfe, and see the order of the worke by an example: and then you may be bold to practise at pleasure without danger of error.

Euma. I am contented to stay and follow your aduise.

Philo. Then this is the rule.

68. But if the diuisor be plurally digits,
With Ciphers or not you must so deuise,
In chusing each quotient, to keepe within limits,
That the remayner may alwayes suffice,
To spare you each vnder digit like oft,
As you tooke the first from numbers aloft,
69. And when the diuisor remoued forth duly,
Exceedes the remayner or sum right aboue it,
If asking the question how many times newly,
It answeres no time, then care you remoue it,
Into the quotient a Cipher downe clappe,
And do as often as like case shall happe.
70. Finally when the diuisor hath past,

48

By

By one place at once euen vnto the end,
Doing his office, so that now at last,
It endeth directly with the diuidend,
Then set the remayner where it may be seene,
Aboue the diuisor with a line betweene.

There is no diuision to be wrought in integers, what number soeuer, be either diuidend or diuisor, but it may be performed either by this rule or by the last, wherefore supposing you vnderstand the former rule and example wel inough, I will without spending moe wordes shew the effect of this rule by an example, which shalbe this. I wil diuide 8206926. by 273. wherefore after that 8206926(I haue setled the numbers as they ought, and made 273(my quotient line thus. I begin with 2. the first figure of my diuisor (as I accompt it in this worke) saying how many times 2. in 8. & I find it 4. times iust, but because my diuisor is not one digit, but plurally digits, before I enter this 4 into my quotient, I foresee whether the remainer by I should leaue wilbe sufficient, to yeeld me euery other vnder digit there (that is euery digit of the diuisor) so oft as I take this first. And I perceiue that it will not, for after I haue taken 4. times 2. that is 8. out of the 8 aboue it, there will remayne ouer the deuisor but 20. wherefore I see that 4. is to great a quotient for me, and therefore abating an vnit thereof, I chuse 3. attempting how that will serue, saying 3. times 2. is 6. which subtracted from the 8. of the diuidend, leaueth 2. remayning: And now I may perceiue that the remayner ouer my diuisor wilbe 220. which wilbe sufficient to yeeld me each other figure of my diuisor, as often as I take this first 2. (that is to say 3. times) wherefore I boldly enter the sayd 3. as an apt and meete 8206926(3
quotient within my crooked quotient line thus 273(3
And now I multiplie euery figure of my diuisor by this quotient 3. saying: first 3. times 2. is 6. which I take out of 8. and there remayneth 2. therefore I dash the 8. & 2. and directly ouer the same 8. & 2. I set downe the 2. remayning thus. Secondly 2
I say 3. times 7. is 21. which I take out the 22. a- 8206926(3
boue it, and there resteth 1. wherefore I deface 273
the 22. and also the 7. and set downe the 1 remayning ouer 7. the fi- 21
gure of my diuisor presently multiplied thus. Thirdly I say 3. times 3. is 9 which I take 8206926(3 21
from the 10. aboue it, & there remai- 273
neth 1. to be set ouer 3. aboue the 0. where I set it 8206926(3
accordingly and deface the old figures thus. 273

And now my Diuisor hauing done his office, once and yet not being come next the end of my diuidend, I remoue it forward one place, as I ought seeking a new quotient with the former cancell, and so the worke standeth as here you see.

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zr1
82o6926 (3
2733
27
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And now my Diuisor being remoued forth duly, it appeareth euidently that it exceedes the remaynder or sum aboue it, for it is manifest that there is but 16. ouer my Diuisor, as appeareth by my notes on the 46. rule, wherefore if I aske how many times 2. (the first figure of my Diuisor) is in the number aboue it, it answeres no time, for there is no nnmber there, but such as are defaced, and therefore are voyd: For which cause I enter 0. into the quotient as this rule willeth me, cancelling my Diuisor wholy, but because there is nothing abated from the nũbers aboue they must remayne as they did, and so the worke appeareth in this sorte.

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zr1
82o6926 (30
2733
27
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Thus my Diuisor hauing done his dutie twise orderly, and yet not come to end leuell with the Diuidend I remoue it agayne one place forward thus.

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zr1
82o6926 (30
27333
277
2
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And here as before, I see that my Diuisor doth exceede the remainer or number ouer it agayne, so that if I say how many times 2. in 1. it answereth me, no time, wherefore I eftsones clap downe another 0. into the quotient & deface my diuisor, not yet medling with the vpper figures, because I haue no occasion to alter any of them, for you know if I should Multiplie euery figure of my Diuisor, by any of these Cyphers, the product would be but 0. And so 0. being subtracted the selfe same remainer must needes continue wholy.

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zr1
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Eu. That appeareth well by the 38. Parim.

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82o6926 (300
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Philo. Well then to returne to the worke which now standeth thus: I remoue my diuisor yet one place further towardes the right hand and so the worke is thus.

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zr1
82o6926 (300
27333
277
2
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And now asking how many times 2. in 16. it answeres iust 8. times, but then I consider that I cannot take 7. the next figure of my Diuisor, 8. times out of 9. wherefore bating an ace I take a lesser quotient by 1. which is 7. And then I see that 7. times 2, wilbe 14. which taken out of 16. will leaue 2. which 2. with

with the 9. following wilbe 29. But yet I see that I cannot take 7. times 7. out of 29, wherefore yet abating another ace, I take a lesse quotient by 1. which is 6. & making proofe what remayneth that will leaue, I finde it will leaue 492. which wilbe sufficient, to spare me euery digit of my Diuisor, as often as I take this first 2. wherefore I am bolde to enter this 6. into my quotient thus.

And so multiplying euery figure of my Diuisor by this quotient 6. I say, first 6. times 2. is 12, which subtracted from 16. leaueth 4. remaining, so I cancell the 16. aboue, and the 2. beneath, and note the 4. which remaineth right ouer the same 2. thus.

Secondly I say, 6. times 7. is 42. which I abate from 49. and there remaineth 7. wherefore I deface both the 49. aboue, and the 7. belowe, and ouer the same 7. I note 7. the remaynder thus.

Thirdly I say, 6. times 3. is 18. which if I should subtract from so great a number as 72. is, would be too troublous. Againe, if I should accompt the 2. for 20. more then it is as the 40. verse teacheth, yet it would be but 12. which were too little to let 18. be abated from it. Wherefore I will (for your example to do the like when occasion shall serue) rate it, for so many times more then it is, as shall serue my turne presently. That is, I will accompt the same 2. for 20. more then it is worth, and so shall it be 22. from which I may abate my 18. and there will remaine 4. wherefore I deface the 2. aboue, and note the same 4. ouer it. And because I borrowed these 2. tens thus in that place, I must by the 41. verse repay two for it againe silently in the next place beneath, saying 2. out of 7. leaueth 5. and so dashing the 7. also, I write downe the 5. remaining ouer it, which done, I deface my diuisor 3. And so the fourme of the workes is thus.

And now I finally set forth my Diuisor once more, which is the last time that I can remoue it, and so it standeth thus. And heere asking how many times 2. in 5. it answereth,

twice and 1. remayning, which 1. with the rest of the diuidend, I perceiue will be sufficient to spare euery other figure of my diuisor twice also wherefore I take that 2. and enter it within the quotient line as a 2. quotiente, and so the worke standeth thus.

And then multiplying euery figure of my diuisor by this quotient, I say first 2. times 2. is 4. which I take out of 5. and 1. remayneth, wherefore I deface the 2. beneath, and the 5. aboue, and set the 1. remaining ouer the head thereof thus.

Secondly I say 2. times 7. is 14. which I take out of the 14. aboue it, & there resteth 0. wherefore I cancell both the 14. aboue & 7. beneath, & note the 0. ouer it thus.

Lastly I say 2. times 3. is 6. which I abate from the 6. aboue it and there resteth 0. wherefore I cancell both the 6. and the 3. and superscribe the 0. and so haue I done, for my diuisor hath passed orderly from place to place by one place at once throughout the Diuidend, till it hath now ended directly with the same thus.

And the quotient of all the Diuision is 30062. as appeareth, wherefore I dare affirme that so often the diuisor is comprehended in the diuidend.

Eu. The first forme of Diuision which you taught & wrought by the 47. rule which was very easie, and I nothing doubt, but that through the sight of your example, and by recording of the verse I cannot do the like, but this second forme doth so occupie the memory, and is so nice in choyse of the quotient, that it seemeth to me, farre more difficult then all the former operations: And in it, is vsed & practised both multiplication & also Subtraction, of both which, the Subtraction, as you worke it) seemeth most obscure.

Phi. Indeed Diuision is esteemed one of the busiest operations of Arithmetick, and such as requireth a mynde not wandering, or setled vppon other matters, but whereas you say that Multiplication and Subtraction are both practised in the worke thereof, me thinkes the same ought to seeme so much the more easie to you because you hauing learned these 2. partes already, are not, or

ought

ought not to be ignorant, or to seeke in the working of them, and as for the obscuritie of my Subtraction, if you remember the 4 1. verse, you shall finde it as light and easie, to rate any digit, 2. tennes 3. tennes, 4. tennes, or 5. tennes more then it is, as to rate it but one 10. more then it is, so that (as the same verse biddeth you) for euery tenne which any such vpper digit so borroweth for the encrease of his credit and estimation, you repay an ace or 1. to the next place beneath, by placing the same 1. there in minde onely, and so subtracting it from the ouer number, aswell as if the same were expressly subscribed there: So that if any figure of the Diuidend, be rated at 20, 30, 40, 50, or 60. &c. more then it is, then must I repay to the next figure of the Diuisor, 2. 3. 4. 5. or 6. vnits or aces or no as the case requireth, as for example, I will diuide this number 8318. by 297. wherefore I place first the numbers 8318 (
geuen as they ought thus, and make my quotient 297
line also as here you see.

This done I aske how many times 2. in 8. and it answereth 4. times, but then I consider that I cannot take 9. the next figure of my Diuisor 4. times out of the remayner 3. wherefore I refuse 4. as an vnmeete quotiente and make a triall how 3. (that is 1. lesse) will serue, and I consider that 3. times 2. wilbe 6. which subtracted from 8. will leaue 2. which 2. with the 3. following wilbe 23. But yet I see I cannot abate 3. times 9. from 23. wherefore I yet take a lesser quotient by 1. that is to say 2. which 2. I set within my quotient line, as an apt and true quotient, and such a one as will leaue a sufficient remainder. for the rest of 8318 (2
my Diuisor thus. 297

And now I go to multiplying of euery figure of my Diuisor by this quotient saying first 2. times 2. is 4. which abated from 8. leaueth 4. also remayning, wherefore defacing the 8 and 2. I note downe the 4. remayning ouer the head thereof thus. 4

Secondly I say, 2. times 9 is 18. which I take 8318 (2
out of 23. (for I esteeme 2. tennes of the 4. last placed, sufficient for my purpose in this behalfe) and there remayneth 297
5. (besides 2. in my minde:) Therefore I write the 5. remayning ouer the 9. and cancell the 9. and 3. and because I haue 2. in my minde to be repayd in the next place of my Diuisor which is vnder the 2. already dashed, I say 2. in minde out of 4. leaueth 4 5
2. and so I deface the 4. and note the 2. ouer it thus. 45

Thirdly I saye 2. times 7. is 14. which 8318 (2
I take out of 21. (so taking the 1. present 297

and borrowing 2. tens of the 10.) and there remaines 7. besides 2. to be kept in minde. Wherefore I vaile out the 1. and put the 7. remaining ouer it, and then going to the next place towards my left hand (as you know I must doe in Subtraction,) I say 2. in minde out of 5. leaueth 3. wherefore I deface the 5. there, and set the 3. remaining ouer the head in his stead, and so the worke standeth thus.

457
8318 (2
2977
23

And now hauing past ouer my Diuisor once, I am to remoue it one place more forwarde, as the verses teach, which thing I do thus.

Euma. I beseech you let me worke out the rest of this diuision.

457
8318 (2
2977
23

Philo. With a good will.

Euma. Then first I aske how many times 2. in 23. and I finde the Dexterous answereth 11. times, and 1. remaining. But I feare I may not take so great a quotient, wherefore I will take it once lesser, that is to say but 10. times.

Phi. Nay, stay, you are out of the way alreadie, for you must neuer put aboue one figure into the Quotient at one time, whereby you may easily coniecture that your Quotient may not exceede a Digit, whereof the greatest is but 9. as you know.

Euma. Then I perceiue that I may take 9. or vnder for my quotient, but aboue 9. I may neuer passe: wherefore prouing how 9. will serue. I say, 9. times 2. is 18. which abated from 23. leaueth 5. which, with the 7. following will make 57. but then 9. times 9. will be 81. which I cannot subtract from 57. wherefore yet I must fall an ace lower, and so taking but 8. for my quotient, I proue how that will serue, and I finde it will leaue 77. remaining, which with the 8. following will suffice to spare all the figures of the Diuisor 8 times also, wherfore entring 8. as an apt and true quotient, I next multiplie it by euery figure of the Diuisor, saying, first 8. times 2. is 16. which I deuet out of the 23. aboue, and there resteth 7. which I note downe right ouer the 2. which I multiplyed aboue the 3. and deface both the 23. aboue, and also the 2. beneath, and so the worke standeth thus.

457
8318 (28
2977
29

Secondly I say, 8 times 9. is 72. which I take out of 77. and there remaineth 5. which 5. I set directly ouer the 9. multiplyed, aboue the 7. and so deface both the 9. beneath, and the 77. aboue, and then

the

the worke appeareth thus.

Lastly I say, 8 times 7. is 56. which abated from 58. leaueth 2. remaining, wherefore I deface all the figures now quite, and write the 2. onely, remaining ouer the figure of my Diuisor presently multiplyed, thus as heere you see. And so it appeareth that the Diuident containeth the Diuisor 28. times. And that there remaineth 2. ouer and besides, which remainer the verses bid me, Set where it may be seene, aboue my diuisor, with a line betweene.

Phi. The meaning thereof is no more but that you must make a fraction of the remainer and the Diuisor, whereof the remainer must be the top terme or Numerator, and the Diuisor must be the balle or denominator, and so that fraction must be adioyned to the quotient, as a perfect ther-of, as I haue heere set it.

Eu. So I should haue vnderstood the verses to signifie, but yet I am glad you haue expounded their meaning so plainly; and hereby I perceiue the quotient is not barely 28. but it is 28 2/7.

Phi. You say truth, and therefore now sithe you haue learned the general rules of diuision, & haue seene the practise thereof by some examples, I would willingly see you worke one example thereof your selfe: but because you shall haue new occasion thereunto ministred in Reduction, I will not staye about it at this time, but will shew you one other way of diuision, no lesse certaine then easie, especially to be vsed in great numbers, wherein the finding of the quotient, with the multiplication of the Diuisor, and the Subtraction thereof would be very tedious, and as the forme of this Diuision varyeth from the common ordinarye forme before taught, which is most generally vsed, so vnoth the rules thereof varie likewise, and are thus.

Another kinde of Diuision.

71. First make a long sword in what place thou wilt,
and pitch downe the point, in forme as it lies.
Neere to whose handle vpon the left hylt,
place the Diuisor your minde is to vse.
But let the right hilt haue a nate on it made,
and all other digits along by the blade.

72. Then let euery digit so setled beneath,
enfold the diuisor each after other.

VVhose seuerall products in placing bequeath,
Right with his own digit like brother to brother:
The least of which products, and next your Remayner,
So pointeth your quotient as nothing is playner.

Euma. These verses haue neede of an Interpreter.

Philo. The sence and meaning of this verse will appeare easie inough by an example, for it is no more to say, but that you should describe such a long crosse as heere you see: which for the similitude, that it hath to a long sworde, I call by that name, accompting the vpper part thereof aboue the crosse, for the handle and that part of the crosse nere the left hand, for the left hilt, and the other part thereof for the right hilt, and all the rest, from the crosse downe warde to the point, I take for the blade. Now sir, this sworde or crosse being made, the verse biddeth me place the Diuisor, which I minde to vse nere to the handle vpon the left hilt. Wherefore minding to diuide 705274. by 234. I do place the same 234. (being the Diuisor which I meane to vse) vpon the left hilt of my sworde nere to the handle thereof thus.

Figure of a long sworde.

Hilts what

234

Then vpon the other hilt, as the rule also biddeth me, I set a vnce or 1. and all other digits (which as you know are but 8. mo in number.) I set along by the blade thus.

234 | 1
2
3
4
5
6
7
8
9

This done, I must let euery digit beneath the hilts, multiplie the Diuisor, and the seuerall products thereof I must set vnder the same diuisor (at the other side of the blade) euery one right against his owne digit: wherefore first, I multiplie 234. by 2. and the product is 468, which I set right with the same 2. as heere you see it.

234 | 1
468 | 2

Then doe I multiplie the same 234. by 3. and next by 4. and so forth till I haue past ouer all the digits, still setting euery seuerall product right against his owne digit, & so the whole standeth thus.

234 | 1
468 | 2
702 | 3
936 | 4
1170 | 5
1404 | 6
1638 | 7
1872 | 8
2100 | 9

This sworde being thus (as it were) sheathed betweene the digits, and their products is ready for our purpose, so that now I will make you worke the diuision your selfe.

Euma. That would I most gladly be doing.

Phil. Then place you the numbers orderly (with their quotient

line ready) as the 46. rule taught you, and tell me what number remaineth aboue ouer your diuisor.

Euma. The numbers are orderly placed thus. 705276(
And the sum aboue the Diuisor is 705. 234

Phi. Well said, it is so, now looke in your scabbard among the products there, and tell me which is the least, but yet drawing nerest to ỹ number 705, remayning aboue your Diuisor. And what digit it hath directly against it.

Eum. That I thinke is 702. and the digit whervnto it dooth as it were directly point, is 3.

Phi. It is euen so, wherfore enter you that 3. for your quotient appointed, & subtract that product 702. from the 705. of the diuidend, & set downe the remainer, as I did in the other workes before, and so hath your diuisor executed his office once, wherfore this being done, you may cancell your diuisor, & remoue it one place more forward, and then looke againe what remaineth ouer your diuisor, which being knowne, seeke among the products of your scabbard (as you did before) for the least next product to that remainer, & do all other things againe in like order as you did before, continuing so from time to time, after euery remouing of your diuisor, till you come to end directly with the diuidend.

Eum. Thē I enter ỹ same 3. for my quotient thus. 3
705276(3
And then abating 702. from 705. there remaineth 234
3. wherefore I deface the 705. and also my Diuisor, setting the 3. remaining ouer the 5. as appeareth in the last example.

And so remouing my diuisor forward thus. 3
There remaineth ouer it now but only 32. which I 705:76(3
seeke for among the products of my scabbard, but I 2343
can neither finde it there, nor any other number so lit- 23
tle as it selfe, for it is I see then the diuisor.

Philo. And do you not know what you haue to do when the diuisor exceedeth the remainder.

Euma. Yes by the 69. verse, after the forme of working there taught, I should now place a cypher in the quotient.

Phi. And euen so must you do heere also: wherefore put 0. in the forth quotient, & cancelling your diuisor where it standeth, remoue it quickly one place more forward, & thē looke what remains ouer it.

Eu. I haue done so & the numbers standeth thus. 3
And now there is 327. aboue my diuisor wherfore 705:76
I seeke on my scabbard for the least product there ap- 23444(30
proching nearest to it, & there is none but exceedeth 233
it, sauing the Diuisor it selfe, whose digit is 1. 2-

wherefore I take that 1. for my quotient, and entering it there I do nert subtract the Diuisor from the 727, and there remayneth 93. which I note downe in their due places, and then I dash out both the 327 and also the diuisor, setting the same forward yet one place more, and so the fourme of the worke standeth this.

727276	
2944444	(301
2733	
22	

Philo. It was well done of you, when you could finde none of the products to serue your turne, but the diuisor, that you tooke the diuisor it self and his digit 1. as your true quotient, for indeed so must you alwayes do in like cases.

Eu. Then to proceed, I find the remayner now about my diuisor is 936. wherfore I seeke in the scabbard of my sword for the least product there, approching neerest to it, which I find to be 702. and the digit belonging to it is 3.

Phi. Nay nay now you are in an error, for the rule saith, The least of which products next to your remainer, so pointeth your quotient as nothing is playner: whereby is not meant that when you finde the remayner it selfe, you should then take any lesser number, for is there any other lesser product to be found that can go neerer to your remayner, then when you finde the very Remayner it selfe.

Euma: No, that is true, I confesse my fault, which came by reason I thought, I must needes take a product lesse then the remayner, and did not consider that that product is the least and neerest to the remainer, which is iust equall with it: but now I perceyue I may, and must, take any such product, though it be equall to the remayner (but not exceeding it,) and therefore in that you doe not allowe 702. to be the lesse product and most neere to the remayner, seemeth now to be good reason. because no 2. numbers can approche neerer together, then such as are equall and both one, and so I must needs confesse that tyough 702. be lesser then the remayner, yet it is not nert to the remaynder, sith 936. is neerer, for indeed the rule biddeth not to take the least product absolutely, but that least, which approcheth neerest in equalitie to the Remainer, which thing from henceforth, I hope I will not forget.

Philo. Well I am glad you conceiue it so well, wherefore now proceede.

Euma. Then finding this remainder 936. it selfe among my products, I take his proper digit, which is 4. and enter it within my quotient lyne, as a true quotient. And then in subtracting the product there of from my remayner, The one consumeth the other iustly

iustly, because they be equall: wherefore I deface both the remainer, and also my diuisor, and so haue I finished the diuision, for the diuisor is come leuell with the diuidend, and the quotient is iust 3014. as heere the worke declareth.

393
705276(3014
234444
2333
22

Philo. You haue done rightly, and the quotient is so as you say: but if you had gone forth with the abating onely of the other product 702. and had taken his digit 3. for your quotient, as you were about to do. The quotient (as you may easily gesse) had not beene so much by 1. as it is now, but then there would haue beene leaft a remainer of 234. after the end of your worke, which remainer had beene iust equall to the diuisor, which thing ought not so to be, nor neuer can chance, but when there is some error cõmitted in the worke: for in all diuisions, the remainer at no time ought to counteruaile ꝑ diuisor, (after ꝑ subtraction is once past,) much lesse may the last remayner so do, which by my notes vpon the 46 rule, is not onely defined to be a percell of the diuidend, lesse then the diuisor, leaft after the diuision is ended, but also by the 63. and 70. verses, the same together with ꝑ diuisor must make a fraction, and so be expressed, which thing you could not haue done, if you had proceeded that wayes, for if you had set the numbers thus. You know by the 21, verse, that they now are an entire vnit, and no true fraction. But in all diuisions truly wrought, the remayner (if any be) with the diuisor, must of necessitie for euer be a true fraction, and therefore note it well.

$\frac{234}{234}$

Euma. I hope to take such heede thereof, that I will neuer be deceiued so agayne.

Phi. Well, thus you haue now learned 2. formes of Diuision: The first ordinary and common, the last not so generally vsed, but more easie, and such as Maister Doctor Recorde thinketh the perfectest and most expert Arithmetician, needs not think scorne at sometimes to vse, especially when the Diuidend and Diuisor be both great numbers, or when many great numbers are to be diuided by one diuisor, which thing (as he saith) commeth very often in vse, in Geometricall and Astronomicall calculations.

Euma. The difference betweene the former forme of worke and this, as seemeth to me, is this: that there the quotient was first sought, and then followed the Multiplication of the diuisor by it, and the subtraction thereof did concurre, and was wrought

wherefore I take that 1. for my quotient, and entering it there I do next subtract the Diuisor from the 1027, and there remayneth 93. which I note downe in their due places, and then I dash out both the 327 and also the diuisor, setting the same forward yet one place more, and so the fourme of the worke standeth thus.

1993
703276
2744446(301
2733
22

Philo. It was well done of you, when you could finde none of the products to serue your turne, but the diuisor, that you tooke the diuisor it self and his digit 1. as your true quotient, for indeed so must you alwayes do in like cases.

Eu. Then to proceed, I find the remayner now aboue my diuisor is 936. wherefore I seeke in the scabbard of my sword for the least product there, approching neerest to it, which I find to be 702. and the digit belonging to it is 3.

Phi. Nay stay, now you are in an error, for the rule saith, The least of which products next to your remainer, so pointeth your quotient, as nothing is playner: whereby is not meant that when you finde the remayner it selfe, you should then take any lesser number, for is there any other lesser product to be found that can go neerer to your remayner, then when you finde the very Remayner it selfe.

Euma: No, that is true, I confesse my fault, which came by reason I thought, I must needes take a product lesse then the remayner, and did not consider that that product is the least and neerest to the remainer, which is full equall with it: but now I perceiue I may, and must, take any such product, though it be equall to the remayner (but not exceeding it,) and therefore in that you doe not allowe 702. to be the lesse product and most neere to the remayner, seemeth now to be good reason, because no 2. numbers can approche neerer together, then such as are equall and both one, and so I must needs confesse that tyough 702. be lesser then the remayner, yet it is not nere to the remaynder, sith 936. is neerer, for indeed the rule bindeth not to take the least product absolutely, but that least, which approcheth neerest in equalitie to the Remainer, which thing from henceforth, I hope I will not forget.

Philo. Well I am glad you conceiue it so well, wherefore now proceede.

Euma. Then finding this remainder 936. it selfe among my products, I take his proper digit, which is 4. and enter it within my quotient lyne, as a true quotient. And then in subtracting the product there of from my remyner, The one consumeth the other iustly

iustly, because they be equall: wherefore I deface both the remainer, and also my diuisor, and so haue I finished the diuision, for the diuisor is come leuell with the diuidend, and the quotient is iust 3014. as heere the worke declareth.

393
702276(3014
234444
2333
22

Philo. You haue done rightly, and the quotient is so as you say: but if you had gone foorth with the abating onely of the other product 702. and had taken his digit 3. for your quotient, as you were about to do. The quotient (as you may easily gesse) had not beene so much by 1. as it is now, but then there would haue beene leaft a remainer of 234. after the end of your worke, which remainer had beene iust equall to the diuisor, which thing ought not so to be, nor neuer can chance, but when there is some error cõmitted in the worke: for in all diuisions, the remainer at no time ought to counteruaile ꝑ diuisor, (after ꝑ subtraction is once past,) much lesse may the last remayner so do, which by my notes vpon the 46 rule, is not onely defined to be a percell of the diuidend, lesse then the diuisor, leaft after the diuision is ended, but also by the 63. and 70. verses, the same together with ꝑ diuisor must make a fraction, and so be expressed, which thing you could not haue done, if you had proceeded that wayes, for if you had set the numbers thus. You know by the 21, verse, that they now are an entire vnit, and no true fraction. But in all diuisions truly wrought, the remayner (if any be) with the diuisor, must of necessitie for euer be a true fraction, and therefore note it well.

$\frac{234}{234}$

Euma. I hope to take such heede thereof, that I will neuer be deceiued so agayne.

Phi. Well, thus you haue now learned 2. formes of Diuision: The first ordinary and common, the last not so generally vsed, but more easie, and such as Maister Doctor Recorde thinketh the perfectest and most expert Arithmetician, needs not think scorne at sometimes to vse, especially when the Diuidend and Diuisor be both great numbers, or when many great numbers are to be diuided by one diuisor, which thing (as he saith) commeth very often in vse, in Geometricall and Astronomicall calculations.

Euma. The difference betweene the former forme of worke and this, as seemeth to me, is this: that there the quotient was first sought, and then followed the Multiplication of the diuisor by it, and the subtraction thereof did concurre, and was wrought

togeather, for assoone as the quotient had multiplied one onely figure of the diuisor, immediately forthwith before it had multiplied the rest. the product of that one figure was subtracted, and then the next figure being likewise multiplied, the product there of was also immediately abated from the remayner or ouer numbers, and so the Multiplication and Subtraction did as it were concurre togeather.

But in this latter forme, the Multiplication is first wrought, for I take the products of the sword made by the digits there, to be nothing els in effect, but the whole product of the Diuisor made at once, by the quotient appointed, the choyse of which quotient, is alwayes annexed to the same product, and that product being euermore lesser then the remayner, or at the vttermost, but equall with it, sheweth by his digit annexed, how often the diuisor may be subtracted from such remayner or ouer number, and so finally the product which in the other kinde of diuision was subtracted by parcels at times, in this worke is ready made before hand, and set by the sword, to be subtracted together in one totall, and at once.

Phi. It seemeth by that you haue sayd, that you vnderstand both these formes of Diuision so well, that I shall not need to say any more therin, wherfore now I will shew you, how in some cases, you may abbridge your labor in the working of diuision, wherein you shall finde great ease, and no lesse speede, the first case is thus.

An abbridgemēt of labor in diuision.

73. And in Diuision, when the Diuisor,
Is nothing but Ciphers saue one single ace,
For euery Cipher, then cutting a figure,
Of from the diuidend by a squire lace,
The figures so cut, do make the remayner,
The rest are the quotient whereof you are gayner. 50

This abbridgement of worke serueth when the diuisor is but an vnit, and all the rest Ciphers, as when the diuisor is 10. 100. 1000. 10000. and such other like, then the diuision is most quickly performed, by cutting of so many figures from the Diuidend, (with a lace or line made squirewise) as there are Ciphers in the Diuisor, for the figures so cut off, shewe the remayner left after Diuision (if any thing do remayne) and all the rest towardes the left hand, represent (and are) the true quotient of that Diuision. Example: I would diuide 3480. by 10. wherefore sith my Diuisor hath but one Cipher, I do with my penne make a squire line, cutting of one figure onely from the Diuidend thus. 348|0

where-

whereupon I say, now that the quotient of this Diuision is 348. iust, without any remayner, for the remayner as you see is 0. but if I would diuide the same 3480. by 100. Now because my diuisor hath 2. Ciphers, I must with a squire line cut off 2. figures from the Diuidend thus. 34|80. And so now the quotient is 34. and the remayner is 80. Unto which if I subscribe the Diuisor, to make it a fraction, and so annex it to the quotient according to the 63. and 70. verses, it will stand thus $34\frac{80}{100}$.

Euma. This rule is passing easie and speedy indeede, I would there were the like for all sortes of Diuisors.

Philo. It is in vayne to wish or craue, the thing a man can no wayes haue, wherefore I say be you content to vse such meanes as god hath lent, the second case is thus.

Another kinde of abridgement.

74. And any Diuisor wherein you do finde,
One Cipher or mo, all outwardly placd,
Set all the figures as longs to their kind,
But such outward Ciphers set by them selues last,
So as they may reach directly and end,
With the last figure of the diuidend,
75. Then worke you forth with those figures alone,
As though they were all the diuisor entire,
Vntill by oft mouing, at length you be grown,
So nigh to the Ciphers you can come no nier,
So haue you your quotient without further payne,
and ouer the Ciphers appeares the remayne. 51

The example of this verse is thus, I haue 1346524. to diuide by 2400. wherefore sith I finde 2. Ciphers in this Diuisor, after that I haue placed the Diuidend, & set downe the 2. signifying figures of the Diuisor, onely at the beginning of my worke in the places of right belonging to them, I set the 2. ciphers by themselues, so as they may reach and end iustly the diuidend thus. 1346524(

And now I must vse the 24. alone, working with 24 00
it as though it were my entire diuisor, till by often remouing forward thereof. I haue brought it so nighe to the 00. that I can remoue it no more, because the ciphers filling the 2. last points, do as it were stop me, and byd me stay there.

Euma. I pray you let me worke this example my selfe, for me thinks I vnderstand both the verses, and all your meaning already.

Philo. Go too then, do so in Gods name.

Euma. First I see, this diuision must be wrought by the 48. rule

ti all poynts, notwithstanding this new position of the cyphers, for I perceiue this location of the ciphers is but to cleare the worke of many idle ciphers, which otherwise would fil & accumber the same, and therfore serueth to like effect in Diuision, as the rule by you prescribed in your notes vpon the 36. rule doth in Multiplication.

Philo. Your iudgement is very good & true, wherefore go on.

Eum. Then the giuen numbers seited thus: I aske how many times 2. in 13. and it answereth 6. times: but then considering that I cannot take 6. times 4. out of the remainer, (which will be but 14.) I take it lesse by once, that is but 5. times, and so often I perceiue I may take it, and leaue a sufficient remainer.

1346524	(
24	00

Therefore entring 5. within my quotient lyne, I forthwith multiply my diuisor by it, saying first. 5. times 2. is 10. which I abate from the 13. and there resteth 3. so I abolish the 1. of the 13. and the 2. of my diuisor, letting the 3. stand as the remainer, (for I were as good to do so, as to deface it, and make a new 3. ouer it) and so the worke is thus.

1346524	(5
24	00

Philo. That was well considered of you, for by that meanes your worke shall be performed with fewer figures.

Euma. Then say I, 5. times 4. is 20. which I take (as you did) not out of the whole 34. but out of so much there of as will serue my present turne, which is 24. and so remaineth 4. (besides 2. kept in my minde,) which 4. sith it is there standing already. I permit so to stand still without defacing, for like cause as I suffered the 3. but euen now. And then going to the nere left hand place, to repay my 2. borrowed, although I finde nothing there at all, yet repaying the 2. in my minde there, I say, 2. in minde out of 3. leaueth 1. wherefore I deface the 4. beneath, and 3. aboue, setting the remainer 1. in his stead, and so the worke standeth thus.

1	
1346524	(5
24	00

And now I must remoue my diuisor forward one place, which being done as heere it appeareth. I aske how many times 2. in 14. and it answereth me 7. times: but streightwayes I consider that I cannot haue 7. times 4. out of 6. which will be the whole remainer ouer my diuisor: wherefore abating an ace, I take it but 6. times, and so often I am sure I may take it, and yet leaue a sufficient remainer: wherefore I enter 6. into the quotient, & by it I multiply

1	
1346524	(5
24	00
2	

euery

euery figure of my diuisor, subtracting the products as I did before, saying, first, 6 times 2. is 12. which subtracted from 14. leaueth 2. remaining, therefore I cancell the 2. beneath, and the 14. aboue, noting downe the 2 remaining ouer the place I am in hand withall, thus.

Secondly, I say, 6. times 4. is 24. which taken from 26. leaueth 2. so I deface the 4. beneath, and 26. aboue, setting downe the 2. remaining ouer 4. the figure of my diuisor, presently wrought thus.

And now I am to remoue my diuisor againe, and to seeke a new quotient, which I do thus.

And heere asking how many times 2. in 2. it answereth me once: wherefore I put 1. into the quotient, and now if I will, I may say 1. time 2. is 2. which I take out of 2. & there remaineth 0. and likewise 1. time 4. is 4. which I take out of 5. and there remaineth 1. or else, because (as you said before) 1. dooth neyther multiplie nor diuide any number, I need not multiply the diuisor at all, sithe it is neuer the more for multiplication by 1. but may presently subtract it from the diuidend, & so both wayes 1. will remaine, which with the 24. following maketh 124. for the last remainer, for now seeing I am come so neere to the cyphers with my diuisor, that I can remoue it no further for them, I iudge by the tenor of this verse, that I haue done, and so the worke standeth thus.

And the whole quotient of this diuision as heere appeareth, is $561\frac{124}{1400}$.

xx	
x346524(56	
24400	
xx	
	xxx
x346524(56	
24400	
x	
	xxx
x346524(56	
24400	
xx	
	xxxx
x346524(561	
24400	
xx	

Philo. It is so, and I cannot but commend your industrious diligence, for you haue wrought this diuision so well and so artificially, that I perceiue therby you do well vnderstand all that I haue taught you hetherto. But heere I must not forget to tell you, that if a diuisor haue any middle cyphers, then may you not alter their situation, to set them by themselues, as you did the vttermost cyphers in this last worke, as if 2004. were your diuisor, you must needes take it all, both figures and cyphers as they stand: but if 2040. were your diuisor, then might you set the vttermost cipher onely by it selfe, after the fashion before vsed, but not the middle 0. and so forth in other numbers like.

Euma. All this I should haue coniectured by common reason, but yet I thanke you for your aduertisement therein, and now if

you thinke it due time, I pray you shew me how I should trie my worke, to the end that when I haue finished any diuision, I may be assured, that I haue done it truly.

Philo. That will I do with a good will, and you moue it in good time. The common triall of diuision is (as it was in the 3. former operations) by casting out all the nines in such order as the rule insuing dooth prescribe. But the surest triall of diuision is by multiplication, and of multiplication by diuision, which now sithe you haue learned both, I would rather wish you to vse: the triall by nines is thus.

The triall of diuisiō.

76. Examine diuision after this fashion,
From your Diuisor all nines first reiect:
Setting the rest as in multiplication,
At side of a + made for that effect:
Next from the quotient, all nines likewise take,
Opposing the rest at the other side there,
Then folding both sides, to the ofcum they make,
Adde the remainer, if any there were:
And from the totall, the nines also cast,
Exalting the rest to the head of the crosse.
All nines likewise nipt, from diuidend last,
The rest to repose at the foote is no losse,
For why if the head and foote now agree,
Then is the worke wrought as well as can be.

Eu. I pray you giue me leaue to examine my last diuision onely by this rule, & I will trouble you with no further trials heercin, for by the light which the 24. 30. & 37. rules, with ỹ examples there insuing, gaue me, I suppose my selfe to vnderstand this rule fully.

Philo. Go to, shew your cunning.

Eum. First the diuisor was 2400. from which casting all the nines there resteth 6. which I place at the one side of the crosse thus.



Secondly, the quotient was 561. from which taking all the nines, there resteth 3. which I set oppositely at the other side of the crosse thus.



Now folding these 2. sides together, that is 6. in 3. the product is 18. to which I adde 124. the remainer of the diuision, and so the totall thereof is 25. from this totall, once againe I cast out the nines, and so resteth 7. which I exalt and place at the head of the crosse, thus.



Lastly

Lastly, I cast out all the nines from 1346524. which was the diuidend, and there resteth 7. also, which I repose at the foote of the crosse thus.



And now sithe the head & foote of this crosse agree in likenesse & equalitie of digits, ỹ worke by this verse is approoued good & true.

Philo. It is so: Now for the other triall by Multiplication, I neede say no more but this. If you multiply your quotient by your diuisor, (and to the product thereof, adde the remainer, if any be) the totall shall returne your diuidend, if your worke were good, for if it do not, then haue you erred somewhere. Also for ỹ triall of multiplication, if you diuide the product by the multiplier, the quotient shall returne the multiplicand, or if you diuide the product by the multiplicand, the quotient shall returns the multiplier, & so each of these operations are the triall of the other. Also if you will, you may proue diuision by diuision, for after any diuision ended, if you diuide the same diuidend by the quotient found, the second quotient shal be your former diuisor, & the selfe same remainer, (if any were) shall remayne still, all which conclusions, you may practise at leasure when you are alone, for seeing you are expert in the working of multiplication & diuision, it cannot be, that you should now be ignorant how to vse them both, for the triall of each by other, and therefore accounting it needlesse to stand about examples in that behalfe, I will proceede to the properties incident to diuision, whereof this is the first.

77. VVhen diuidend and diuisor is equall,
Whether the numbers be whole or broken
A measure once taken doth measure it all,
VVhose quotient is 1. well fare a good token.

The properties of diuision.

So 4. measureth 4, but once, and 6. measureth 6. likwise but once, by reason that both the diuidend and diuisor are equall, and therefore the quotient at each time is but 1. as heere you see.

Likewise if ½ be measured by ½ the quotient wilbe 1. as you shall better perceiue when we come to diuision of fractions.

Euma. What meane you by measuring, for I not remember that I haue heard that worde of you before now.

Phi. That number is generally sayd to measure another, which being subtracted frõ it as often as may be, leaueth nothing remaining: and this is the proper definition of the measure of a number, whereby it followeth necessarily, that the greater number cannot measure the lesser, because the greater (by the 27. rule) cannot be-

Eucl. 1. d. 5 & 1. d. 10. What a measure is & what is is to measure a nũber.

be subtracted from the lesser, and yet the lesser may be diuided by the greater, as appeareth by my notes vpon the 45. rule, and yet will appeare better to you anon. But forasmuch as by the notes of your 46. rule it appeareth, that diuision serueth in stead of many subtractions, therefore to measure any number, is but as much to say, as to diuide it: and although euery measuring be a diuision, yet euery diuision is not a measuring, nor euery diuisor a measure, but such diuisor onely, is called both a diuisor, and a measure, as diuiding any number, leaueth nothing.

The measuring of euery nũber, is the diuisor of it, but eũry diuisor is not a measuring nor euery diuisor, a measure.

Euma. I vnderstand you thus: that if 20. be diuided by 3. the quotient willbe 6. and there will remayne 2. Likewise, if it be diuided by 8. the quotient wil be 2. & there will remayne 4. wherefore both these numbers 6. and 8. are diuisors to 20. but they are not measures thereto, but if 20. be diuided by 4. the quotient is iustly 5. or if it be diuided by 5. the quotient is iust 4. or if it be diuided by 10. the quotient is iust 2. and there is nothing leaft at any time, wherefore both 4. 5. and also 10 are as well measures as diuisors to 20.

Phi. You vnderstand it euen as I meane, wherefore I will recite the next proprietie which is thus.

78. If diuidend lesse then diuisor you see,
The quotient of force a fraction must be:
Whereof the diuidend shallbe numerator,
And the diuisor the Denominator,
But euery diuisor lesse then his diuidend,
Some part or parts thereof doth comprehend. } 4

Eucl. 4. prop. 7.

As for example, (when the diuidend is least) if you diuide 5 by 8. the quotient is $\frac{5}{8}$. For such diuision is accomplished onely by diuiding the termes with a lyne, and so making a fraction of them whereof as you see the diuidend is numerator, and the diuisor denominator.

Note how the lesse number is diuided by the greater.

Euc. 3. d. 7

But if the diuisor be least, then doth it conteyne eyther one single part, or else many parts, of the diuidend, as if the diuidend be 6. and the diuisor 3. Now doth this diuisor comprehend one single part thereof, that is to say, $\frac{1}{2}$. of it, and such single parte alwayes whose numerator is 1. is generally called an aliquot part of the whole.

Euc. 4. d. 7 What an aliquot part is.

But if 4. be the diuisor to 6. then doth the same diuisor comprehend $\frac{2}{3}$. or $\frac{4}{6}$. of 6. which are not called singularly one part onely thereof, but plurally parts: because the numerators of such parts are not a vnit, but a number.

The difference betweene a part and parts.

Eu. All this is very plaine, manifest, and agreeable to reason.

Philo.

Phi. Then ſhall you heere the third propertie, which is this.

79. If a diuiſor be part, but not parts,
then is it ſuch part as the quotient imparts:
For why the quotient ſet vnder a line,
with 1. aboue it, that part doth define.

Euel. 39. prop. 7. How to find, what Aliquot part, the diuiſor is of the diuidend.

That diuiſor is called a part of his diuidend, (that is to ſay) ſuch an aliquot part therof as is mẽtioned in the notes of the laſt verſe, which meaſureth his diuidend by an whole number leſſe then it ſelfe: As in your owne example following the 53. rule, where 4. meaſured 20. by 5. Therefore by this verſe, if 5. being the quotient, be ſet vnder a lyne with 1. aboue it thus. $\frac{1}{5}$. the ſame fraction doth define and ſhew that 4. the diuiſor is an aliquot part of 20. that is to ſay, $\frac{1}{5}$. thereof.

Euma. So then if 20. be meaſured by 10. the quotient wilbe 2. which is one whole number leſſe then 20. wherefore I may conclude that 10. is an aliquot part of 20. which by this rule is $\frac{1}{2}$ thereof.

Philo. You may do ſo: Now another propertie of diuiſion is thus.

80. If any diuiſor to mo parts extend,
Then one ſingle part of his diuidend,
Then that diuiſor made numerator,
And eke that diuidend denomenator,
The fraction ſo framd doth tell and not mis,
What parts the diuiſor of diuidend is.

56

How to find what Aliquot parts the diuiſor is of the diuidend.

That Diuiſor is parts of his diuidend, which being leſſe then it, dooth not meaſure it, but being by diuiſion ſubtracted from the diuidend, as often as it may, leaueth a remainer leſſe then it ſelfe, as in your former example, when you diuided 20. by 3. the quotient was 6. and you had 2. left remaining, which is leſſe then 3. the diuiſor. Therefore by this verſe, the ſame diuiſor 3. ſhall be $\frac{1}{6}$. of 20. and ſo when you diuided 20. by 8. the quotient was 2. and there remained 4. which is leſſe then 8: the diuiſor. Wherefore by this verſe the ſame diuiſor 8. ſhall be $\frac{1}{2}$. or $\frac{2}{5}$. or $\frac{4}{10}$. of 20. all which Fractions are equall by the 40. rule or marim.

The difference betweene a part and parts. And note that thoſe parts are called partes Conſtituentes. Integrum ſine totum.

Euma. All this I vnderſtand well, and it is ſo apparent that it needeth no further proofe.

Philo. Thus much concerning the properties growing of one ſingle diuiſion onely. Now followeth the properties incident to many diuiſions, whereof this is firſt.

The properties of diuers diuiſions together.

57. As diuers diuiſors diuiding a number,
Are equall together, or differ a ſunder.
So ſhall the quotients by each of them quoted,
Be equall, or differ, which is to be noted.

Eum. I vnderſtand this verſe thus, that if 12. be diuided by 6. and 6. which are equall diuiſors, the quotients ſhallbe 2. and 2. which are allſo equall numbers, but if the ſame 12. be diuided by 6 and 3. which are vnequal diuiſors, the quotients ſhallbe 2. and 4. which are alſo vnequall and different in quantitie.

Philo. You take it well, wherefore I proceede thus.

58. If the quotient of a number diuided,
do newly diuide that number once more,
Then ſhall the quotient, ſo laſtly deſcribed,
Be that which was the diuiſor before:
Wherefore when one number will meaſure another,
The quotient will do the ſame like a brother.

So if 3. diuiding 12. make the quotient 4. then the ſame quotient 4. newly diuiding 12. againe, ſhal now make the quotient 3. which was the diuiſor before, as this rule importeth, wherefore if 3. meaſure 12. by 4. (or 4. times,) then 4. ſhall meaſure the ſame 12. by 3. that is to ſay 3. times.

Eum. All this is very true and plaine.

Phi. Then how do you vnderſtand this next propertie, which is thus.

A corrolary to the 1. prop. 7. of Euclid.

59. If a number meaſure many numbers mo,
particulerly one after another,
And afterwards meaſure the totall alſo,
of thoſe particulers, ioyned all together,
Then is the quotient of that totall tride
Iuſt equall to all the quotients beſide.

Eum. I will declare how I vnderſtand it by this example. Firſt I ſet theſe 4. particuler numbers, 6. 15. 18. 24. to be meaſured by this number 3. and ſo the particuler quotients of their meaſuring, are 2. 5. 6. and 8. & the totall of theſe particuler quotients is 21.

Secondly, I let the ſame number 3. meaſure the totall of 6. 15. 18. & 24. (which is 63.) and ſo the quotient is alſo 21. equall to the ſum of all ỹ former particuler quotients, is not this your meaning.

Phi. Yes indeed, & heereby you may perceiue, that to diuide any nũber, by another nũber taken wholy together, or by all the parts therof ſeuerally, bringeth one effect, and is all one thing. Againe.

84. A number that measures sundry numbers geuen,
VVill measure their totall aswell and as euen,
And also their ofcum produced in due fashion,
Of each in to other by multiplication,
} 60

6. common sentences added to Euclid. by Campane & Flussates fol. 188.

Eum. The first part of this rule is manifest by the last verse foregoing, & by the example thereof: for as 3. measured sundry numbers there giuen: to wit. 6. 15. 18. and 24. so did it also measure their totall 63. as euenly. But now whether it will also measure their product or not, that resteth yet to proue: wherefore for triall thereof, I multiplie 6. into 15. and it produceth 90. which I multiply next into 18. and the product thereof is 1620. which finally infolded in 24. produceth 38880. And now diuiding this last product by 3. I finde the same 3. measureth it iustly, as heere appeareth.

ꝏxxꝏ
38880(12900
33333

Philo. Because you are so ready in conceiuing these rules, beholde another.

85. If any number be aptly enclind,
to measure another number assignde,
And some part besides thereto apperteining,
Then will it measure the part eke remayning.
} 61

4. cõmon, sentences added to Euclyd: by Campan and Flussates, fol. 187. and in a corollary to the 2. prop. 7. of Euclid.

As thus, suppose that 3. is a number aptly enclinde (that is to say) able to measure this whole number 12. + 6. that is 18. and suppose also that the same 3. will likewise serue to measure 6. the one part of 18. Then I say (by this rule) it shall also measure 12. the other part of ỹ same 18. Or let the same 3. be a mesure to 9 + 6 that is 15. and to 6. the one part of the same 15. then I say, it shall also measure 9. the other part of 15. remaining.

Eu. This is very euident of it selfe. Phi. Then I proceede thus.

86. That number will measure a number assignd,
which measures his measure, for so is his kinde.
} 62

So 18. being measurable by 9. the same 9. is also measurable by 3. wherfore I say by this maxim, sith 3. wil measure 9 (the measure of 18.) therefore it shall also measure the whole number 18. which being so apparant, I go in hand with the next rule, which is thus.

A theoreme of Montaurens annexed to the 3 pro. 10. of Eu.

87. That number which measures a number assignd,
is greatest measure common to them both,
And to ther number so measurd by kinde,
is dividend least, like common of troth.
} 63

The definition of a commõ measure.

Heere I must tell you that a common measure, is such a number as will serue to measure diuers giuen numbers, & that is called properly

properly the greatest, when as no other common measure to the giuen numbers can be found greater. As for example: Let 9. measure 18. Now because 9. dooth measure 18. therefore by this verse I say, that 9. is the greatest common measure both to 18. and 9. and first I say, that it is a common measure, for by the 53. Maxim and glose thereof, it measureth it selfe, and it measureth 18. by supposition.

Demonstration. Moreouer it is the greatest common measure to the said numbers, for if any number greater then 9. were common measure to 18. and 9 Then should a greater number then 9. measure the same 9. which should be contrary to the former cited glosse, and therfore not true.

Secondly, I say that 18. is the least common diuidend (that is to say, the least number measurable) by 18. and 9. and first, that it is a common diuidend to both appeareth, for it is diuidend to 9. by supposition, and that it is diuidend to it selfe, is verified by the foresaid 53. rule. But if it were possible that a lesse number then 18. could be common diuidend to 18. and 9. then should the same 18. measure a number lesser then it selfe, which is both absurd, and contrary to the glosse or notes of the said 53. Maxim.

Eu. This doctrine is so plainely & perfectly prooued by demonstration leading to absurditie, that no reasonable man can deny it.

Philo. You say truely, and therefore you shall haue another rule euen much like, being as it were a consequent or corollarie thereto, and it is thus.

A coroll to the 2. prop. 7. & a corollary to the 3. prop. 10. of Eucl. And a coroll: to the 4. prop. 10. of Eucl.

88. A number which measures 2. numbers or mo,
will measure also, if you applie it,
Their greatest common measure also,
which is so apparant, no man can deny it. } 64

Euma. Indeed the truth of this rule is very manifest, euen by the last verse before: for I may easilie gesse, that forasmuch as in the example there giuen, 9. measured both 18. and 9. and that the same 9. is there prooued to be the greatest cōmon measure to 18 & 9. therfore I say it must necessarilie follow, that 9. measureth their greatest common measure, because in so doing it dooth but measure it selfe, which thing by the 77. verse euery number can do.

Philo. And the like commeth to passe, if there were 3. numbers or moe, for let 9. be (as indeed it is) the greatest common measure to these 3. numbers, 9. 18. 45. Now, you see heere as before, that 9. being their greatest common measure, ceaseth not therefore to

to be a measure to it selfe, and so therefore measureth the greatest measure to the geuen numbers as the rule affirmeth, which sith you vnderstand I will go forward.

89. If 2. numbers geuen will easily afford,
Each of them aptly to measure a thurd,
Then the least diuident common to both,
Will also measure that third of a troth. } 65

Euc. 37. prop. 7.

As for example let the 2. numbers geuen or assigned be 4. and 6. and let each of them measure this third number 24. now the least common diuident to 4. and 6. is 12. wherefore by this rule, the same 12. shall also measure this third number 24. which being manifest, needeth no further proofe, wherefore I go forward thus.

90. If 2. numbers be, both like proportionall,
Vnto any third, then are they both equall,
And contrarily 2. numbers coequall,
Are to any third both like proportionall. } 66

Euc. 7. prop. 5.

As for example I say that as 12. is to 4. so is twise 6. to the same 4. wherefore by this verse 12. and twise 6. are equall, because 12. and twise 6. are both like proportionall vnto 4. a third number, and contrarily, sith 12. and twise 6. are equall, therefore by this verse 12. and twise 6. are both like proportionall to 4. or to any other third number.

Euma. And so I thinke I may say, that if 9. be to 3. as the product of 3. by it selfe is to 3. then 9. and the product of 3. by it selfe is equall, or els thus, if 9. be to 3. as 5 + 4. is to 3. then 9. and 5 + 4. are equall.

Philo. You may say so very well and truely, and you haue described both the meaning of this Maxim and your owne minde as well, wherefore to proceede.

91. If 4. numbers be in proportion direct,
They keepe a proportion in euery respect,
As well interchanged as also inuearsed,
Or any way els that can be rehearsed. } 67

Euc. 16. prop. 5. and 13. prop. 7.

This rule belonging onely vnto proportionals, teacheth that when any 4. numbers are proportionals directly, then are they also proportionals still, whether you take them interchangeably (which otherwise is called alternatly) or inuersedly or any other way orderly.

Euma. I remember that in your notes vpon the 33. and 45. Maxims, you taught me what numbers were proportionals, and

also that the 42. & 43. maxims, taught me a fine cattle rule, whereby I might alwayes know, when any 4. numbers propounded were true proportionals, but I neuer heard of any interchanging, altering or inuersion before, wherefore I pray you expound the meaning of these termes more playnely.

Philo. Do you not remember that in the notes of the 33. rule, I told you that when 4 termes or numbers assigned are in such rate or proportion together, that the first couple is rated like the latter couple as were these 4 termes, 24. 6. 4. 1. there mentioned, that then such 4. termes are called proportionals.

Euma. Yes I remember that very well.

Proportion direct defined

Phi. Then I say that the same 4. termes, as they stood there directly in order thus, 24. 6. 4. 1. or rather thus, $\frac{24}{6}$. and $\frac{4}{1}$. are proportionals directly, or in proportion direct, because the first couple, viz. $\frac{24}{6}$. are compared together by them selues each to his owne fellow, & the latter couple viz. $\frac{4}{1}$. are compared likewise by thẽselues each to his own fellow, the one couple not intermedling with the other, & so both Antecedents are compared to their owne consequentes.

But when one of the Antecedentes onely of such 4. proportionals being changed from an Antecedent, is conferred and vsed as a consequent, then are the same proportionals sayd to be alternated or interchanged.

As when the former numbers standing as they did thus $\frac{24}{6}$. and $\frac{4}{1}$. If now you will compare the 2. vpper numbers together by them selues, and the 2. lower by them selues, saying thus, as 24. is to 4. so is 6. to 1.

Or thus as 4. is to 24. so is 1. to 6. in both these cases they are sayd to be alternated or interchanged, because one of the termes of each couple, is compared to another of the other couple, and not to his owne fellow as in proportion direct, and yet as you see they are proportionals still aswell, so, as directly,

Euma. I see that very well by the 42. and 43. rules or Maxims, and here I perceiue also that you call the number first named of each couple of proportionals the Antecedent, & the second, or that which followeth, you name the consequent.

Phi. Yea or thus, the Antecedent is the number compared, and the consequent is it to whom the same is compared, wherfore when both the vpper numbers are so compared together as is aforesayd, then one of the Antecedentes is compared to the other, wherby the whole proportionallity, is called by ỹ name of proportion alternate

or

or interchanged, as appeareth by the definition therof insuing.

92. Sith Antecedent to Antecedent,
And consequent to consequent compard,
Are in proportion aswell correspondent,
As were the proportions directly declared,
Therefore proportions so changing their state,
Are called interchanged or alternate. 68

Euclid 12. d. 5. proportion alternate defined.

Fuma. Very good, I thinke I vnderstand now, what is ment by the alternation or interchanging of a proportionalitie wherefore I pray you declare what you meane by proportion inuersed.

93. Inuerst proportion returneth backward,
The same proportion which earst was direct,
So that each consequent now is compard,
With his Antecedent, and so in effect,
The number which was Antecedent before,
Is consequent now, I neede say no more. 69

Euc 13. def. 5. proportion inuersed defined.

As for example, let the former proportionalitie of numbers proportionall yet stand as they did thus. 2. and 4. now if you will conferre them (as it were) backwardly, to wit, the consequentes to the Antecedents, saying thus, as 1. is to 4. so is 6. to 24. or thus, as 6. is to 24. so is 1. to 4. In both these cases the proportions is sayd to be inuersed, for although heere each seuerall coople of termes is compared together by them selues, the one not intermedling with the other coople, yet for that their conference is not directly according to their naturall situation, but the consequentes are now made Antecedentes, and the Antecedentes vsed as consequentes, therefore the same proportionallitie so made, is called inuersed.

Fuma. Very well, I hope to remember this.

Philo. Thus hauing vnfolded the properties of integers, rising by Multiplication and Diuision seuerally: now I will likewise shewe you their notable properties deriued from Multiplication and Diuision ioyntly together.

Of the properties of Integers rising by Multiplication and Diuision ioyntly together. Chap. 8.

94. If any number which you would diuide,
Be the product made of any 2. other,
Then each of those 2. if it be applied,
diuiding that product shall quote out the tother. 70

As thus, let 12. be the product of 3. by 4. now I saye by this Maxim that 12. diuided by 3. shall quote 4. the other number in the quotient: for here both by supposition, and by the 33. rule, as 12. is to 3. so is 4. to 1. and alternatly by the 68. rule as 4. is to 12. so is 1. to 3. wherefore by the 45. rule 3. diuiding 12. shall quote 4. or yeeld 4. in the quotient, and agayne.

95. If any one number do measure another,
The product made by that quotient and measure,
Shall be the same number iust and none other,
Which was so measured at first for your pleasure. } 71

For sith by the demonstration last proceeding 3. measuring 12. did quote 4. in the quotient, then I affirme 12. to be the product of 3. by 4. for here by supposition and by the 45. Maxim as 4. is to 12. so is 1. to 3. and by the 69. rule inuersedly as 12. is to 4. so is 3. to 1. wherefore by the 33. rule, 12. is the product of 3. by 4. which 12. is the number first geuen to be measured as the verse present intendeth.

Euma. These 2. last maxims are very playne and easie to be vnderstood, especially as your demonstrations haue lightned them.

Philo. And the next is also as intelligible, which is thus.

Corollary to the 15. prop. 10. of Euclid: prop. Montau.

96. A number produced of any 2. others,
Is subiect to both in commensuration,
And euery number so hauing 2. measures,
Is made of those 2. by multiplication. } 72

As thus you see by the glosse of the 94. and 95. verses proceeding, that 12. is there proued to be the product of 2. other numbers that is to wit, of 3. & 4. wherefore now this verse affirmeth, that any such number so produced of 2. others, is subiect to them both in commensuration, which is almuch to say, as that it is measurable by eyther of both the same 2. numbers whereof it was produced.

Wherefore aswell 3. as 4. shall measure 12. and agayne that euery number which is subiect so to 2. measures is made and produced of the same 2. measures by Multiplication, wherefore 3. times 4. shall make or produce the same number 12. all which is so apparant that it needeth no demonstration, for it is proued by the glosses aforesaide.

Euma. It is so, wherefore I pray you stay no longer about it.

Philo. 97. If of 3. numbers successiuely couched,
The first do diuide any of the rest,

Then

Then if the tother left yet vntouched,
do multiplie but that quotient expreſt:
The product ſhall be directly in generall,
vnto the firſt 3. a fourth proportionall. 73

Euma. You ſhall ſee how I conceiue this rule by this example. Firſt I couche or ſet theſe 3. numbers ſucceſſiuely, viz. 3. 15. 12. and then I make 3. being the firſt to diuide 15. one of the other, and the quotient is 5. which quotient I multiplie by 12. the other number yet not touched, and the product is 60. wherefore by this verſe or maxim, as $\frac{3}{15}$. ſo $\frac{12}{60}$. which I finde to be ſo by the 42. and 43. rules.

Philo. They are ſo: and yet for further demonſtration thereof, you know that both by ſuppoſition & alſo by the 94. verſe, the quotient of 60. diuided by 12 is 5. and againe, that by ſuppoſition, the quotient of 15. by 3 is alſo the ſame 5. wherefore as $\frac{3}{15}$. ſo is $\frac{12}{60}$. that is, as 3. is $\frac{1}{5}$ of 15. ſo is 12. $\frac{1}{5}$ of 60. which being plaine inough, I proceede thus.

98. Euery 3. numbers conſent and agree,
that if the product of the former twaine,
Be but diuided by laſt of the 3.
the quotient ſhall ſhew a 4. number plaine.
VVhich to the ſecond ſhalbe in ſuch rate,
as firſt to the 3. and ſtand in like ſtate. 74

As for example, let 18. 4. and 12. be three numbers taken at aduentures, in ſuch order as I name them. So are 18. and 4. the former twaine, or firſt coople of them, which being multiplied together, produce 72. and that product diuided by 12. (the laſt of them) quoteth 6. wherefore by this verſe I conclude, that as 18. the firſt number is in rate and proportion to 12. the third number, going forward or directly, ſo is this quotient 6. in like ſtate and proportion to 4. the ſecond number, returning backward or conuerſedly.

Euma. I trowe you called ſuch backward proportion euen now, proportion inuerſed, wherefore I pray you let me know why now you name it conuerſed, or is there no difference betweene them?

Philo. Although indeed both names generally ſignifie but one thing as moſt authors vſe, yet I make this difference betweene them, that when both the conſequents are referred or compared backwardly, each to his owne Antecedent, then do I account ſuch proportion to be moſt properly ſaid inuerſed, as I tolde you before in the 69. Maxim, and the gloſſe thereof. But when the forepart or

The diuerſitie betweene proportiõ inuerſed and conuerſed.

first comparison of the termes, is made directlye from the first terme to the third, and the latter part onely is conferred backwardlie (as this is) from the fourth to the second, then doe I name it proportion conuerted, and so my minde is that you should take it.

Euma. Indeede I perceiue by the 69. rule, that if these proportionals were placed in such order as the 3. first of them were giuen, and the last found. Then they should stand thus 18. 4. 12. 6. where, looke how 18. the first number is to 12. (the third, forword or directly) so is 6. the last number (found by this verse) to 4. the second backwardly, or as you now call it, conuertedlie: and therefore they are not triable by the 43. rule, which was woonte to be my common triall, for if the bias termes were multiplyed, as there is taught, their two products would be nothing like equall.

Philo. You say true, for in this case the product of the 2. first termes, and the product of the 2. last, make that equalitie, because that by the very vtterance of the numbers (as they are heere to be pronounced) the first couple are the extreames, and the latter couple are the meanes, as it would plainely appeare vnto you, if they were placed in order like proportionals directly, but loe another like rule.

99. If of 3. numbers set forth to the veiw,
the last diuide one of the other twayne,
Then if the quotient, which so doth ensue,
Multiplie tother which yet doth remayne,
The product shall beare such rate to the second,
As first to the 3. els haue you misrecond. } 75

Euma. This appeareth true by the 98. verse, and the exposition thereof.

Phio. You consider it well, for indeed both that verse and this, tend both to one end, and bring both one effect, wherefore I will proceede.

Euclid: 19 prop. 7. Boetius li. 2. ca. 44

100. If 4. numbers be truly proportionall,
The product produced of both the extremes
Shall neither be more nor lesse but iust equall,
vnto the product made of the 2. meanes. } 76

This verse or rule is a conuerse to the 42. rule.

Euma. What meane you by a conuerse.

The definition of a conuers.

Philo. A conuerse or conuersion, is as much to saye, as the turning backwarde of the sentence. So that, that which in the former

mer 42. rule, was the conclusion, is heere made the supposition, and that which there was the supposition, is heere made the conclusion.

Euma. I thinke I vnderstand you now, for I perceiue that in the 42. marim, 4. numbers being first supposed to make the offcum of their meanes, equall to the offcum of the 2. extreames, are thereby concluded to be 4. true proportionals.

But heere 4. numbers proposed, being first supposed to be true proportionals: it is thereby concluded, that therefore the product of their extreames, shall be iust equall to the product of their 2. meanes.

And so it seemeth to me, that a conuerse is no more in effect but thus. If you will affirme 10. groates to be equall to 40. pence, then by conuersion I may aswell say, that 40. pence is equall to 10. groates, and therefore in my minde such conuerses needeth no demonstration for their proofe, for he that will denye that 40 pence is equall to 10. groates, cannot by reason affirme that 10. groates is equall to 40. pence, because that in graunting one, hee graunteth bothe, and in denying the one, hee denyeth both.

These cõuerses are not like the cõuersions in Logicke, where it were as foule an absurdity to say, Omnis homo est animall. Ergo, Omne animall est homo: as it were to say, that a mil horse, and an horse mil is all one. Euc. 19. prop. 9.

Philo. You haue saide reasonable well to the matter, wherefore sithe you confesse, this rule needeth no demonstration, I say foorth.

101. If any 3. numbers what euer they be,
Haue the product made of the latter twane,
Diuided forth with, by first of the 3.
The quotient doth shew the 4. number plaine,
In right proportion to these 3. propounded,
And on this verse the golden rule is grounded. } 77

Although the 70. rule make this verse manifest inough, yet for example take these 3. numbers. 6. 10. and 15. and let the product made of 10. and 15. the 2. latter numbers be 150. then let 6. (being the first of the giuen numbers) diuide this product 150. and the quotient (which is 25.) shall be the fourth number proportionall to the former 3. numbers, in right proportion, that is to say, in direct proportion, for as 6. 10. so is 15. 25. the triall whereof is euident by the 42 43. and 76. Marims.

And vpon this present verse dependeth the vsuall and common practise of the rule of proportions, commonly called the golden rule, or rule of 3. direct, which is the chiefest and most excellent of all the rules of Arithmaticke, as you will easilye

confesse, when you shall know the manifolde vses and applications thereof, but the reason of it riseth and is deriued from the 100. verse last before.

Eum. I thinke long till I come at it, though I dare not say so, but will abide such time therefore as you thinke meete.

Philo. You must needs do so, else shall your learning prosper like his husbandry, which thinking to haue a forward harueſt, reapeth vppe his corne, assoone as he seeth the eares shot out of the hose or blade. But to my purpose.

A coroll. to the 17. prop. 7. by Flussates.

102. If 2. or 3. fractions haue numerators,
all in one like proportion and rate:
Vnto their proper denominators.
Then are they equall and all of a state.
And by conuersion all fractions coequall,
assuredly haue their termes proportionall. 78

This verse as you see, conteineth both a rule & the conuerse thereof, so that if the first part be prooued, the conuerse needeth no proofe by your own confession in the glosse of the 76. marim, but the first part is manifestly prooued both by the 40. 42. 43. and 76. maxims, wherefore I seeke no further proofe, but go forward thus.

103. If a number be compounded or growne,
of 2. other numbers by Multiplication,
Then his owne measure and quotient once knowne,
are meane proportionals hauing relation
Vnto those numbers from whom they first grew,
betweene which coople they claime their place dew. 79

The truth whereof appeareth sufficiently by the 70. marim, and yet for more cleare vnderstanding thereof, let 12. the number there giuen be a number produced or growne by the multiplication of 3. by 4. then by the 72. Marim both of them, that is aswell 3. as 4. shall measure 12. and by the said 70. rule, if 3. measure it, then the quotient shall be 4. wherefore by this verse 3. being the measure, and 4. the quotient, shalbe meane proportionals to the same 3. & 4. from whose multiplication and diuision they first grew, and so I may set them betweene the first 2. numbers thus. 3. 3. 4. 4. and say, as 3. is to 3. so is 4. to 4.

But now let 12. haue some other measure, and then will necessarilie follow another quotient, and let 2. measuring 12. bring 6. in the quotient. Therefore by this verse, this 2. being the measure, and 6. the quotient, are also meane proportionals

to

to thesame 3. and 4. the numbers from whence they first grewe, and challenge their places betweene 3. and 4. the 2. extreames, thus 3. 2. 6. 4. or thus, $\frac{1}{1}$. and $\frac{6}{2}$.

Eu. This is playne inough, for any man that hath hard so much as you haue taught me, may perceiue the meaning of this Maxim.

Philo. Very well, I am glad you conceiue it so well, wherefore I say.

104. When numbers in any proportion assign'd,
Are of all others the smallest and lesse,
Then shall they measure the greater of their kinde,
All by one number in quotient expresst. } 80

Euclid. 21. prop. 7.

Although this propertie be derined onely from Diuision, yet could it not be perfectly demonstrated without knowledge of some of the other properties before explicated, and specially of the 78. maxim, wherefore it hath not bene deferred hetherto without good cause, but now admit 3. 7. and 9. to be numbers hauing none lesser in the like proportion to them, but let 12. 28. and 36. be greater numbers in like proportion to them.

Then by this verse I say, that 3. 7. and 9. are measures to the same greater numbers 12. 28. & 36. for if they be not, let the quotientes of 12. diuided by 3. of 28. diuided by 7. and of 36. diuided by 9. be 3. $\frac{2}{3}$. 3. $\frac{6}{7}$. and 3. $\frac{8}{9}$. Now here by this supposition these quotients 3 $\frac{2}{3}$. 3. $\frac{6}{7}$. and 3. $\frac{8}{9}$. are equal together, for by supposall 3. 7. and 9. are in the same proportion together, that are the greater numbers 12. 28. and 36. wherefore in these quotients being integers mixt with fractions, the integers 3. 3. and 3 are equall, for by supposition 12. 28. and 36. are greater proportionals to 3. 7. and 9. and therefore the integers of these quotientes being equall, if they be abated from the fractions, then shall the fractions by the 45. verse, remayne also equall together: And then by the 78. maxim as 2. the numerator of the first is to 3. his owne denominator, so is 6. the numerator of the second to 7. his owne denominator, and so is 8. the numerator of the third to 9. his propper denominator, but in these fractions $\frac{2}{3}$. $\frac{6}{7}$. and $\frac{8}{9}$. the tops, viz. 2. 6. & 8. are lesser then the bases, viz. 3. 7. and 9. by the glosse of the 56. maxim, whereby it should follow that 3. 7. and 9. haue lesser numbers proportionall to them, which is flat contrarie to our supposition, and therefore absurd and vntrue, wherefore I conclude that 3. 7. and 9. shal measure their proportionals greater then themselues.

And not onely measure them, but also they shall measure them

all by one selfe same number of times, that is, the quotient of euery such mensuration, shalbe equall or all one, which appeareth as well by this conclusion as by the supposition, for 12. measured by 3. and 28. by 7. and 36. by 9. make euery of their quotientes iustly 4.

Wherefore I conclude that as $\frac{3}{12}$ so is $\frac{7}{28}$. and so is $\frac{9}{36}$. but there are no other lesser numbers that can be in such like proportion to the same 3.7.& 9. as they the same 3.7. and 9. are to their proper consequents 12.28. and 36. Do you vnderstand this.

Euma. Yea truly very well, for I cannot chuse but vnderstand it as you haue expounded the verse and demonstrated it.

Philo. Then in going forward I say thus.

The conuerse of the former proposition.

105. Such number in any proportion assign'd
As measure the greater of their owne kind,
All by like quotient where euer they rest,
In that proportion are alwayes the least. } 81

Euma. I perceiue plainely that this verse is the conuerse of the former, for which cause I know the other being proued, this must needes be true.

Phi. It is euen so indeede as you say, wherefore I proceed thus.

106. If that 4. numbers being proportionall,
Do multiplie 4. proportionals more,
Then shall the products produced be all
Proportionals still like the rest before. } 82

This is a very notable marim, and such as shalbe hereafter applied to the multiplication of proportions: but for a present example for the better vnderstanding hereof, set you your selfe downe 4. proportionals in a line, & vnder them subscribe 4. other proportionals.

Euma. Then at all aduentures I take these, 8. 4. 6. 3.
vnto whom I subscribe these other as you see. 6. 4. 3. 2.

Philo. Now draw a line vnder them all, and let euery inferior multiplie his superior, and subscribe their products, and you shal see this marim verified in that those products shalbe proportionals also.

8. 4. 6. 3.
6. 4. 3. 2.
48. 16. 18. 6.

Euma. Then neuer writing the numbers as here you see, I haue done as you willed me, and the products are 48. 16. 18. and 6. as it appeareth, which indeede are proportionals also, by the 42. 43. and 76. rules.

Philo. This marim I would willingly haue demonstrated vnto you, and the rather for that I haue not found it demonstrated in

Euclid

Euclid, but because it would be somwhat difficult for you to vnderstand, & no lesse tedious for my self, I omit it at this present, because that now (the notable and necessarie properties of Integers rising by Multiplication and Diuision, aswell seuerally as ioyntly, being fully declared and explaned) I purpose forthwith, in one table to shewe you all the needfull distinctions, diuisions, names, and definitions of numbers, with their further properties, passions, and lawes, as cause shall minister occasion, and withall to expound the members and branches of the same table by the way, so farre forth as shall be necessarie for our purpose in this first part of vulgar Arithmeticke, wherefore behold the table.

Of the distinctions, diuisions, definitions, and names of true Integers, or entire numbers. Chap. 9.

107. Number is first diuided as you see,
In number abstract, and number contract,
And numbers abstract, are such as stand free,
from euery substance so cleare and exact,
That they haue no firname or demonstration,
saue the pure vnits of their numeration. } 83

Cos Record. f. 2.

This rule following the order of my generall talke of numbers sheweth you that numbers are first diuided into two sortes, viz. numbers abstract, and numbers contract, which thing the table it selfe sheweth also: Then it further declareth vnto you what maner of numbers, are numbers abstract, affirming them to be such as stand free, and are not tied, coupled, or restrained to any kinde of substance, and therefore voyde of any firname or denomination, sauing that the Aces or vnits, contayned in such numbers, may bee in such case reputed as a firname or denomination: as for example, these numbers 4.6.10. standing as I name them, are free from all maner of substances, or substantiues (for I thinke you are not ignorant that all numbers generally according to the rules of Gramer are adiectiues, and naturally require some substantiue to be ioyned with them, to expresse their signification.

The first diuision of numbers is into numbers abstract and numbers contract.

All numbers are adiectiues.

Euma. I haue been taught so:

Phil. But these abstract numbers being purely Mathematicall are not referred to any substantiue or firname, for when I saye 4.6.10. or that 10. and 10. make 20. I neyther couple nor restraine these numbers, to any weight, mesure, time, coyne, men nor

braſſes, nor to any other thing in the worlde, except their owne vnits whereof they are compoſed, in which reſpect I may ſay, that by 4. 6. and 10. is vnderſtoode 4. vnits, 6. vnits, and 10. vnits, and ſo the number of their owne vnits, ſtandeth in ſtead of a ſirname or denomination.

The ſirname, the ſubſtantiue and the denomination ſignifie all one thing.

Euma. So then I perceiue, that by the ſubſtantiue, the ſirname and the denomination, you meane all one thing.

Philo. Truly I do ſo, for although I willingly would, and generally do preferre & vſe the common termes onely proper to this art, yet I am compelled ſometimes for breuieties ſake, but more oft for meeters ſake, to vſe ſome other woordes then are generally accuſtomed thereunto, which neuertheleſſe are not ſo ſtrange, but that after you haue bene a little acquainted with them, ſhall appeare no leſſe apt to expreſſe the nature of the thing ſignified by them, then are thoſe others which (for the cauſes aforeſaide) I am conſtrayned to omit, yea and more eaſie for ſuch as are altogether ignorant of the latine tongue: As for example, that which is generally called the numerator of a fraction, I ſometimes call the toppe or toppe part of the fraction, and each of theſe woordes ſignifie as playnely and as aptly, what is meant thereby, as doth the word numerator.

Vnitie, an vnit, an ace &. 1. betoken all one thing.

Likewiſe, that which in this art is generally called vnitie, or an vnit, or an integer, I many times name an ace, and yet I doubt not but you vnderſtand me aſwell when I ſay an ace, as when I ſay an vnit or an Integer.

To fold, infold, and multiplie, haue all one ſignification.

Euma. I muſt needes confeſſe that to be true, and ſo when you bid me fold any one number in another, I know it is as much to ſay, as if you bid me multiplie them together, for as both an ace and an vnit ſignifie but 1. So to multiplie ſignifieth nothing els but an often folding.

Philo. Well then concerning this woorde ſirname (becauſe to ſome, it may ſeeme a woorde farre fetched to be vſed in this arte) I pray you tell me, what do we Engliſh men moſt commonly vnderſtand by it, or what do you your ſelfe take it to be.

Sirname qd.

Euma. I take a ſirname to be a name of difference, whereby diuers perſons of one like forename or like chriſtian name, are diſtincted and knowne one from another: as when I ſay, Marcus Marcellus, Marcus Antonius, Marcus Aurelius, beere be 3. ſeuerall men, all whoſe forenames are like, and all one, viz. Marcus, but their ſirnames being vnlike maketh the diſtinction and diuerſitie

sitie of their persons: so likewise whē I say Peter Bembus, Peter Martir, Peter Ramus, here Peter is their christian name cōmon to them all three and the one is Peter aswell as the other, but their sirnames being vnlike, make the differēce betweene them, for though Peter Bembus be Peter, yet is he not Peter Martir, nor Peter Ramus.

Phi. You haue said very well, wherfore euen so, when any numbers are contracted, that is to say, when numbers being abridged from their generall liberty of signification are restrained & bound onely to signifie some one perticuler substantiue, or some speciall parts of a fraction, be it proper or improper, then do I call the substantiues whereto such numbers are so bound and restreyned, and such speciall parts of fractions, their sirnames, (vsing that metaphorical speech for the similitude that is between them), as when I say 10. men 10. horses, or 10. dayes. Here you see the number 10. resembleth a forename or christian name, common both to the men, horses, and dayes, euen as Marcus was common forename to your 3. Romaynes. But the men, horses, and dayes, being vnlike, resembleth their sirnames, and distinguisheth the one ten, from the other. So that though all be tennes, yet the 10 men, signifie nothing els but men, and neyther horses nor dayes, no more then Peter Bembus, signifieth eyther Peter Martir or Peter Ramus.

And so likewise in these fractions $\frac{1}{4}$. $\frac{1}{5}$. $\frac{1}{7}$. you see the number of the parts expresst, to be all like, that is to say, 3. of euery sorte. but their sirnames are not like: for the first is quarters, the second fift parts, and the third seauenth parts: and so the first 3. signifieth onely $\frac{1}{4}$. and not fift parts, nor seauenth parts.

And againe, as there may be sundry betweren or diuers men of like sirname, which yet differ in Christian name, as Lawrence Valla, and George Valla, and such other: euen so, aswell Integers as the numerators of fractions may differ in their forenames, (that is in their likenesse or equalitie,) and yet agree in one sirname, as thus, 6. men, 8. men, 10. men: or thus, $\frac{3}{7}$. $\frac{4}{7}$. $\frac{5}{7}$. where you see the numbers which represent the forename to differ, for the first is but 6. the second 8. and the third 10. but their sirnames are all one and like, viz. men.

The denominator of a fraction, the basse of a fraction, & the sirname of a fraction, are al one.

And likewise the numerators of the fractions, which resemble their forenames, differ each from other, but their sirnames are all one and like, videlicet, fift parts. And this sirname in whole numbers, is generally by the greater part of writers (and by me also

braſſes, nor to any other thing in the world, except their owne vnits whereof they are compoſed, in which reſpect I may ſay, that by 4. 6. and 10. is vnderſtoode 4. vnits, 6. vnits, and 10. vnits, and ſo the number of their owne vnits, ſtandeth in ſteade of a ſirname or denomination.

The ſirname, the ſubſtantiue and the denomination ſignifie all one thing.

Euma. So then I perceiue, that by the ſubſtantiue, the ſirname and the denomination, you meane all one thing.

Philo. Truly I do ſo, for although I willingly would, and generally do preferre & vſe the common termes onely proper to this art, yet I am compelled ſometimes for breuieties ſake, but more oft for meeters ſake, to vſe ſome other woordes then are generally accuſtomed therevnto, which neuertheleſſe are not ſo ſtrange, but that after you haue bene a little acquainted with them, ſhall appeare no leſſe apt to expreſſe the nature of the thing ſignified by them, then are thoſe others which (for the cauſes aforeſaide) I am conſtrayned to omit, yea and more eaſie for ſuch as are altogether ignorant of the latine tongue: As for example, that which is generally called the numerator of a fraction, I ſometimes call the toppe or toppe part of the fraction, and each of theſe woordes ſignifie as playnely and as aptly, what is meant thereby, as doth the woord numerator.

Vnitie, an vnit, an ace &. 1. betoken all one thing.

Likewiſe, that which in this art is generally called vnitie, or an vnit, or an integer, I many times name an ace, and yet I doubt not but you vnderſtand me alſwell when I ſay an ace, as when I ſay an vnit or an Integer.

Euma. I muſt needes confeſſe that to be true, and ſo when you bid me fold any one number in another, I know it is as muth to ſay, as if you bid me multiplie them togeather, for as both an ace and an vnit ſignifie but 1. So to multiplie ſignifieth nothing els but an often folding.

To fold, infold, and multiplie, haue all one ſignification.

Philo. Well then concerning this woorde ſirname (becauſe to ſome, it may ſeeme a woorde farre fetched to be vſed in this arte) I pray you tell me, what do we Engliſh men moſt commonly vnderſtand by it, or what do you your ſelfe take it to be.

Euma. I take a ſirname to be a name of difference, whereby diuers perſons of one like forename or like chriſtian name, are diſtincted and knowne one from another: as when I ſay, Marcus Marcellus, Marcus Antonius, Marcus Aurelius, beere be 3. ſeuerall men, all whoſe forenames are like, and all one, viz. Marcus, but their ſirnames being vnlike maketh the diſtinction and diuerſitie

Sirname qd.

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Phi. You haue said very well, wherefore euen so, when any numbers are contracted, that is to say, when numbers being abridged from their generall liberty of signification are restrained & bound onely to signifie some one perticuler substantiue, or some speciall parts of a fraction, be it proper or improper, then do I call the substantiues whereto such numbers are so bound and restreyned, and such speciall parts of fractions, their sirnames, (vsing that metaphoricall speech for the similitude that is betweene them), as when I say 10. men 10. horses, or 10. dayes. Here you see the number 10. resembleth a fore name or christian name, common both to the men, horses, and dayes, euen as Marcus was common forename to your 3. Romaynes. But the men, horses, and dayes, being vnlike, resembleth their sirnames, and distinguisheth the one ten, from the other. So that though all be tennes, yet the 10 men, signifie nothing els but men, and neyther horses nor dayes, no more then Peter Bembus, signifieth eyther Peter Martir or Peter Ramus.

And so likewise in these fractions $\frac{1}{4}.\frac{1}{5}.\frac{1}{7}.$ you see the number of the parts expresst, to be all like, that is to say, 3. of euery sorte. but their sirnames are not like: for the first is quarters, the second fift parts, and the third seauenth parts: and so the first 3. signifieth onely $\frac{1}{4}.$ and not fift parts, nor seauenth parts.

And againe, as there may be sundry bretheren or diuers men of like sirname, which yet differ in Christian name, as Lawrence Valla, and George Valla, and such other: euen so, aswell Integers as the numerators of fractions may differ in their forenames, (that is in their likenesse or equalitie,) and yet agree in one sirname, as thus, 6. men, 8. men, 10. men: or thus, $\frac{2}{3}.\frac{4}{5}.\frac{5}{7}.$ where you see the numbers which represent the forename to differ, for the first is but 6. the second 8. and the third 10. but their sirnames are all one and like, viz. men.

The denominator of a fraction, the basse of a fraction, & the sirname of a fraction, are al one.

And likewise the numerators of the fractions, which resemble their forenames, differ each from other, but their sirnames are all one and like, videlicet, fift parts. And this sirname in whole numbers, is generally by the greater part of writers (and by me also

also many times,) named and called the denomination, and in fractions it is sometimes called the denominator, and somtimes the basse as you learned by the 19. verse, all which I tell you before hand, to the end you shall not misunderstand what I meane, whensoeuer I shall hereafter vse any of those names.

Eu. I thanke you, for your good promonition, whereof I trust to be mindfull, and here by this one rule and your large exposition thereof, I learne not onely the difinition of numbers abstract, but also of numbers contract, for it is a good consequent, that if numbers which be not restrained nor bound to any substantiue or sirname, be therefore onely said to be numbers abstract, then such as are coupled or bound to any substantiue or sirname, must of necessitie be numbers contract, as are those 10. men, 10. horses, 10. dayes, which you rehearst, and such other like.

Numbers contract.

Phi. You say truth indeed, those and all such other, are called numbers contract vulgarlie, which as they cause the like sequele and effect, in all manner of operations and applications Arithmeticall, and haue the selfe same passions and properties that they haue when they are abstract, so doe they very often concurre, & are much intermixt, or rather more generally vsed in all examples and questions Arithmeticall, because in such questions and applications, the numbers generally are diuers wayes contracted, either to weight, measure, time, coynes, men, beasts, or some other thing, and therefore although it be true, that all the rules and preceptes of Arithmaticke, ought to consist of numbers abstract, as Saligniacus affirmeth: yet I dare auouche, that if no examples in numbers contract be annexed withall, the learner shall neither conceiue the rules so readely, nor remember them so perfectlye, nor yet passe through the operations with such pleasure and delight, as otherwise he should doe. But I will proceede to the diuision of numbers Abstract as I proposed.

Cos Recordi fo. 2

3. Kindes of numbers abstract, to wit, absolute, Relatiue, and figurall.

108. Of numbers abstract, are 3. sundry sorts
whereof the first of which I must intreate,
Are absolute numbers, which in truth imports,
but numbers abstract, absolutely set,
Loose from comparision or any relation,
to number or figure, in rate or in fashion. } 84.

Fu ma. What meane you by numbers abstract, set absolutely loose from comparison, or any relation to number or figure.

Phi. There are as appeareth, aswell by the table as by this rule,

3. sorts

3. sorts of abstract numbers, whereof the first sort are called numbers absolute, as when I say, 4. is an euen number, or 5. is an odde number, not referring nor comparing them to any other number, nor to any Geometricall figure, then such maner of numbers are not onely abstract from all sirnames or denominations, but also they are called absolute, because they are loose both from comparison to any other number, and also from application to any kinde of figure, for if I should say, that 4. is double to 2. or that 2. is halfe 4. then were 4. and 2. numbers relatiue together in rate or proportion, because as you see there is a comparison or relation between them, for 4. cannot be double but in respect of 2. neither can 2. be halfe, but in respect of 4. Likewie if I should say, that 4. is a square number, then were 4. a figurall number, because it is referd to a Geometricall figure named a square, and so in neither of those cases are to be accompted numbers absolute, although in both cases they yet remaine numbers abstract.

Numbers absolute defined.

Numbers relatiue defined.

Numbers figurall defined.

Eu. Very good, this distinctiō of nūbers absolute is plain inough.

109. *Phi.* And absolute numbers are parted agayne,
first as they soly, are euen or odd,
And second as they togethers do claime,
to lack or to haue some common metrod,
An euen number, is that (to be plaine,)
which may be euer iust measur'd, by twaine.

85

Cos Record. fol. 3 Eucl. 6. defin tion 7. Boetius. cap. 4. liber 1.

Numbers absolute are eyther euen or odd.

Hereby is declared vnto you, that absolute numbers are 2. wayes diuided and considered, to wit, first as they soly or alone by themselues, are eyther euen or odde. And secondly, as they are together or among thēselues, in which respect they either lack or haue some common meatrod, ỹ is to say, some number which may be a cōmon mesure to thē: furthermore by this rule an euen nūber is defined to be such a nūber as may allwayes be mesured or iustly diuided by 2.

definition of an euen number.

Eu. So then, 2. 4. 6. 8. 20. 42. 100. &c. are euen numbers by this definition, for euery of thē may be measured or iustly diuided by 2.

Phi. They are so: euen numbers are further distinguished thus.

110. And euen numbers are parted likewise,
into prime numbers, and numbers compound,
Where of the prime, is that which doth arise,
by 2. vnits onely vnited and bound,
Into one number by pure addition,
Besides which, none hath the like Condition.

86

Eucl. 12. d. 7. Euen nūbers are either prime or compounds.

This rule sheweth you, that euen numbers are of 2. sorts, whereof

the first is called a prime, & the second sort are called compounds.

Definitō of a prime number

No number euen, but are compounds except 2.

Now a prime number generally (whether it be euen or odde,) is that which is made onely by addition of vnits together, and not by the multiplication of any 2 numbers together, and therefore there is no mo primes of euen numbers but onely this number 2. which is made by the addition of one vnit to another, for 4. 6. 8. and so all other euen numbers whatsoeuer, are compounds, as you shall perceiue by the nert rule.

Eum. Very well, I finde no great difficultie in all this hetherto.

Philo. Then will I shew you the definition of compound numbers, which is thus.

Euclid. 14. d. 7. Compound numbers defined & distincted into 4. kinds.

111. All compound numbers, are such as are made,
not by adition but multiplication:
As twice 2. twice 3, and so by like trade,
whose knowledge is easie by this demonstration,
And euen compounds, in 4. kinds are driuen,
whereof the first is called euenly euen. } 87

Now following the order of my table, this rule sheweth you that such numbers are called compound numbers, as are made or produced, or possible to be made or produced, by the multiplication of any 2. nūbers together, as twice 2. which make 4. or twice 3. which make 6. for though 3. & 1. added together make 4. and so likewise though 4. & 2. make 6. yet is both 4. & 6. accompted truely for compound numbers, made by such multiplication, as was before expresst, and not by addition, in such forme as is heere last rehearst.

How nūbers are said to be compounded.

Euma. So then I perceiue, that although 8. & 4. make 12. and likewise though 10. and 2. make 12. yet I must not accompt 12. to be made of 8. and 4. neither yet of 10. and 2. for 8 times 4. will make 32. and 10. times 2. make 20. both which products far exceed 12. but I may say, that 12. is made or compounded of 2. by 6 or of 3. by 4. because twice 6. make 12. and so doth 3. times 4. also.

Philo. That was well considered of you, and as you see that 12. may be compounded by any of those 2. couple of numbers, so may you say, that it is made or compounded of which couple you will, & so I may say, that 36. is compounded eyther of 2. times 18. or of 3. times 12. or of 4. times 9. or of 6. times 6. for euery way there is 36. produced. Now these euen compound numbers are driuen or seperated into 4. distinct kindes, as the rule telleth you, whereof the first, are called numbers euenly euen.

Eum. I vnderstand not what you meane by euenly euen.

Philo.

Euc.8.d.7. Ard.32. prop.9. Boetius. 9.cha.1. book.

Phi. 112. Euenly euen, by those numbers soly,
which yeeld euen halues till 1. be left wholy. } 88

Lo heere is a short definition of numbers euenly euen, and it is as playne as it is short, for those numbers onely are truly named euenly euen, whose halues are alwayes euen and neuer odde till they come to an ace, as 16. whose halues are 8. & 8. And the halues of 8. are 4. and 4. And the halues of 4. are 2. and 2. And finally the halues of 2. is 1. and 1. And so now 1. onely being left, nature it selfe forceth a stay, because vnitie is indiuisible by nature.

The first kinde of euē compound nūbers defined.

Eu. Then hereby I gather that though 12. be an euen number by the 85. rule, yet it is not of this kinde, for the halues thereof are 6. and 6. and the halues of 6. are 3. and 3. but now 3. hath no halue in true Integers or whole numbers, wherefore sith heere is a stay before we come to vnitie I conclude that 12. is no number euenly euen.

Phi. Your conclusion is good and true, wherefore I will handle the next sort of euen compounds whose definition followeth.

Eucli.9.d. 7.

113. The next sort are nam'd, vneuenly, euen,
and this is the token whereby they are knowne,
They may be measur'd, as measure is giuen,
by euen or odd, and so els can none. } 89

The secōd sorte of euen compound nūbers defined.

Euma. This rule and definition is plaine inough of it selfe, for first it sheweth that the second sort of euen compound numbers are called numbers vneuenly euen. And secondly, that they may bee knowne from all other numbers, because they are measurable both by an euen number and an odde, as occasion shall serue, and so are no others, but numbers of their kinde, as this rule affirmeth, whereby I gesse, that 12. whom euen now I prooued to be no number euenly euen: will prooue it selfe to be of this second kinde a number vneuenly euen, for it may be measured both by 3. which is an odde number, and also by 4. which is an euen number, and so likewise may 6. be measured by 2. or 3. whereof the one is euen and the other is odde. Wherefore if I vnderstand you rightly, 12. and 6. are numbers vneuenly euen.

Philo. You vnderstand me very well, wherefore I will goe to the definition of the third sort of euen compounds.

Eucli.23. d.7.

114. The 3. sort are named perfect numbers soly,
because all their parts, make no more nor lesse,
But their owne number, iustly and wholy,
without any want or any excesse. } 90

The third sort of euē compoūds are numbers perfect.

As 6. whose halfe part is 3. the third part 2. and the sixth part 1. (for as for quarter or fift part it hath none) now 3. 2. and 1. being all the parts of 6. added together, make also iust 6. and neyther more nor lesse, wherefore 6. is a perfect number.

Eu. Very well, I pray you proceed to the fourth kinde of these euen cõpounds, which by your table are called numbers imperfect.

Records Whetston of wit, fol. 4. The 4. sort of euẽ compoũd numbers.

115. *Phi.* The 4. kinde, are call'd numbers imperfect,
for none other cause, but onely in respect,
Of perfect numbers, whose iustice they neglect,
by their aboundance or by their defect,
For take all their parts, by iust and true title,
and still they will make too much, or too little. } 91

Numbers imperfect

As for example, take 12. the halfe whereof is 6. the third part 4. the fourth part 3. the sixth part 2. and the twelueth part 1. (and as for any other part it hath not) now add together all these parts that is to say 6. 4. 3. 2. and 1. and they make 16. which is 4. more then the whole number 12. wherefore such a number is named an imperfect number abounding.

Numbers aboũding.

Againe, take 8. and you shall finde $\frac{1}{2}$. thereof to be 4. and $\frac{1}{4}$. thereof to be 2. and $\frac{1}{8}$. to be 1. (and as for any other part it hath none) now put all those parts which it hath together, that is to say, 4. 2. and 1. and they make but 7. and so is too little by 1. to make the whole number 8. wherefore the same 8. and all such like numbers, are named numbers imperfect defectiue.

Numbers defectiue.

Eu. Both ỹ names & definitions of these numbers, are agreeable to reason, but now by ỹ order of your table, od nũbers should follow.

Philo. They do so, and the definition of an odde number is thus.

Euc. 7. d. 7 Odde numbers defined.

116. An od number cannot be measur'd by twaine,
but some od vnit will allwayes remayne,
Wherefore the difference, betweene od and euen,
is but an vnit abated or giuen. } 92

You learned before by the 109. verse, touching the definition of an euen number, that it was such a number as might alwayes be measured by 2. now heere by this verse, an od number is contrarily defined to be such a number as cannot be measured by 2. wherfore al numbers that are not euen, must needs be od, for either they may be measured by 2. or they may not: if they may, then are they euen by that 85. rule. If they may not, then are they odde numbers by this rule. And further this rule saith, that the difference betweene odde and euen, is but an ace abated or giuen. As for example, 11. is an od number,

number, for it cannot be measured by 2. yet the difference between it, and an euen number, is but an ace abated from it, or giuen to it, for if you abate 1. from 11. the residue is 10. which is an euen number by the 85. rule: or if you put but 1. to 11. it maketh 12. which is an euen number by the same rule also.

Euma. All this I perceiue well, wherfore I pray you go forward.

117. *Phi* Od numbers are all either primes or compounds,
the ptimes arise onely by meere addition,
Of vnits togeather, not breaking that bounds,
and so 3. and 2. make 5. by tradition. } 93

Eucli. 11. d. 7. Od numbers diuided into primes & cõpounds.

Eu. This definition is easie inough to vnderstand by the 86. rule. and the annotations thereof, where it was said, that ptime numbers generally, whether they be euen or odde, are such as are made only by the addition of vnits together, & not by multiplication, which definition it seemeth, extendeth principally to od ptime nũbers, sith there is but one only ptime number euen, as there it was declared.

Philo. You haue said very truly, for indeed there are no mo euen ptimes but the number 2. but the multitude of od ptimes is infinite, & so 3. is an od ptime number made by addition of 3. single vnits, or of 2. & 1. and likewise 5. is an od ptime number made by the addition of 5 single vnits, or of 3. and 2. or els of 4. & 1. and so fooyth of 7. 11. 13. 17. 19. and such others, for none of all these kinde of of numbers can arise by multiplication of any Integers.

A corollary to the 20. pro. 9.

Eu. I vnderstand you thus. That there can be no 2. whole numbers found to be multiplied together, to make or produce 2. 3. 5. 7. 11. 13. 17. 19. or any such others, & therefore they are aptly called ptimes, as those that are first originally of themselues, or of single vnits, and not compounded or made of other numbers.

Phi. You vnderstand me right ly, wherfore now we wil go in hand with the definition of odde numbers compounded, which is thus.

118. Odde numbers compound, are such as amount,
of 2. odde numbers by Multiplication,
As 3. times 3. which yeeld 9. by account,
and is a patterne to know all their nation. } 94

Euc. 11. d. 7. calleth this, a nũber odlye odde.

Eu. This I suppose I vnderstand wel inough, for hereby I iudge 15. to be an od compound number, made of 3. by 5. & likewise 21. which is made of 3. by 7. & so 25. produced of 5. by it self, & 35. rising of 5. by 7. are all od compound numbers, for they are all cõpounded of 2. od nũbers, & withal I perceiue ỳ thogh al ptime nõbers (except 2.) are generally od, yet all od are not therefore generally ptimes.

Od numbers compound defined.

Philo. You conceyue all these thinges well, and the numbers by you rehearsed are such, and so made, as you haue sayde, and thus haue we ended the first branch of our diuision of numbers abstract absolutely, as they are alone by themselues, wherefore now I will vtter to you certayne notable proprieties naturally incident to this kinde of numbers, and then I will prosecute the second branche.

Of the properties of numbers absolute, as they are alone, or by themselues, euen, or odde onely. Chap 10.

Euclid. 21. pro. 9.

119. If euen numbers be added togeather,
The totall shalbe as euen for euer. } 95

As for example, let 2. 4. 6. and 8. be euen numbers by the 85. rule, and let their totall be 20. now I affirme 20. to be an euen number also, which thing appeareth true by the same 85. rule.

Euclid. 24. pro. 9.

Eu. This is playne inough.

120. *Phi.* An euen number from euen abated,
Leaues the remayner as euenly rated, } 96

Euclid. 28. pro. 9.

Eu. This followeth consequently by the former verse.

121. *Phi.* Let euen multiplie what number you will,
The product thereof shalbe euen still. } 97

As for example, let 4. which by the 85. rule is an euen number multiplie 6. which is also euen, and the product shalbe 24. also let the same 4. multiplie 3. and 5. which are odde numbers, and the products shalbe 12. and 20. all which products are euen numbers by the sayd 85. rule.

Euma. All this is manifest, wherefore I pray you proceede.

122. *Phi.* If euen do measure a number assign'd,
The number so measured is euen by kinde. } 98

Eu. The reason of this rule, and of that next before is both one.

123. *Phi.* If euen and odde be made but one summe,
The totall is odde that thereof doth come. } 99

Euma. That must needes be truely so, for if it should be euen, then the difference betwene an euen number & the next odde number should not be an vnit as the 92. rule affirmeth.

Euclid. 21. and 23. prop 9.

124. *Phi.* If many odde numbers be added togeather,
VVhose multitude is likewise odde also,
Then shall the totall of them for euer,
Be an odde number who euer say no,
But if the multitude of them be euen,
Then so shall the totall be of them giuen, } 100

For

For if you take but an odde vnit from each odde number, the same numbers will become euen numbers by the 116. verse, & so in ỳ case their total shalbe euen by ỳ 95. rule, wherfore (if abating nothing) you ad them as you finde them, their total shalbe od by the 99. rule.

Eu. I perceiue well it is so.

125. Odde taken from euen, or euen from odde,
Leaues odde remayning as round as a roode. } 101

Euclid. 25. & 27 prop. 9.

For if euen taken from odde, should leaue the remayner euen, then the abatement & remayner contoined againe by addition should yeelde the totall euen, by the 119. verse, which were contrarie to our supposition, and agayne if odde taken from euen, should leaue an euen remayner, then the same abatement added agayne to that remayner should make the totall odde by the 99. rule, but that is also contrarie to the supposition, and therefore absurde.

Eu. Very good I pray you go on.

126. *Phi.* All numbers of 2. odde numbers compounded,
Shall also be odde the rule is so grounded. } 102

Euclid. 29. prop. 9.

Eu. That must neeves be so by consequence of the 94. rule.

127. *Phi.* All the true measures of odde numbers be,
Odde numbers themselues by natures decree. } 103

Which thing appeareth both by the former rule, and also by the 62. rule, for if any one of them should be euen, then 2. in measuring it should also measure an odde number, which should be contrarie to the 92. rule.

Campanus in 2. prop. added to Eu. fol. 224. d.

Eu. It should be so as you say, and therefore could not be true.

128. *Phi.* If an odde number, measure an euen,
The quotient shall euer be euen also:
And eke that number for measure so giuen,
shall measure halfe of the diuidend too. } 104

Euclid. 30. prop. 9.

The first part of this maxim is manifest, for if the quotient of an euen number diuided by an od number should be od, then should the diuidend or number diuided be od also by the 126. verse, but ỳ were contrarie to our supposition, now for the second part, let 12. be an euen number, and let 3. being an odde number measure the same by 4. here 4. being the quotient is an euen number, by the 85. rule, agayne, by supposition 6. is halfe of the diuidend 12. wherfore I affirme that 3. ỳ od nũber first so geuen for a measure, shal also measure the same 6. which thing appeareth sufficiently by my notes on the 53. rule.

Eu. In all these rules I see no doubt nor difficultie,

129. *Phi.*

Euc. 12. d. 7.

129. *Phi.* Now note that each number alone by it selfe,
Is eyther a prime or a number compound,
Whereof the prime number is such a pore elfe,
That his greatest measure which is to be found,
Is eyther it selfe or els but an ace,
For other measure it hath in no case. } 105

So 2.3.5.7.11.13.17. and infinite such like, are prime numbers, for they are measurable by no number, but by thē selues or by an vnit.

Euc. 14. d. 7.

103. But numbers compounded, as they arise duly,
Of two other numbers by Multiplication,
So their greatest measure besides themselues truly,
Is alwayes plurall, and this is their fashion,
There is no number compound but it may,
Be measured by some prime number alway. } 106

For by the 87. rule let 48. be a number compounded of 6. by 8. or of 4. by 12. Now by this rule I affirme that some prime number shall measure this compound number 48. for let some other nūber measure the measure of this compound number, and so the measure of that measure as often as it may, yet here because 48. is a cōpound number, whose vnits by the 10. rule are not infinite, therefore the last quotient exceeding an ace, shall finally be a prime number by the 105. rule, and then the same prime number measuring any measure of 48. the given number shall also measure the same 48. if selfe by the 62. rule, which is the thing I entended to proue.

Eu. This proofe seemeth vnerpugnable in my slender iudgement.

Phi. Well thus hauing finished the explication of the first branch of numbers absolute, and sufficiently shewed their seuerall names, kindes and definitions, according to the order of my generall table wᵗʰ an appendix contayning their notable properties, very necessary to be knowen, now wil I returne to ẏ explicatiō of the other branch to wit, as they are together among themselues, as I promised.

Of numbers among themselues togeather, that is, as they be either Primes togeather, or compounds together. Chap. 11.

Each nūber to other is either commensureable or incommensurable togethers.

131. All numbers among themselues are eyther,
Primes togeather or compounds togeather. } 107

That is to say, as all numbers absolute, whatsoeuer they be, whether they be euen or odde, perfect or imperfect, being taken & considered alone by themselues, are numbers either primes or cōpounds, so if there be mo then one taken together, then are those numbers (among themselues) either primes together, or cōpounds together.

Euma.

Eu. I must confesse I neyther vnderstand this verse, nor your meaning in this behalfe, but if you did propound vnto me any certayne numbers, as these, 5. 17. 9. 15. & 20. and did demaund of me what kinde of numbers euery of them are, I could answere by the 92. and 93. rules, that 5. and 17. be odde prime numbers, and by the 92. and 94. rules, that 9. and 15. be odde compound numbers: and by the 87. rule, that 20. is an euen compound number, and further if I were demaunded what kinde of euen compound number 20. is, I could answere by the 89. & 91. rules, that it is a number vneuenly euen, imperfect abounding.

Philo. You seeme both to vnderstand, and as perfectly to remember the rules which you haue hetherto learned, for those numbers by you propounded, are such indeed as you haue affirmed them to be.

But all this is nothing touching this present verse, for you consider still onely what euery number is alone by it selfe, which thing indeed was incident to the former branch of numbers absolute whereof I last intreated, but in this second branche, both the meaning of the verse, and of my wordes also, is to consider 2. or mo numbers, as they are togethers, one to another, among themselues and not severally, for there cannot be fewer then 2. numbers called primes together, or compounds together.

Eu. Very well, if you would vouchsafe to define what maner of numbers you call primes together, & which you call compounds together, I doubt not but I should conceiue ỳ matter much ỳ better.

Philo. That will I willingly doe: and first primes together, are thus defined.

Euc. 13. d. 7. Numbers prime together defined.

132. All primes together, haue no common measure, } 108
Exceeding an ace which is all their treasure. }

Eu. Lo sir, Now I perceiue euen already by this definition that 5. 17. & 9 the three first of my late propounded numbers, are primes together, for I see that no one number or measure exceeding an ace or vnit, can be common measure to them all three.

Phi. No nor yet to any 2. of them, but 5. and 15. or (if ye will take 3. together) 5. 15. and 20. haue a common measure exceeding an ace, whereby they may all be measured, that is to say 5. one of the geuen numbers themselues, for euery number measureth it selfe by the 53. rule, wherefore by this verse or maxim, 5. 15. and 20. are no primes together.

Eu. And so if I take but 9. and 15. together, I thinke they are no primes together, for they may both be measured by one

common

common measure 3. which is more then an ace.

Philo. You thinke euen as the truthe is, and yet if you had taken 9. 15. and 20. togethers, then should you finde them primes together, because that no one number exceeding an ace, can measure them all three: for although 9. and 15. may both be measured by 3. yet the same 3. can in no wise measure 20.

Eum. So then I perceiue, that if any one among many giuen numbers, cannot be measured by that common number, which measureth all the rest, such impossibilitie maketh them all to be primes together, because the same one number is not subiect to that common measure, & yet were it not in respect of that common measure, peraduenture none of them all are primes.

As these 3. former numbers, 9. 15. and 20. euery of the which being taken alone by it selfe, is a compound number, & no prime.

Phi. You haue said well and truly to the matter, and therefore now seeing you vnderstand it so well, I will rehearse the definition of numbers compound togethers, which is thus.

Euel. 15. 6. 7.

133. Compounds together are numbers which haue,
some common measure exceeding an ace,
And alwayes to chuse they specially craue,
some one prime number to serue in that case. } 109

Numbers compound together defined.

Eum. The first part of this rule is very plaine, for thereby I perceiue that all those numbers which haue any common measure aboue an ace, are such as you call compounds together, and therefore 5. 15. and 20. being three of the numbers before propounded by me, are compounds together, because they haue 5. for a common measure to them all.

Phi. You might as easily haue perceiued that by the first verse of this chap. for sithe by the last rule they were found not to be primes together, they must of necessitie be cõpoũds together by ỳ same vers.

Eum. It is very true that the same verse teacheth so, which I acknowledge I did not remẽber, or at the least, not consider til now. But now withall I also see further, that such numbers as by themselues are meere compoũds, yet among themselues may be primes together, as are those nũbers 9. 15. & 20. before rehearsed, & againe that such numbers as alone by themselues are meere primes, yet among thẽselues with others may be compounds together, as here 5. 15. & 20. which are truly compounds together, & yet 5. by it self alone is a meere prime nũber, as appereth by ỳ 86. rule, & 93. rule.

Phi. They are so as you say, wherfore seeing you vnderstand their

diuersities

diuersities so well. I will explaine the latter part of this rule, which as yet we haue not touched, and therefore as the first part contayneth a perfect definition of numbers compound togeather, shewing you that they are such numbers, as haue some common measure togethers aboue an ace or vnit, so the latter part assureth you, that euermore infayliibly some one prime number may be that common measure, seruing to measure such compounds together.

For if a prime number measure 2. or 3. geuen numbers, then you know the truth thereof appeareth euen sufficiently of it selfe, but if a compound number measure them, yet some prime number may measure that common measure by the 106. rule, wherefore by the 62. rule the same prime number shall also measure those compounds together, which was to be proued.

Euma. The proofe is sufficient.

Philo. And here by the way, I must not forget to tell you that all primes togeather, are generally called (by Arithmaticians) numbers incommensurable, which is as much to say, as numbers not able to be measured togeather by any one number, for although all true numbers vniuersally are measurable togeather, by an vnit, yet sith vnitie causeth no alteration, neyther by Diuision nor yet by Multiplication, but the numbers measured or multiplied by it, alwayes returne immutably the selfe same, both for quotiene and product, that they themselues were before, therefore they are named numbers incommensurable.

But compounds together, because they are measurable together among themselues, by some one number being a common measure to them all, therefore they are as generally named numbers commensurable, or communicants, or symmitrials, and these last kinde of numbers haue farre greater facilitie in many operations and applications Arithmaticall, then haue numbers incommensurable or primes togeather, as you shall finde hereafter.

Eu. Very good, all this I trust to remember.

Phi. Thus haue we fully ouerrunne all the branches of numbers abstract absolutely, with their seuerall names, kindes, definitions, and proprieties, as they are comprised within the first part of Arithmatiche, sauing that there resteth onely to declare the proprieties of these numbers, primes, and compoundes togeather, vpon whose knowledge dependeth many principall poyntes of this arte, which therefore I will shew you forthwith, and that being done, I will stay from proceeding any further, into the diuisions

and

and distinctions of nũbers, after the ordꝛ prescribed in his latter table of numbers vntil I haue taughe you reductions, & the operations of fractions as they now follow in my first table.

Euma. I pray you do euen as you thinke best.

Of the properties of numbers among themselues, viz of Primes together, and compoundes together, otherwise named numbers incommensurable, and numbers commensurable. Chap. 12.

Euclid. 23 prop. 7. Euc. 24. prop. 7.

134. *Philo.* If numbers geuen be primes togeather,
Then haue they no lesse proportionals neuer,
And such as haue no proportionals lesse,
Are primes together as all men confesse. } 110

Demonstration.

The latter part of this verse is the conuerse of the first part, wherefore the first part being verified, the latter is not to be doubted, therefore let 4.6. and 9. be primes together: Now I say by this verse that they haue no proportionals lesse then themselues. for if any man will anouch that 2.3. and 5. (being lesse then they) are proportionals to them: that is to say, in like proportion togeather, as 4.6. and 9. are, then by the 104. verse 2.3. and 5. must measure 46. and 9. all by one number in the quotient: and so by this supposition, primes together shall haue a common measure exceeding an ace, which is contrarie to the 132. verse, and therefore impossible, wherefore I say agayne by the latter part of this verse, that sith 4.6. and 9. haue no lesser proportionals in the same proportion that they themselues are, therefore the same 4.6. and 9. are primes togeather, and incommensurable, by any one number.

Euma. All this must needes be vndoubted true.

135. *Phil.* All numbers hauing proportionals lesse,
Are compounds togeather by any mans gesse,
And contrarily all compounds togeather,
Assuredly haue proportionals lesser. } 111

These are conuerses to the former.

Euma. The truth hereof appeareth manifestly by the last verse, for the consequents of contrarieties are so contrarie, wherefore I pray you goe forward.

Euclid. 36 porp. 7.

136. *Philo.* The product made of 2. primes togeather,
Is diuident least and common to both,
Which thing by a triall may soone be seene whether,
It be so or not, it needeth no oth. } 112

Demonstration.

As for example, let 3. and 5. be primes togeather, and let their product be 15. hereupon I affirme presently, that 15. is the least common diuident or number measurable, both by 3. and 5. which

is

is so apparant that it needeth no further proofe also.

Phi. 137. If numbers measured by one measure make,
Their quotientes in primes together to fall,
Then is that measure I dare vndertake,
The greatest measure common to them all. } 113

Campanus a corallary to the 35. prop. 7.

For example whereof let euery of these 3. numbers 16. 24. and 36. be measured by this one number 4. so are the quotientes of their commensuration 4. 6. 9. which by the 134. verse are primes together, wherefore I affirme by this verse, that 4 is the greatest common measure to the same, 16. 24. and 36. the numbers geuen which is so manifest that without further proofe thereof, I proceede thus,

Demonstration

138. If a measure meating numbers whatsoeuer,
Be the greatest measure common to them all,
Then shall their quotients be all primes togeather,
And also the least numbers proportionall,
To those measured numbers at first so assign'd,
Which propertie failes not, it comes so by kinde. } 114

This is the conuerse of the next before.

The example and proofe of this verse is thus. First it appeareth by the 137. verse, that 4. is the greatest common measure to 16. 24. and 36. because the quotientes of their commensuration by 4. were 4. 6. 9. which are primes together, wherefore now by this verse, I affirme also, that 4. 6. 9. are not onely primes togeather, but also (by the 134. verse) I say further, that because they are primes togeather, therefore they are the least numbers proportionall to the same 16. 24. and 36.

Demonstration.

Euma. That is euident for as $\frac{16}{12}$. so is $\frac{4}{3}$. and as $\frac{24}{18}$. so is $\frac{4}{3}$. but so are not any other numbers lesser then 4. 6. 9.

Ph. You say truly, now another propertie is thus. vt in versu. 139

139. If a number being absolutely prime,
To measure some other make any deniall,
Then both those numbers from time vnto time,
Are primes togeather, well knowne by such triall. } 115

As thus, let 5. be a prime number, denying to measure 12. that is to say, which cannot measure it, now I affirme by this verse that these geuen numbers 5. and 12. are primes together, which if it be gainsaid, then the same 5. and 12. by the 10. & 109. rules are compounds togeather and haue some common measure, exceeding an ace, & so by this contradiction, the prime number 5. shall haue another measure besides it selfe and vnitie, which is agaynst the 129. verse, therefore absurd, agayne.

Demonstration.

Euclid. 13. prop. 7.

140. If a prime number proposed will measure,
the product of any one couple in troth,
Then will it measure, at your like pleasure,
one of that couple at least, if not both. } 116

Demonstration. For such a prime number shall be compound together, with one of the numbers proposed at the least by the 133. verse, and heere I say with one of that couple at the least, because sometimes it is compounded to both, as may appeare by 3. which being a prime number, will not onely measure the product of 6. by 9. that is to say 54. but it will also measure both 6. and 9. but now suppose 4. and 6. to be a couple of giuen numbers, whose product is 24. and let 3. be a prime number, measuring the same 24. by 8. Now although the same 3. cannot measure 4. one of the giuen couple of numbers, yet I say, that it shall measure 6. the other of the same couple: for 3. being a prime number, not measuring 4. by supposition, shall therefore be prime to it by the 139. verse. Moreouer, sithe 3. is supposed to measure 24. by 8. the product of 3 by 8. shall also be 24. by the 96. verse, and by supposition the product of 4. by 6. is likewise 24. Therefore by the 59. verse, as 3. is to 4. so is 6. to 8. but 3. and 4. being primes together, as appeared are the least numbers in their proportions, by the 134. verse, wherefore by the 104. verse, 3. & 4. shall measure 6. and 8. by one selfe same number or quotient, and so consequently 3. shall measure 6. one of the giuen couple, which was the thing to be proued, & vpon his verse dependeth the finding of all the measures to any compound number assigned, the manner whereof is this.

Peter Ramus in the 7. ca. of his 1. booke of Arith. Salig. 13. cha. of his first book

141. To finde all the measures to numbers compound,
2. seuerall workes of force must be wrought,
For first the prime measure require to be found,
as those by whose helpe the compounds are sought,
Wherefore of prime numbers, first loke out the least,
most meetest to measure your number assignd,
VVhereby diuide it and so let it rest,
diuiding the quotient that thereof you finde,
And euery quotient from time vnto time,
as long as a number doth therein remayne,
Renuing diuision by prime after prime,
till lastly an vnit doth bid you refraine,
Then take those diuisors disperfed a sunder,
for all the prime measures vnto your first number. } 117

Euma.

Eum. This rule is very darke without an exposition.

Phi. I will lay open the meaning therfore to you by an example so plainely, that I trust you shall not chuse but vnderstand it, and also practise it, and that very easily: and for example, let 420. be a compound number, all whole measures I desire to know: Now by this verse I must first seeke the prime numbers that measure it, before I can finde the compound numbers that measure it, and to do ỳ, I must also loke out the least of all prime numbers, that will serue to measure the same, 420. but the least prime number, by the 110. verse is 2. therefore by it I diuide the geuen number 420. And the quotient is 210. as here you see.

420 (210
zzz

Now according to the propoyt of the verse, I di- uide this quotient 210. agayne by 2. and so the second quotient is 105. as here appeareth also.

210 (105
zzz

This being done, it is apparant to sight that this prime number 2. cannot measure the quotient now remayning, wherefore I must renew my measuring or diuision of the quotient, by the next least prime number, which as you knowe by the 117. verse is 3. therfore renewing diuision by this prime 3. it maketh in the quotient 35. as here you see.

105 (35
x
zz

But now I may easily gesse agayne, that 3. cannot measure the 35. remayning, wherfore I must yet agayne seeke for the next least prime number that can measure it, but the least prime number nert aboue 3. is 5. as you know by the 117. verse, wherfore I diuide this last quotient 35. by 5. and the quotient is 7. as here appeareth.

35 (7
5

And now 5. cannot measure this 7. remayning in the quotient as your selfe can readily gesse, wherfore I must yet agayne renew my diuision, by the next least prime number aboue 5. which is 7. it selfe, and so diuiding 7. by 7. the quotient is 1. by the 77. verse, and this 1. now left thus in the quotient doth by this verse bid me refrayne, or leaue, as a token that I haue done with diuision, and that I must take all the diuisors disperſed a sunder, as all the prime measures vnto the first number, 420.

Euma. The diuisors I trowe were but 2. 3. 5. and 7. and therefore by this rule 2. 3. 5. and 7. are all the prime numbers that can measure 420.

Philo. And euen so are they, for as touching an vnit or 1. though it be a common measure generally to all numbers, aswell primes as compounds, yet because it selfe is no number, neyther prime

nor compound, by the 15. verse, I make no mention thereof among these measures, but now you shal here how by ỹ knowledge of these prime measures all the compound measures of the said number 420. shalbe likewise found and knowne.

The 2. part of the probleme

142. Which done set them downe, all new in a rowe,
each of them as oft as it serued for measure,
VVhereof the greatest first highest bestowe,
and so the next great right vnder at pleasure,
Proceeding forth so, till least be set last,
then hauing those primes in order so plac'd,
Euen then I say if the highest saue one,
do multiplie that which highest doth rise,
And next if the rest of primes euery 'chone,
do multiplie all aboue them likewise,
And allso the products produced of those primes,
collaterally, but once and no more,
Then all those offcomes shall measure at times,
your compound number first giuen before,
VVhich offcomes thus had, and primes before found,
are all his measures, both primes and compound. } 118

I doubt not but you perceiue by the last verse afore this, and by the example thereupon giuen, how all the prime measures to the giuen number were found, wherefore now by this verse is taught the finding of all the compound numbers that shall measure the same giuen numbers.

And first this verse biddeth me to set all the prime measures found, downe in a rowe, not once onely, but each of them so often as it serued for measure, and that in such order, as the greatest may be first and highest, and so the next great, and then the next great to that, and so forth till the least be set last. Wherefore I begin first with 7. the greatest prime measure to our number 420. which because it serued for measure but once, I set downe but once, and sith 5. which is the next great prime measure we had, did serue for measure but once also, I set downe the same but once also, & that right vnder the 7. & vnder that I likwise set 3. once, which being the nert prime to 5. serued for measure once also, finally comming to 2. the least of our prime measures, I set it downe last right vnder 3. twice, because it serued for measure twice, and so the numbers stand as heere you see.

7.
5.
3.
2.
2.

But now to proceede, I must make 5. the highest saue one

to

to multiply 7. the highest of all, whereby is produced 35. which product I set collaterally at the right hand of this multiplier 5. as heere you see, at the right side of this page.

And now must euery one of the rest of the primes multiply all his fellowes aboue him, both directly and also collaterally but once onely, and their products must be placed still at the right hand of that prime, which is the present multiplier, wherefore I cause 3. being now the next in order of all the rest) to multiply 7. and 5. being directly aboue it, and also 35. being collaterally aboue it, whereby I produce 21. 15. and 105. which 3. products, I place directlye at the right side of the same 3. my present multiplier, as the rule biddeth thus.

And so, both 5. & 3. haue done their offices, wherfore now comming to take 2. for my multiplyer, I doe by it first multiply 7. and then 5. with his collaterall 35. and next 3. with all his collaterals, 21. 15. and 105. and so do I produce these seuerall products 14. 10. 70. 6. 42. 30. & 210. all which I set directly at the right side of 2. my present multiplier, and so they stand thus.

Finally, comming to vse the last 2. for multiplier, I must not vse the same vpon any of the numbers multiplied by the former 2. because the verse saith, that neither the primes nor their products, must be multiplied by any one number aboue once, or but once and no more.

7.
5. 35.
3.
2.
2.
7.
5. 35.
3. 21. 15. 105
2.
2.
7.
5. 35.
3. 21. 15. 105.
2. 14. 10. 70. 6. 42. 30. 210
2.

Eu. That is true, & againe if you should multiply 7. 5. 35. 3. 21. 15. & 105. by the same latter 2. you should make but the same products againe, which you made before by ỹ former 2. ỹ is to say, 14. 10. 70 6. 42. 30. & 210. which I think were but superfluous labour & losse of time, seeing ỹ same nũbers are once placed as measures already.

Philo. It was well considered of you, and it is euen as you say but lost labour, to make two workes of one. Wherefore I must make this last 2. to multiply, onely all such of the said primes and products, as the former 2. did not multiply, that is, the 2. aboue it selfe, with all his collaterals, which are 14. 10. 70. 6. 42. 30. & 210. which done, I produce 4 28. 20. 140. 12. 84. 60. & finally 420. the giuen number it selfe,

7.
5. 35.
3. 21. 15. 105.
2. 14. 10. 70. 6. 42. 30. 210.
2. 4. 28. 20. 140. 12. 84. 60. 420.

(for euery number is a measure to it selfe by the 77. verse) all which products serued in their due places directly, with that 2. which is the present multiplier, make the fabrike of the worke, as before appeareth. Wherefore now I affirme by these 141. & 142. verses, that those 23. seuerall numbers heere described, are all the numbers or measures that can measure the giuen number 420. Neither is there any other number besides them, that can measure it, the certaintie whereof I could demonstrate vnto you, but I omit the demonstration thereof, because it would be very tedious: wherefore according to my promise, I will returne againe to the rest of the properties of numbers, primes together, and compounds together, in order insuing.

Euclid. 25. prop. 7.

143. If 2. numbers giuen, be primes each to other,
Then all the measures of them had or found,
Shall also be primes, each one to another, 119
whether those measures be primes or compound.
VVherefore when 2. numbers be primes together,
the measure of th'one is prime to the other.

As thus, let 8. and 15. be primes together, and let all the measures of 8. be 1. 2. 4. and 8. and likewise let all the measures of 15. be 1. 3. 5. and 15. Now I affirme by this verse, that all the measures of 8. and all the measures of 15. are primes together: the proofe whereof is manifest. Againe.

Euclid. 26. prop. 7.

144. If any 2. numbers be primes to a third,
The product produced, between the first twaine,
Shalbe to the prime left standing vnstird, 120
still prime as before, and keepe his olde vaine.

For let 6 & 8. be primes to 5. and let the product of 6. by 8. be 48. Now I say by this verse, that 48. and 5. are primes together, but if it be denyed that they are primes together, then by the 131. and 133. verses, some prime number shall measure 5. and 48. which for examples sake, we will suppose to be 2. but if 2. measure 48. it shall also measure 6. or 8. (the numbers whereof it is compounded) by the 140. verse, and the same 2. measureth 5. also by the consequence of the former veniall, & contrary supposition, which 2. forasmuch as it exceedeth an ace & yet measureth 5. arguеth that either 5 & 6 or els 5. & 8. are cõpounds together, by ỹ first part of the 133. verse. but ỹ is contrary to ỹ 1. supposition, & therfore cõcludeth absurdly.

Euma The proofe heereof is very certaine, and the demonstration is plaine.

145. If

145. *Phi.* If 2. numbers giuen be primes each to other,
then shall the product of each of those two,
Made by it selfe, be prime to the tother,
still keeping his kinde as it loues to do. 121

Eucli. 27. prop. 7.

The certaynte of this verse is manifestly demonstrated by the former verse last before, for any product so made as this verse importeth, must needes be all one with the product of 2. numbers being prime to a third number.

Eum. It is euidently so, for of like causes, commeth like effects in matter like.

146. *Phi.* If 2. numbers giuen be each of them primes,
to some other couple of numbers assign'd,
The products, made by each couple at times,
shall also be primes together by kinde. 122

Eucli. 28. prop. 7.

As for example, suppose that 3. and 5. be each of them seuerally primes to 4. and 8. the couple of numbers assigned, that is 3. to 4. and 8. and likewise 5. to 4. and 8. now by this verse, I say that the product of 3. by 5. is prime to the product of 4. by 8. for here 3. is prime to 4 and 8. by supposition, and therefore by the 144. verse the product of 4. by 8. shall also be prime to the same 3. and by the same cause, the same product shall also be prime to 5. then siche 3. & 5. are primes to the product of 4. by 8. Therefore againe by the same 144. verse, the product of 3. by 5. shall be prime to the product of 4. by 8. which was to be proued.

Demonstration.

Euma. This proofe is so certaine that it cannot be denyed.

147. *Phi.* If 2. numbers giuen, together be prime,
Th'offcome of each of them by it selfe made,
And next by that ofcome, and so euery time,
still infinitly obseruing that trade,
Shall still be all primes together by nature,
as were their parents, but greater of stature. 123

Eucl. 29. prop. 7.

For let 3. & 4. be 2. numbers prime together, & let the products of 3. by it self, & of 4. by it self, be 9. & 16. & againe, let the products of 9. by 3. & of 16. by 4. be 27. & 64. Now I affirme by this verse, that 9. and 16. and also that 27. and 64 are primes together.

Demonstration.

But now vpon the properties of these numbers among themselues, or one to another, (viz. primes together and compounds together) groweth occasion to search & finde out the greatest cõmon measure, and least diuident to any 2. or moe numbers giuen, both which are verye often times requisite to be knowne in this arte.

And therefore I will (as it were by the way) briefely shew you the state thereof, which I haue deferred hetherto, because I could not demonstrate the finding of the least diuipend vnto you, till now that you are instructed in the properties of primes together, whereof you before were ignorant.

Eu. Very well, I pray you vse such order as you thinke best.

Euclid. 2 prop. 7. & 3. pro. 10. The finding of the greatest common measure to 2. numbers.

148. *Phi.* Of 2. numbers the greatest common measure,
but diuidend least, and common to both,
Is sought out for neede, aswell as for pleasure,
as that which to lack, all logists are loath,
Which greatest measure, by this meanes is found,
diuide the greater number by the lesse,
By whose remayner, if any abound,
diuide your diuisor, and so neuer ceasse,
Till nothing remayne, so shall that diuisor,
measure both numbers, no measure precisor. 124.

Eum. I pray you let me my selfe put this verse in practise by an example.

Phi. Go to, do so then, trie how you can find out the greatest common measure to these 2. numbers 3420. and 930.

Eu. First the verses bid me diuide the greater number by the lesse, wherefore in diuiding 3420. which is the greater, by 930. the lesser, the quotient is 3 & the remainer 630. now by this remainer, I must diuide my diuisor, & so at euery time, neuer ceasing till nothing remaine, wherefore to do accordingly, I diuide 930. which was my present diuisor by this 630. remaining, and the quotient is 1. but the remainer is 300. wherefore I diuide my present diuisor 630. by the same 300. & so is the quotient 2. & the remainer 30. wherfore by the same 30. I diuide 300. my last diuisor, & now the quotient is 10. iust, & nothing remaineth, wherfore 30. being the onely diuisor which brought ỳ quotient cleere without any remainer, by this rule should be ỳ greatest cõmon measure to ỳ giuen nũbers 3420. & 930.

Philo. You conceiue the thing very well, and 30. is indeed their greatest common measure, as you haue said.

Eu. But yet if the greatest cõmon measure to 3. or 4. numbers or mo, were required, I see not how ỳ same could be done by this rule.

Philo. You say truth, but therefore serueth the next rule, which teacheth the same thus.

149. The greatest measure common to a couple,
first found by diuision and knowne as before;

May well be confer'd without any scruple,
to any third number, and then by like lore,
Diuide the greater of both by the lesse,
by whose remaynder if any appeare,
Diuide that diuisor, and so neuer ceasse,
till nothing rest but the quotient comes cleare.
Then take that diuisor what euer it be,
for greatest measure, vnto them all three. 125

Euclid. 7. prop. 7. & in a corollarie to the 3. prop. 7. Euclid, & Euclid. 4. prop. 10. The finding of the greatest common measure to 3. numbers or moe.

As if I would know the greatest common measure to these 3. numbers 3420. 930. & 648. First I say, that 30. is already proued to be the greatest common measure to 3420. & 930. wherfore now according to the contents of this verse, I confer the same common measure 30. to this third number 648. by diuision in such manner as I did the former 2. numbers 3420. & 930. before, that is to say, euen so diuiding the greater by the lesser, & that diuisor by the remainer, continually til ỳ quotient come cleere without any remainer, wherfore first diuiding 648. the greater of these 2. giuen numbers by 30. the lesser, I finde the quotient of this diuision 21. & the remainer 18. wherfore by this remainer 18. I eftsoone diuide 30. my last diuisor, wherby the quotient yeeldeth 1. but the remainer is 6. wherefore by the same remainer 6. I also diuide 18. my last diuisor and so the quotient is 3. cleere, without any remainer: whereupon I affirme now by this verse, that 6. the only diuisor, bringing the quotient cleere so without any remainer, is the greatest common measure to all the 3. numbers, 3420. 930. & 648 And againe, sithe 6. by the 148. verse, is the greatest common measure to 30. & 648. I now say further, that it shall also measure the giuen numbers 3420. 930. & 648. for first 6. measureth 648. by supposition, & then the same 6. measuring 30. shall also measure 3420. & 930. by the 86. verse, therefore 6. shall truly be the greatest common measure to the giuen numbers 3420. 930. and 648.

And so after this manner the greatest measure, to 4. 5. or 6. numbers, or to as many mo as you will, may be found and knowne.

Eum. Indeed this verse, with your example thereof, hath fully supplied the want which I found, touching the finding of the greatest common measure to many numbers, wherfore I pray you now shew me the inuention of the least common diuidend to numbers.

Euclid. 36. prop. 7. The finding of the least common diuidend to 2. numbers.

150. *Phi.* For diuidend least to 2. numbers giuen,
diuide any one of both so assign'd,
By greatest measure, diuiding both euen,
whose quotient infold in tother behinde, 126

The product so made is diuidend least,
to both those numbers iust as you request.

As first for example of the practise, I require to know the least common diuidend or number measurable by 6. and 8. wherefore to finde the same, I may diuide which of them I will by the greatest measure common to them both, and therefore taking 2. for my diuisor, which by the 148. verse I finde to be greatest common measure to the same 6. and 8. I doe thereby diuide 6. one of the giuen numbers, & the quotient is 3. which quotient as the verse willeth me, I multiply or infolde in 8. the other giuen number, (left as it were behind hether to vnoccupyed) & so the product is 24. wherfore I say by this verse, that 24. is the least common diuidend or number measurable by 6. & 8. But heere before I stay, I will shew you also the finding of the least common diuidend to 3. numbers or mo, & so make an end of these numbers, primes & compounds together.

Euclid. 38. prop. 7.

The finding of the least common diuidend to 3. nũbers or mo.

151. If the least diuidend measureable duely,
by 2. numbers pointed, be secondly compared
With any third number, in former forme newly,
and one of them diuided, as there was declared,
By greatest common measure, both of them diuiding,
and then if that quotient, be multiplied also,
By his fellow number, least as yet abiding,
th'offcome thereof rising least diuidend shall go,
To all 3. numbers common, and so you may find mo. } 127

As thus, I would know the least cõmon diuidend to these 3. numbers, 6. 8. & 15. wherfore, forasmuch as 24. is (by the last verse) alreadie knowne to be the least cõmon diuidend to 6. & 8. Now therfore according to the effect of this verse, I doe newly compare the same 24. with this third number 15. in the former forme of worke prescribed by the 150. verse, which is to diuide either the same 24. or els the 15. (which I will) by their greatest common measure, & to multiply that quotient by the other number omitted, but by the 124. maxim, I finde their greatest common measure to be 3. wherfore by the same 3. I diuide 24. one of the giuen numbers, and the quotient bringeth 8. which quotient I forthwith multiply by 15. the other giuen number omitted before, and so I produce 120. Wherefore by this present verse I affirme 120. to be the least diuidend, or number commensurable by 6. 8. and 15.

And after this manner the least diuidend to 4. 5. 6. or as many numbers as you will, may be found and knowne.

And

And thus haue we passed ouer the diuisions and definitions of numbers, as they are among themselues, one to another, eyther pr̃imes togeathers, or compounds togeathers: wherefore now as my first table requireth, I will teach you the seate of Reduction in Integers, and then we will go in hande with the operation of fractions, and first the verse here following, shall shew you the office and end of Reduction, which is thus.

Of Reduction. Chap. 13.

152. Reduction doth change number, weight, & measure,
Time and money to such denomination,
As cause requireth for profit or pleasure,
And changeth fractions by abreuiation,
Vnto their least termes, still keeping proportion,
And that without error or doing extortion.

The office or dutie of Reduction. 128

This verse doth fully declare the office of reduction, and to what purpose it especially serueth, that is to say, to change or alter numbers, weight, measure, time or money, from one denomination or surname to another, and also to abridge fractions to their least tearmes, and this it doth so precisely that the first proportion remayneth still without any iot of error or wrong committed, so that it belongeth aswell to fractions as to Integers.

Yet here to speake exactly of Reduction, I must needes say it is no seuerall operation or part of Arithmaticke, but rather a mixt operation or exercise of the 4. former partes together, especially of Multiplication & diuision, by meanes whereof, things of grosser denominatiõ or surname, are reduced to a more subtiler or lesser surname, & contrarily things of the more subtil or lesser surname, may be altered vnto a more grosser surname, and yet the things whose denominations or surnames are so changed or altered remayne equall in valure, though they change in number, being neyther added nor subtracted, multiplied nor diuided, as touching their first valure signified, and therefore although M. D. Recorde and some other, doth both teach it, and allow it as a seuerall operation Arithmaticall, yet diuers others neuer speake more of it, at least wise if they do, they teach it but as a thing accidentall, & not essentiall, nor as any seuerall operation of Arithmaticke, neuertheless forasmuch as Reduction requireth the notice of sundry things very necessary to be knowen to all thẽ that shall exercise this arte, I haue thought it not amisse to make a seuerall Chapter of it as you see, to the end that both you and they may be the better able by such short instruc-

tions

tions as I haue set downe, to passe through al maner of operations and questions nedyng reduction, as occasion shall serue.

Euma. I thanke you for it, but I pray you which denomination or sirname is that you call grosse, and which call you subtill or lesser.

Philo. That is called a grosse denomination or a grosse sirname which being compared to a lesser, containeth many of those lesser, and that is called a subtill sirname, whereof many make but one of the grosser, as thus first in weightes, for example, 1. pound compared to ounces, is a grosse sirname, because 1. pound containeth many ounces, viz. sometimes 12. ounces, & sometimes 16. ounces, wherefore contrarily, ounces compared to pounds is a subtill sirname, because many of them make but 1. pound.

Secondly in measures 1. gallon compared to pintes, is a grosse sirname, because 1. gallon containeth many pintes viz. 8. and contrarily pintes in respect of gallons is a subtill sirname, because many of them make but 1. gallon.

Agayne, yardes compared to feete or Inches, is a grosse sirname, because 1. yarde containeth many feet or many Inches, and feete or Inches, in respect of yardes are the subtils, because many of them are comprehended in one yarde.

Likewise in partes of time, 1. yeere compared to monethes, is a grosse sirname, and so is monthes to weekes, and weekes to dayes, or houres, because 1. yeere comprizeth many monthes, and 1. moneth many weekes, and 1. wecke many dayes, and 1. day many houres, and so by like reason, houres, dayes, weekes, and monethes, are euery of them subtils, or of subtill denomination in comparison of yeeres, because many of them are containeth in 1. yeere.

Eu. All this I vnderstand very well, and further I perceiue that one selfe sirname may be both grosse and subtill, in some respects as weekes, which in respect of dayes & houres is a grosse sirname, but in respect of yeeres or monethes they are a subtill denomination.

Phi. You haue sayd very well and truly, wherefore by this which I haue sayd, you may conceiue the like in all kinde of monyes and coynes: as pounds, crownes, shillings, grotes, pence, and such like, wherein if shillings be compared to grotes or pence they are a grosse denomination, containing many grotes, and many pence, but if shillings be referred to crownes or pounds, they are but subtils, as those whereof many are containeth in one of the grosser, and

because

because you vnderstand all the effect hereof so readily, I would presently shew you the working of Reduction, but to geue speciall rules or examples, for euery perticuler thing would be both tedious and superfluous, wherefore I will first draw you a table of the names and contentes of the principall weightes, measures, times, and coynes, as they are presently named, vsed, and alowed, within this realme of England: I meane, such as are most generally in vse with vs Englishmen: and that done, I will compyse the rules of Reduction touching Integers, in 2. verses onely, and shewing you the practise thereof in 2. or 3. examples, will proceede to the operation of fractions, wherein I shall haue new occasion to entreate thereof agayne.

And sith Reduction of the partes of number pertayneth onely to fractions, whose operations you haue not as yet learned, I will beginne my table first with the partes of weight, whereof with vs Englishmen the least is a grayne, that is to say, the weight of a coyne or grayne of wheate vse, gathered not partially, but out of the middest of the eare, whereupon is framed this table of weightes following.

The table of weightes.

1. Grayne.	weight maketh	1. Grayne.	And 15. ounces of Troy weight should by the statute make 1. pound of hauerdepoise, but ỳ same pound weyeth cõmonly but 14. ounces ½. Troy or 14. ounces ¾. at the beter most. Statute de compositione ponderum 51 H. 3.
10. Graynes ½.		1. Peny.	
12 Pence.		1. Shilling.	
5. Shillings.		1. Ounce Troy.	
12. ounces Troy		1. Pound Troy.	

16. Ounces of hauerdepoise	weight maketh	1. Pound of hauerdepoise.
8. Pounds hauerdepoise.		1. Naile or Stone.
14. Pounds.		1. halfe quarter of an C.
28. Pounds.		1. quar. called also a Tod.
56. Pounds.		1. halfe hundred.
112. Pounds.		1. hundred weight.
5. of those hundreds.		1. Hogshead weight.
10. of those hundreds.		1. halfe Tunne.
20. of those hundreds.		1. Tunne weight.

Now

Now from these weightes lying measures, both of things Liquid and drie, for 1 pound of Troy weight maketh a pinte in measure, of which pinte growe these measures insuing. 12. H. 7. ca. 5.

Liquid measures.

1. Pound weight	in measure make	1. Pinte.
2. Pintes.		1. Quart.
2. Quartes.		1. Pottell.
2. Pottels.		1. Gallon.
8. Gallons.		1. Firkin of ale, soye, herring.
9. Gallons.		1. Firkin of Beere.
10. ½. Gallons.		1. Firkin of salmon or Eeles.
2. Firkins.		1. Kilderkin.
2. Kilderkins.		1. Barrell.
42. Gallons.		1. Tierce of Wine.
63. Gallons.		1. Hogshead.
2. Hogsheades.		1. Pipe or Butte.
2. Pipes or Buts.		1. Tunne of Wine.

The Table of drie measures.

1. Pinte.	in drie measure make	1. Pinte.
2. Pintes.		1. Quarte.
2. Quartes.		1. Pottell.
2. Pottels.		1. Gallon.
2. Gallons.		1. Pecke.
4. Peckes.		1. Bushell land measure.
5. Peckes		1. Bushell water measure.
8. Bushels.		1. Quarter.
4. Quarters.		1. Chalder.
5. Quarters.		1. Wey.

Also like as the table of weight began at a graine or corne of Wheate, so the measures of length, breadth and thicknesse, with the table thereof, beginneth with the length of a Barly corne as here appeareth.

The

The Table of long measures.

3. Barly Cornes.	in length make	1. Inch.
12. Inches.		1. Foote.
3. Foote.		1. Yarde.
3. Foote 9. Inches.		1. Ell.
6. Foote.		1. Fadome.
5. Yardes ½.		1. Perch.

1. Perch in breadth, and 40. in length, 1. farthendele or roode of lande.

4. Farthendales, or 4. perches in breadth, & 40. in length make, 1. Acre of lande.

Item 40. Roodes in length maketh a furlong, and 8. of those furlongs make an English mile, so that an English mile contayneth 5280. foote, which is somewhat more then an Italian mile, as appeareth hereafter.

Agayne.

4. Barly cornes in breath.	make	1. Fingerbreadth.
4. Fingers.		1. Handbreadth.
3. Handbreadthes.		1. Spanne.
4. Handbreadthes.		1. Foote.
6. Handbreadthes.		1. Cubit.
2. Foote.		1. Single pace.
5. Foote.		1. Geometricall pace.
125. Paces.		1. Furlong.
1000. Paces.		1. Italian mile, which is somwhat shorter then ours.

The Table of Time.

1. Minute.	in time maketh	1. Minute.	13. moneths	make one yere
60. Minutes.		1. Houre.	1. Day and 6. Houres.	
24. Houres.		1. Day naturall.		
7. Dayes.		1. Weeke.		
4. Weekes.		1. Moneth.		

Now for the partes of Coyne or money, the least in name is a farthing, but there are none extant in coyne at this day to my knowledge.

The

The Table of Coynes or Monyes.

1. Farthing
2. Farthings. } maketh 1. Halfpence.
2. Halfpence. } maketh 1. Penye.
3. Halfpence. } maketh 1. Dandipzate.
2. pence. } maketh 1. Halfegrote.
2. Dandipzates. } maketh 1. Trippe.
3. Pence. } maketh 1. Trippe.
4. Pence. } maketh 1. grote.

1. Farthing.
1. Halfpeny.
1. Peny.
1. Dandipzate.
1. Halfegrote.
1. Trippe.
1. Trippe.
1. grote.

6. pence oz 4. dandipzats oz 3. halfe grotes oz 2. trippes. } maketh 1. Testerne oz halfe shilling

12. Pence oz
8. Dandipzats oz
6. Halfe grotes oz
4. Trippes. oz
3. Grotes oz
2. Testernes
} make 1. shilling.

5. Testernes. } make 1. halfe crowne of gold.
5. Shillings. } make 1. crowne oz halfe angell in golde.
10. Shillings. } make 1. Angell oz soueraigne of gold.
13. s 4. d. } make 1. marke oz old noble of gold.
15. shillings. } make 1. Rose noble oz olde royall of gold
20. Shillings. } make 1. pound oz double soueraine in gold

in siluer

Also 3. Nobles.
4. Crownes oz
2. Angels oz
2. soueraignes oz
1. double soueraigne
} in gold make 20. shillings, oz 1. pound of English money.

But of all the coynes here named, the pounds, shillings, and pence, are generally most vsed in reductions, for most commonly the occasion serueth so, that eyther pounds must be changed into shillings oz pence, oz els, eyther pence oz shillings must be reduced to pounds.

Euma. Truely I vnderstand all that you haue sayd very well, but it seemeth a worke of vnfitnes to me, to intermedle with so many thyngs of so diuerse qualities, in the learning of our Arte.

Philo.

Philo. Why, hereby you may see how this arte runneth through all arts and things, and how all artes haue need of Arithmaticke, and yet Arithmaticke it selfe consisteth onely of it selfe, and of his owne grounds, rules, and principles, craning no aide of any other arte, neyther for the practise, nor yet for ỳ demonstrations or proofes therof, but now let vs go to the rules of Reduction.

Reduction of grosse to subtill.

153. All simames grosse are easily constreyn'd,
to turne to subtils as little as you lust,
For marking how many subtills are conteyn'd,
in one of the grosser, let so many iust,
Infold the number so grossely simamed,
and forthwith the subtills are readily framed. 829

As for example first in weight I haue 56. poundes of pepper which I would alter or reduce to ounces, wherfore I consider how many ounces (which are the subtils) are contayned in 1. pound being the simame of the grosser, and in this case because the weight is hauerdepoise, I find by the table of weights, that 16. ounces are contayned in 1. pound, wherefore as this rule willeth me, I let so many iust viz. 16. infold or multiplie 56. the number so grossely simamed, and the product is 896 as here appeareth.

Wherefore I conclude by this verse that 56. poundes of hauerdepoyse weight contayneth 896. ounces, agayne for example, in liquid measures, I would know how many gallons is in 12. hogsheads of wine, now by the table of liquid measures, I finde that 1. hogsheade contayneth 63. gallons, wherefore I multiplie 12. the number of the hogsheades (which is the grosser simame,) by 63. the number of gallons contayned in 1. grosse, hogshead and the product is 756. as here appeareth.

Wherefore I affirme that 12. hogsheades containe 756. gallons of wine.

	56
	16
	336
	56
	896

	63
	12
	126
	63
	756

Fuma. If there be nothing els to do but this, I doubt not, but I can euen alreadie reduce any grosse simame to a number of subtill simame, as you shall see if you will geue me leaue.

Phi. Go to let me see.

Fuma. I will vse an example of dzie measures, supposing that I haue 100 quarters of wheate and would know how many bushels they were: here the grosse simame is quarters, and the subtill or lesser simame is bushels: Now whilst I consider howe many

many subtills, viz. how many bushels are cõperied in one quarter, that is in 1. grosse, I find by the table that there is iust 8. bushells, and therfore I multiplie the 100. quarters by 8. and the product is 800. And so 100. quarters, make 800. bushels.

Phi. It is euen so, and now if you will, you may go forward to alter or change the same 800. bushels into a yet more subtill or lesser sirname, as namely into gallons or pintes.

En. That will I also indeuor, and to performe it, I consider that 8. gallons are comprehended in 1. bushell wherefore by the same 8. I multiplie 800. and the product is 6400. which therfore is the number of the gallons conteyned in 100. quarters of wheate and because 1. gallon conteyneth 8. pintes, I eftsones new multiplie the same 6400 by 8. and the product is 51200 wherefore I finally conclude that 100. quarters of wheate conteyneth 51200. pintes, or 6400. gallons or 800. bushels.

Phi. You haue said truly, but here, that which you did at 2. seuerall multiplications, might haue beene done at once for if you had marked that 1. bushell doth conteyne 84. pintes, and so had folded 800. in 84: the product would haue yelded 51200. pintes immediatly without naming any gallons, how be it, as there is no error in that forme of worke which you vsed so many times, when you know not readely, how many of the least subtills are conteyned in 1. of the grosser immediatly, it shall not be amysse to vse such meanes, by 2 or 3. multiplications so to attayne your purpose, as it were by steppes or degrees. as if it were demanded of you how many farthings are in 100. pound. Heare the grosser sirname being pounds and the sirname of the subtile being farthings, if you presently know how many farthings are in 1. pound, there needeth no more but to multiplie the number of the one by the number of the other, and the product immediatly sheweth the thing desired, but if the number of farthings be not readily knowne, then the next way to worke is to turne the 100 first into shillings, by multiplying the same 100. by 20. because 1. pound contaynes so many shillings, and so shall you haue 2000. shillings: and secondly to change the same 2000. shillings into pence by multiplying them with 12. because 1. shilling contayneth 12. pence, and so shall you haue 24000. pence. and then finally to conuert those pence into farthings, by multiplying their number by 4. (because 1. penny contayneth 4. farthings) and so shall you finde that 100. pounds turned into farthings, maketh 96000. farthings, and as I haue

wrought

wroughe this reduction in coynes, so may you do the like with weights, measures, tyme, or any thing els subiect to reduction.

Ed. I perceiue it very well, and thanke you for this easie example, which I thinke is sufficient for the reduction of any number of grosse sirname into subtills, but yet I see not how things of Subtill sirname may be reduced or transformed to a grosser sirname if neede be.

Phi. You say troth, for the former rule serueth only to alter grosse denomination to subtill, but the next rule shall contrarily teach you to reduce subtill denominations to grosser, and it is thus.

154. All subtill sirnames or denominations,
may be reduced to such grosse as you lust,
By finding first what number of their nation,
will serue to make 1. of the grosser iust,
For by that number diuiding their totall,
the quotient culs out the grosser most plaine,
But if ought be left as some perhaps shall,
thereby is declared what subtills remayne. 130

Reduct. of subtills to grosse.

As thus, 188. ounces of plate, is required to be turned into pounds Troye, wherefore finding that 12. ounces of Troy weight make 1. pound, I do by the same number 12. diuide 188. the totall summe of ounces, and the quotient is 15. and there is a remainer of 8. besides as heere appeareth.

	188(15
	122
	8
	6

Wherefore I say, that 188. ounces of plate, maketh in pounds of Troy weight 15. pound & 8. ounces, euer & besides, because the rule saith, þ if ought be left, (that is to say, if there be any remainer after diuision finished) the same remainer declareth what number of subtils remaine, not sufficient to make 1. of the grosser: yet if I will, I may make the same remainer also in a fraction thus, $\frac{8}{12}$ 1. or $\frac{2}{3}$ 1. both which fractions are all of one valure by the 56. verse, and equall with 8. ounces: and in like maner, if you will, you may returne your 51200. pintes of Coyne into bushels, & frō bushels into quarters, againe. (which you late turned frō quarters to pints) for if you diuide 51200. by 64. (the nūber of pints in one bushell) the quotient shalbe 800. bushels. And then knowing that 8. bushels make 1. quarter, if you secondly diuide the same 800. bushels by 8. the quotient will returne the 100. quarters iustly, as they were at the first, as heere apeareth.

	30
	51200(800. bush.
	6444
	66
	800
	888(100. q.

And this I thinke to be a sufficient instruction for reduction in weight, measures, time, and money, besides which I know nothing subiect to reduction, but number & the parts thereof, which shall be shewed anon in place more conuenient.

Euma. I pray you giue me leaue to practise this verse but in one example before you proceed any further, and I wil trouble you no more therein.

Phi. Go to then do it quickly, for I haue beene longer about this worke then my minde was to be.

Eu. It is thus, there are 8000. pence, to be reduced into pounds wherfore considering, that I know not presently how many pence make 1. r. I reduce them first to shillings, diuiding their totall by 12. (because so many pence make 1. shilling) & the quotient peeldeth 666. s. and there remaines 8. subtils, that is to say, 8. pence, which are vnsufficient to make 1. shilling, as appeareth in the fabrique aboue. Secondly, because that 20. shillings make 1. pound, I diuide the 666. shillings by 20. and the quotient is 33. r. and there remaineth 6. subtils (which now are shillings,) and are likewise vnsufficient to make 1. pound, as heere appeares.

	zz
288.8.d. .	
8zzz (666.s.	
xzzz	
xr	
	666 (33. r.
zzo	

And so the whole 8000. pence reduced, with their seuerall remayners after euery diuision (which reason willeth me to adde together) maketh 33. pounds 6. shillings and 8. pence.

Philo. I perceiue you are cunning inough heerein, wherefore now I will go in hand with the operations of fractions, but yet if you had knowne presently that 240. d. make 1. r. you might haue diuided 8000. thereby immediately, and then the quotient would haue beene presently 33. at the first chop, and there would haue remayned 80. subtils viz. 80. pence, which euery childe knoweth to be a noble, or 6. shillings and 8. pence.

Euma. I doubt not, but with vse, to see further and further into these matters, wherefore when it pleaseth you, I pray you let vs begin with fractions.

Of the Addition and subtraction of simple fractions, with the Reduction proceeding. Chap. 14.

155. Addition of fractions, and likewise subtraction, requireth that first they all haue like basses.

Which

VVhich by reduction is brought to perfection,
and being once done as ought in like cases,
Then adde or subtract their tops and no more,
Subscribing the basse made common before. } 131

This rule teacheth you, that both Addition and Subtraction of fractions, requireth first, that all the fractions to be added together or to be subtracted, haue like basses: that is to say, that they be all of one common surname or denomination, for if they be so, then may you adde their tops or numerators onely, and subscribe their common basse to the totall, and so is the addition ended.

And likewise you may subtract the lesser Numerator from the greater, subscribing also the common basse to the remayner, and so is the subtraction finished. As for example. $\frac{1}{3} + \frac{1}{3}$, make the totall $\frac{2}{3}$. And againe, $\frac{2}{7} + \frac{4}{7} + \frac{5}{7}$, make the totall $\frac{11}{7}$. And so likewise in Subtraction $\frac{7}{9} - \frac{5}{9}$, maketh the remainer $\frac{2}{9}$. And $\frac{11}{20} - \frac{5}{20}$, leaueth remaining $\frac{6}{20}$. Vnderstand you this?

Euma. Yea very well as I thinke, for heere I note thee things which I see in all your examples propounded. First that the giuen fractions haue alwayes all like basses, or bee all of like surname. And secondly, that both the Addition and also the Subtraction of the same fractions is wrought wholy and onely with their numerators, as if they were Integers or whole numbers. And thirdly, that their common basse or surname, is euermore restored and subscribed, aswell to the totall of the Addition, as to the remainer of the Subtraction, as the tenor of this rule present importeth, whereby the same returneth againe to the shape of a fraction.

Philo. You haue obserued good notes, but now if fractions hauing vnlike basses, or being of diuers surnames or denominations, were to be added, or to be subtracted: then by the wordes of this verse, they must first be brought to one common surname or likenesse of basses by reduction, before eyther Addition or Subtraction can be wrought, which reduction being once done, as necessarilie it must be and ought to be in such cases. Then are you to proceede to their Addition or Subtraction, after the former order without further impediment.

So that if you had learned the reduction requisite, then had you euen already learned both the operations of addition, and also of subtraction of simple fractions.

But yet by the way I must tell you, that all fractions cannot be

reduced to like basses presently, at one onely reduction, for sometimes the ayde of 2. reductions is required. Namely if any of the giuen fractions be particles, which we commonly call a fraction of fractions, except it hap so that the same particles, or fractions of fractions, at their first reduction, chaunce to haue like basse to the other giuen fraction. As for example, I would adde $\frac{7}{24}$ to $\frac{1}{2}$. $\frac{2}{3}$. $\frac{3}{4}$. or I would subtract $\frac{1}{2}$. $\frac{2}{3}$. $\frac{3}{4}$. from $\frac{7}{24}$. Heere in both operations the particles must be first reduced to one single fraction, by particuler reduction, the rule whereof is thus.

Particuler Reductiõ.

156. Fractions of fractions soone are resolued,
into one fraction after this fashion,
Their Numerators being all involued,
each into other by Multiplication,
Note downe their product for new numerator,
and so vse their basses for denominator. } 132

Eum. The sence of this verse is very plaine: but I pray you, why do you name this reduction, Particular reduction?

Philo. Because there are diuers kindes of reduction, I purposely giue euery kinde a seuerall name, (as neere as I can) agreeable eyther to the nature therof, or to the effect that it bringeth, which I do for none other cause, but to put a difference betweene each kinde, to the end, that when I name any one kinde, by such seuerall name, you may the better vnderstand forthwith, which kinde of reduction is specially meant.

And so, because the reduction which this verse teacheth is the reduction of fractions of fractions, which otherwise are generally named particles, therefore I call it particuler reduction.

But now to reduce these particles $\frac{1}{2}$. $\frac{2}{3}$. $\frac{3}{4}$. to one fraction according to the prescript of this rule, I multiply all their numerators, viz. 1. 2. & 3. each into other, and their product is 6. which I note down for a new numerator thus $\frac{6}{\ }$. & then vling their denominators likewise, that is, folding 2. 3. and 4. together, each in other, I finde their product 24. which I subscribe for denominator to 6. the former numerator thus $\frac{6}{24}$.

And so haue I reduced all the same particles to one simple fraction equall to them all, (which as it hapneth here) is of like surname to the other giuen fraction whereto it should be added, or frõ whom it should be subtracted: wherefore now if I will adde, I neede do no more but say, $\frac{7}{24} + \frac{6}{24}$. maketh $\frac{13}{24}$. Or if I will subtract, I may say, $\frac{7}{24} - \frac{6}{24}$. leaueth the remainer $\frac{1}{24}$.

Euma,

Eu. This is no hard matter to do, for I presume I vnderstand both the verse and forme of this reduction, and likwise the reason of the name now also very perfectlie.

Ph. Indeed it is the easier, because the particles being reduced, chaunced forthwith to haue like basse, to the other giuen fraction, which chance, chanceth but seldome: but by that meanes the fractions here (as you see) were subiect to both opperations at the first reduction only, which thing you shall see fal out otherwise by and by in the next example, which is thus.

Let $\frac{1}{2}$. and $\frac{3}{4}$ $\frac{4}{5}$. be added together, here the particles being first reduced by this present rule, yeeld in one fraction $\frac{12}{20}$. which agreeth not in surname with $\frac{1}{2}$. wherefore both $\frac{1}{2}$. and $\frac{12}{20}$. must now be secondly reduced to like basses by conforming reduction, the rule whereof is thus.

Cõforming Reduction.

157. Reduction to one like denomination,
in vulgar fractions by this meanes is wrought.
Take the whole product by multiplication,
of all their basses and note downe as oft,
As you haue fractions in all to reduce,
so haue you the common basse for your vse. 133
158. Then next to finde out each numerator,
infold the topterme of the first fraction,
Crosse in the seconds denominator,
and that in the third, and so by that action,
The first numerator forthwith doth redound,
the rest by like reason are byasse found.

Wherefore now to reduce these fractions secondly to one common basse as the 155. verse byddeth, and this rule present teacheth: I multiply 5. & 40. together, which are the basses of the same fractions, and their product is 240. which I note downe twise thus, $\frac{\cdot}{240}$ $\frac{\cdot}{240}$. (because I haue 2. fractions to reduce). Now this rule telleth me, that the same 240. is the cõmon basse that must serue for my vse to both the giuen fractions. Wherefore now next, as the 158 verse present teacheth, to finde out each numerator to these common basses. I multiply 5. the numerator of the first fraction crosse, or byasly in 40. the denominator of the second, and it produceth 200. and so by this action I haue the first numerator to one of the common basses before found, ouer whom I therefore set it accordingle, thus, $\frac{200}{240}$ $\frac{\cdot}{240}$.

And then sith the rule saith, that the rest by like reason are byasly

found, I likewise folde 12. the numerator of the second fraction bi-ssue in 6. the denominator of the first, and the product is 72. which I subscribe ouer the other common basse, as due numerator thereto thus. $\frac{30}{140}$. $\frac{72}{140}$. and so in stead of $\frac{5}{7}$. and $\frac{12}{20}$. the former fractions, being of diuers surnames or vnlike basses, I haue now these other fractions $\frac{30}{140}$. and $\frac{72}{140}$. being not onely of like surname and denomination, but also equall in value to the former, wherefore now the totall of their addition (by the 155. verse) shalbe $\frac{272}{140}$.

Eu. This I vnderstand reasonablye well touching the reduction and addition of 2. fractions onelye.

Phi. If you vnderstand it touching the reduction and addition of 2. fractions, then can you not misconceiue the worke in the reduction and addition of 3. 4. 5. or mo fractions together, for the rule is generall, and yet neuertheless to remoue all doubts, I will shew you one example more, wherein I will adde together these foure fractions $\frac{1}{2}\frac{1}{3}\frac{3}{4}\frac{2}{5}$. which being as you see of diuers surnames or hauing vnlike basses, must first be reduced to like basses or to one common surname as the 155. verse affirmeth, & this present verse teacheth.

Wherefore to performe the same accordingly. I take the whole product of all their basses, which is 120. (for you know that twice 4. is 8. and 8. times 3. is 24. and that multiplyed by 5. maketh 120) which product I note downe 4. times thus, $\frac{120}{}$. $\frac{120}{}$. $\frac{120}{}$. $\frac{120}{}$. because I haue 4. fractions now to reduce. And thus hauing gotten 4. like basses to the 4 giuen fractions. Now by the 158. verse present (to finde out the numerator proper to euery of them) I infolde or multiply 1. the top or numerator of the first fraction in 4. the denominator of the second fraction, which done the product is 4.

And this product 4. infolded in 4. the denominator of the third fraction, produceth 12. which 12. also infolded in 5. the denominator of the fourth fraction (for so I must continue forth the multiplication still as often as there be Fractions) produceth lastlie 60.

So by this action (as the verse saith) this same product 60. shalbe the first numerator to 120. the common basse before found, ouer whom therefore I subscribe it thus, $\frac{60}{120}$. $\frac{}{120}$. $\frac{}{120}$. $\frac{}{120}$.

And now this rule saith further, that the rest of the numerators by like reason are biasly found, that is to say, by multiplying the numerator of euery seuerall fraction biasly, by euery denominator there, (hauing his owne) wherefore to find the second numerator,

I now beginne with 1. the numerator of the second fraction, and folding it in 2. the denominator of the first fraction, it maketh 2. which estsoones folded in 3. the denominator of the third fraction produceth 6. and lastly the same 6. infolded in 5. the denominator of the fourth produceth 30. the numerator to the second common basse, wherevnto I therefore ascribe it thus. $\frac{60}{120}$. $\frac{30}{120}$. $\frac{}{120}$. $\frac{}{120}$.

Eu. Now I pray you let me proue how I can finde the numerators to the other 2. basses, or fractions remayning.

Phi. With a good wil for so I shall see what bred you haue take.

Eu. Then I beginning at the third fraction, infolde 2 the numerator thereof biasly first in 2. the denominator of the first fraction, & the product is 4. which 4. I new multiplie by 4. the denominator of the second, and it produceth 16. then (omitting 3. his owne denominator) I infolde the same 16. in 5. the denominator of the fourth and last fraction, and it produceth 80. which product I take for the top or numerator to the third common basse, placing it ouer the same accordingly thus. $\frac{60}{120}$, $\frac{30}{120}$, $\frac{80}{120}$, $\frac{}{120}$.

This done, I come to the fourth fraction, whose numerator is 3 which I multiply biasly by all the denominators of the 4. fractions (sauing his owne) that is to say by 2. 4. and 3. and the product is 72. which I take for the propper top or numerator to the fourth fraction or last common basse, ouer whom I therefore subscribe it thus, $\frac{60}{120}$, $\frac{30}{120}$, $\frac{80}{120}$, $\frac{72}{120}$.

Philo. You haue done very well: & thus you see, that in stead of these former fractions $\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{5}$ & $\frac{3}{5}$ being of diuers sirnames or denominations, you haue now these 4. other fractions $\frac{60}{120}$, $\frac{30}{120}$, $\frac{80}{120}$ & $\frac{72}{120}$. which are equal in valure to the former, & yet are all of one sirname or like denomination, wherfore now if you would adde them together, you must adde onely their tops or numerators and no more, subscribing thereto the common basse thus found out before, as the 15. rule biddeth.

Euma. I remember that well, and therefore for my exercise, I adde all their numerators together, euen as if they were meere integers thus, as heere appeareth.

60
30
80
72
242

And their totall is 242. whereto I subscribe their former common basse, and so it returneth againe to the forme of a fraction thus. $\frac{242}{120}$. which improper fraction you say is the totall, eyther of the said 4. last fractions, or of the 4. first giuen fractions, and I see it must needs be so by the 36. verse.

For if the 4. first giuen fractions, & these 4. last be equall together

as they be indeed both by the 56. 59. and 102. verses,) then must their seuerall totals be also equall by the same 36. verse.

Phi. You remember it well: but now if you would subtract the 2. first fractions from the 2. last, you may likewise do it as easilye, but yet it shalbe most conuenient, first to adde each couple together as before, so to make them one entire fraction, and then to subtract the one numerator from the other. As thus: I would subtract $\frac{60}{120} + \frac{30}{120}$. from $\frac{80}{120} + \frac{72}{120}$. wherefore first I adde $\frac{60}{120}$ to $\frac{30}{120}$, and they make in one fraction $\frac{90}{120}$. Then secondly I adde $\frac{80}{120}$ to $\frac{72}{120}$. and the totall is $\frac{152}{120}$.

Now subtracting 90. from 152. that is, numerator from numerator, there remaineth 62. as here appeareth by the worke, vnto whom the former balle being subscribed and restored, the remainer is $\frac{62}{120}$.

	152
	90
	62

And note, that when you haue but 2. fractions at once to be added, or the one to be subtracted from the other, after that you haue by this present rule reduced them both to like balles, then if your purpose be to add them, you may forthwith couple their numerators with a + euen as they stand, for a note that they are to be added.

As in the former example, where the first 2. fractions $\frac{1}{2}$, and $\frac{1}{3}$. being reduced to like balles, stood thus. $\frac{120}{240}$, $\frac{72}{240}$. whome sithe I purpose to adde together, I couple with the signe of addition presently euen as they stand, thus $\frac{120}{240} + \frac{72}{240}$, or thus, $\frac{120}{240} + \frac{72}{240}$.

Or els, if I will new write them, I may couple the numerators onely with the +, & subscribe the balle but once onely in the middle as common & indifferently pertaining to them both thus $\frac{120 + 72}{240}$.

And in like maner, if after reduction, my entent be to subtract, I may presently linke them with a —— the signe of subtraction, in remembrance thereof thus, $\frac{120}{240} —— \frac{72}{240}$, or thus, $\frac{120}{240} —— \frac{72}{240}$, or thirdly after this maner, $\frac{120 —— 72}{240}$ which in effect represēteth but $\frac{48}{240}$.

Euma. I perceiue this well, and see that euery of these 3. seuerall descriptions, aswell of the totall, as of the remainer, tend to one end: and seeing 240. is common denominator to both numerators, it seemeth by reason, that it may be aswell represented by once writing after the last fashion, as oftner, although in pronunciation I thinke it must be so often repeated as there are numerators.

Phi. It must be so, for in pronouncing them, if they be coupled with a + howsoeuer they be described, you must say $\frac{120}{240}$ more $\frac{72}{240}$. but if they be linked with a ——, you must say $\frac{120}{240}$. lesse $\frac{72}{240}$.

Eu. If this be all that belongeth to the Addition & subtraction of

fractions

fractions, me thinkes their operations are not so difficult as I feared, but the reduction in my iudgement is harder then eyther the addition or subtraction.

Phi. Indeede when there be many fractions to be reduced, the reduction is both the harder to doe, and also longer in performing then the operations themselues.

And although the rules past are sufficient for the addition and subtraction of simple fractions, yet sith there is another rule of conforming reduction, which sometimes doth it in lesser tearmes, I haue thought good to impart it to you also, that so you may take your choyse to vse, which way you like best, and loe it is thus.

Confirming reduction otherwise

159. Reducing fractions to surnames alike,
Take the least diuident to all their basses,
For that common basse or surname you seeke,
And then for new tops to fill the old places,
Let euery fractions particuler basse,
Diuide that common denominator,
And let each quotient forthwith ere it passe,
Be multiplied by his owne numerator,
Then place each product produced by this action,
For new numerator, vnto his owne fraction.

As for example, I will by this rule eftsoones reduce the selfe same former fractions, viz. $\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{3}$. and $\frac{3}{5}$. to like surnames or one common basse, and to the intent, I may therein not onely shew the meaning of this rule, but also the very apt forme of the working, after that I haue placed the fractions each by other orderly, I draw one line all along ouer them all, and another beneath vnder them all thus.

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{5}$

Then as the rule willeth me, first I take the least diuident to all the basses of these fractions (which by the 126. and 127. rules I finde to be 60.) which I subscribe in the middest beneath the lower line as the common surname or denominator to all the sayd fractions thus.

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{2}{3}$ $\frac{3}{5}$

60

Then secondly to finde the new toppes or numerators to this common basse (which shal stand in steade of the old as it were to fill their places) I diuide this common denominator 60. by 2. the particuler basse of the first fraction, and it quoteth 30. which quotient forthwith ere it passe further, I multiplie by 1. the numerator of the same fraction, and it continueth 30. wherefore I cancell the same first fraction wholy, and place the same product 30. aboue the

vpper line (right ouer the sayde cancelled fraction) as the new numerator to supplie the place of the old thus.

Secondly I cause 4. the particuler basse of the second fraction, to diuide this common basse 60. and the quotient is 15. which quotient, I forthwith multiplie by 1. his owne numerator & it continueth 15. wherefore I take the same 15. for new numerator to that second fraction noting it ouer the head thereof, aboue the line, and so cancell the olde fraction thus.

Thirdly I cause 3. the particuler basse of the third fraction to diuide the common basse 60. and the quotient is 2. which I forthwith multiplie by 2. his owne numerator, and the product is 40. which I take for new numerator to the same third fraction, writing it right ouer the same aboue the line there, and so cancell the olde fraction thus.

Finally I cause 5. the particuler basse of the fourth or last fraction to diuide 60. the common sirname, and it quoteth 12. which I multiplie by 3. the propper numerator of that fraction, and the product is 36. which I accept for new numerator to that fourth fraction, setting it ouer the head thereof insteade of the olde, and so reface the same old fraction as here you see.

	30			
x	1	2	3	
2	4	3	5	
	60			
	30	15		
x	x	2	3	
2	4	3	5	
	60			
	30	15	40	
x	x	2	3	
2	4	3	5	
	60			
	30	15	40	36
x	x	2	3	
2	4	3	5	
	60			

And thus insteade of the first 4. fractions being of diuers sirnames or vnlike basses, I haue by this reduction gotten now as you see these 4 other fractions, to wit $\frac{30}{60}$. $\frac{15}{60}$. $\frac{40}{60}$. and $\frac{36}{60}$. being equall to the former, but changed all to one like sirname or common basse, so that they are readie to be applied eyther to addition or subtraction.

Eu. Truly this kinde of reduction is not vneasie to do, if a man remember the 126. and 127. rules, but I pray you how commeth it to passe that the operation of the last rule before, brought forth $\frac{60}{120}$. $\frac{30}{120}$. $\frac{80}{120}$. and $\frac{72}{120}$. and this rule bringeth fourth $\frac{30}{60}$. $\frac{15}{60}$. $\frac{40}{60}$. and $\frac{36}{60}$.

Phi. The very cause thereof was because 120. which was there assigned and put for the common denominator, was not the least diuident to all the particuler denominators of those fractions.

Euma. Indede that seemeth to be a good reason And here I perceiue by the 56. 59. 102. verses, that euery of these 3. fractions viz. $\frac{1}{2}$. $\frac{30}{60}$. and $\frac{72}{120}$. are each equall to other, and so are these, $\frac{1}{4}$. $\frac{15}{60}$.

and

and $\frac{2}{120}$. and also these $\frac{2}{7}$. $\frac{3}{40}$. and $\frac{3}{120}$. and likewise these, $\frac{1}{3}$. $\frac{10}{60}$. and $\frac{22}{110}$. but because this last fourme of conforming reduction bringeth the fractions reduced in lesser termes then did the former, therefore it seemeth to me to be the better forme.

And herewithall I note that this forme of reduction subscribeth the common denominators but once onely, to all the fractions reduced, so that if they should be applied eyther to addition or to subtraction, or to both, they may be presently signd as occasion doth serue, as I haue done here, where I haue added the 3. first fractions with the signe of addition, and subtracted the last with the badge of subtraction, and so the totall here is but $\frac{30}{60}$.

$$\frac{30}{60}+\frac{15}{60}+\frac{40}{60}-\frac{36}{60}$$

Philo. That note was not amisse obserued, and now seeing you know both these formes of reduction you may vse which of them you like best, but yet at this present I would haue you reduce these fractions, $\frac{1}{3}$. $\frac{2}{5}$. and $\frac{3}{7}$. to like surname after both formes.

Eu. To doe that, I begin first after the forme prescribed by the 132. rule, taking the product of all the bases of these giuen fractions, which is 315. & that I note downe thrice (according to ꝑ number of the giuen fractions) thus, $\frac{\ }{315}$ $\frac{\ }{315}$ $\frac{\ }{315}$ and then I infold 3. the numerator of the first fraction, crosse in 9. the denominator of the second and it produceth 27 which I cause infold in 7. the denominator of the third, and so haue I 189. to be numerator to the first my new common bases or denominators thus, $\frac{189}{315}$ $\frac{\ }{315}$ $\frac{\ }{315}$.

Then doe I multiplie 5. the numerator of the second fraction, biasse both with 5. the denominator of the first, and also with 7. the denominator of the third, (neuer medling with his owne denominator) & it produceth 175. for the numerator to the second common denominate, vnto whom therefore I apply it thus, $\frac{189}{315}$ $\frac{175}{315}$ $\frac{\ }{315}$.

And now to get a third numerator I fold 3. the numerator of the third fraction crosse byesly, in 9. the base of the second, and the product thereof, in 5. the base of the first, and so I produce 125. which I write ouer the last common base for the numerator thereof, which done these fractions are become all of one surname or like bases, and are thus: $\frac{189}{315}$ $\frac{175}{315}$ $\frac{135}{315}$.

Phi. You haue done well, wherefore now trie the other forme.

Eu. To doe that, first by the 126. and 127. rules I seeke the least common Diuidend to 5. 9. & 7. the particuler denominators of the giuen fractions, but by the 108. rule, I finde them to be all primes together

together, & therefore haue no common measure exceeding an ace.

Wherefore I wot not what to doe, for if I should diuide any one of these basses by 1. (which is their greatest common measure) and then multiplie the quotient thereof by the other, as the 126. rule willeth, the selfe same numbers will remayne still immutably by the glosse of the 133. verse, and so such diuision and multiplication serueth to no vse nor purpose.

Philo. I perceiue you haue forgotten the 112. rule, which sayth that the product of 2. primes together is diuidend least and common to both, wherefore if 5. and 9. be primes together, as you say they are, then 45. being the product of them is least diuidend to them both by that rule.

Numa. I cannot denie my fault, that rule indeede was not presently in my minde, but seeing 45. (by that rule is the least diuidend to 5. and 9. and that the same product 45. and 7. the other denominator remayning, are also primes together, therefore 315. being the product of 45. by the same 7. shall by the same 112. rule be the least diuidend to 5. 9. & 7. wherefore now if I should appoint this 315. for the new common basse to the former giuen fractions, and should proceede with the rest of ỳ worke after the order prescribed by this present verse, it seemeth for as much as ỳ basse presently found, is ỳ selfe same which it was before, by the other worke, ỳ therfore the new numerators, wil also be the selfe same ỳ they were before. & so the fractiõs reduced wilbe $\frac{150}{315}$ $\frac{175}{315}$ $\frac{135}{315}$. as they were before.

Phi. It is most certayne they wilbe so, and the reason is, because the denominators of the giuen fractiõs are primes together, wherfore their whole product, taken after the first forme, is the least diuidend to them all.

And so now you see that sometimes, the one way is as nigh as the other, & that is especially when the denominators of the giuen fractions are primes together, yet if you would adde but 2. fractions only togeather, or would subtract one only from the other, then if their basses be compounds together, there is another rule also, somewhat briefer in operation then the last, because it nameth the common diuidend presently without any search thereof by the 87. or 150. verses, and it is thus.

Reduct 6 concurring.

160. At once to reduce, and adde or subtract,
2. fractions whose basses, are compounds together.
The greater of both those basses elect,
and take it for common basse vnto either.

VVhich

VVhich done diuide the common Basse so got,
By each former basse distinctly in order,
Then fold each quotient, next in his owne toppe,
Placing their products in steade of the former. } 135

As for example, (first of Addition) I will adde together these 2. fractions ⅓. and ⅕. whose basses by the 133. verse appeare plainely to be compounds together.

Wherefore after that I haue placed them orderly with their signe of addition, and enclosed them with 2. lines, then I presently take 9. the greater of both the said basses and subscribe it vnder the lower line, for common basse to both the fractions thus,

Which done, I diuide the same 9. twice distinctly to wit, first by 3. the former basse of the first fraction, and so is the quotient 3. and next by 9. the former priuate basse of the second fraction, whereby the quotient is 1.

Lastly I infolde each of these quotientes, in his owne toppe or Numerator, that is 3. in 2. and 1. in 5. & their products are 6. and 5. which products I place aboue the vpermost line, instead of 2. and 5. their former tops or Numerators, coupled also with their former signe thus.

which done I cancell all betwene the lines as here you see it, leauing the rest to expresse the totall of this addition, which is ⅘ and thus these 2. fractions are both reduced and added together, as it were at once, and their totall in one fraction is ⅘.

So likewise in Subtraction, if from ⅔ I would subtract ⅖ after that the fractions are once orderly placed with their lines and signe of Subtraction thus: sith their basses are apparant compounds together, I forthwith take 15. the greater of both their basses, subscribing it vnder the lower line for common basse to both the fractions thus.

Which done, I diuide the same 15. seuerally or distinctly, that is to say first by 5. the former priuate basse of the first fraction, and it quoteth 3. and next by 15. the priuate basse of the second, and the quotient is 1.

Finally I infolde each of these quotients in his owne proper toppe or Numerator, that is 3. in 4. and 1. in 6. and their seuerall products are 12. & 6. which products I place aboue the vpper line in

in steade of 4. and 6. their former tops, linked or coupled also as they were before; cancelling all betweene the lines thus.

$$\begin{array}{r|l} 12 & 6 \\ 4 & 6 \\ \hline 8 & 15 \\ \hline & 15 \end{array}$$

And so the rest which is $\frac{12}{15}$. representeth the remaynder of this subtraction, and thus these 2. fractions are both reduced, and also the one subtracted from the other, as it were at once, and their remaynder in one fraction is $\frac{8}{15}$.

Eu. I perceiue the order of this worke very well, both in the Addition and also in the subtraction, and I see withall, that this verse present is all one thing in effect with the 159. verse, for although here is no mention made of taking the least diuident to the basses of the fractions, for the common basse, yet I see it is silently performed, because I finde by the 87. verse, that alwayes the greater of 2. such basses, is the very least common diuident to them both.

Philo. You haue sayd very truly.

And now touching Abridging Reduction, which abridgeth fractions by reducing them to their least tearmes, I thinke I shall neede to say nothing, for that I suppose you vnderstand it sufficiently by the 148. verse.

Eu. Indeede I remember that verse teacheth to finde the greatest common measure to any 2. numbers giuen, but I know not how to applie it to the abridging of any fractions.

Philo. Why if you can by that verse finde the greatest measure to any 2. numbers, then if you will esteeme the two tearmes of a fraction, for 2. numbers, after that you haue found the greatest cõmon measure to them both, the quotients of their diuision by the same measure, shalbe primes together, and the least numbers proportionall to the same tearmes by the 138. verse.

Eu. I know very well that it is so as you say, for I remember I tried the experience of the 148. verse my selfe by mine owne practise, when you taught it me, but yet I pray you, for my better remembrance, make me a rule purposely touching this abridging Reduction.

Philo. It is needelesse so to doe, but to satisfie your request, take here the selfe same verse somewhat altered from the generalitie whereto it extendeth there, and applied more specially to this reduction in order following.

Abridging reduction. 3

161. Abridging reduction, is done by diuiding the greater terme onely of both by the lesse,

By

By whose remayner, if any be byding,
diuide that diuisor and so neuer ceasse,
Till nothing remayne, then take that diuisor,
for measure which makes both termes the preciser. } 136

Euma. Although this verse be all one in effect with the 148. yet it satisfieth me better for this purpose, and therefore now I am out of doubt touching the practise and application of it, as you shal see by the abridging of this fraction $\frac{162}{270}$. which I will reduce to least termes presently.

For performance wherof, first I diuide 270. being the greater terme by 162. being the lesse, and the remayner is 108. (for as for the quotient, here is not regarded, neyther needs so much as once to be mentioned).

Secondly by this 108. remayning, I diuide 162. my late diuisor, and now the remayner is 54. by which remayner, I thirdly diuide 108. my last diuisor before, & now there remayneth nothing, wherefore as the verse present importeth I take the same last diuisor 54. for the greatest common measure to both the termes of the fraction.

And so measuring both the same termes by this 54. as the verse requireth, the quotient of the numerator yeeldeth 3. and the basse yeeldeth 5. both which being restored to supplie the places of their predecessors maketh $\frac{3}{5}$. and so the same $\frac{3}{5}$. and $\frac{162}{270}$. are equall together and proportionals by the 40. 42. 43. and 78. maxims, and the same $\frac{3}{5}$. are the least proportionals to $\frac{162}{270}$ by the 105. verse.

Phi. You haue sayd well, & confirmed your worke with good authorities of maxims before proued, wherfore I perceiue you are perfect inough in this reduction, & as you haue reduced this fraction, so may you reduce al other, whose termes are commensurable, but if they be incommensurable, then is such a fraction already in the least termes, by the 134. verse: & here learne this by the way, that when both termes of any fraction haue any outward ciphers, then may such a fraction be much abridged speedily by cutting off or tolling like number of ciphers, from both termes, as here $\frac{300}{900}$. hauing the ciphers cut off thus, $\frac{3}{9}$ $\frac{00}{00}$ is forthwith reduced to $\frac{3}{9}$.

Fu. Then so $\frac{140}{420}$. hauing the vtter ciphers onely cut off thus, $\frac{14}{42}$ $\frac{0}{0}$. maketh forthwith $\frac{14}{42}$. which abridged further, by this verse commeth finally in least termes to $\frac{1}{3}$.

Phi. You vnderstand it well, & thus haue you learned the addition and subtraction of simple fractions in diuers fourmes, with their

Reductions

Reductions both proceeding and ſucceeding, touching whoſe ſeuerall trials I neede ſay nothing, ſith as I told you before, in the gloſſe of the 30. rule, the one operation is the triall of the other, and ſo the 22. and 27. rules alſo impart in effect, wherefore now the like opperations in mixt numbers (viz. Integers commixt with fractions) ſhall follow in their order.

Of the Addition and Subtraction of ſecondarie mixt numbers. Chap. 15.

162. Adde ſecond mixt numbers ſeuerally,
The Integers firſt and then next the fractions,
And ſo ſubtract them except ſome cauſe why,
Vrge you ſometime to change theſe directions,
And force you with fractions firſt to begin,
For then you muſt vſe diſcretion therein.

This verſe ſheweth that ſecondarie mixt numbers muſt be added and ſubtracted ſeuerally: that is, the Integers or whole numbers firſt by themſelues, and then the fractions afterwarde by themſelues, except ſometimes, when occaſion may be ſuch that the fractions muſt be firſt added or ſubtracted before the Integers, wherein there is a diſcretion to be vſed, after as the worke ſhall require.

Now for example of the practiſe, I would adde 4. ½. to 5. ⅓. here by this rule I adde the Integers firſt ſaying 4 + 5. make 9. and thē next by the 155. verſe ½ + ⅓. make ⅚. which fraction annexed to the ſumme of the Integers maketh the totall of this addition 9. ⅚.

And ſo if I would ſubtract 4. ⅔. from 5. ¾. firſt I ſubtract the Integers ſaying 5 —— 4 is. 1. and then ¾ —— ⅔ is 1/12. by the ſayd 155. verſe, both which remayners put together, make the whole remayner 1. 1/12.

But if I would adde 4. ⅔. to 3. ¼. here by the 155. verſe, theſe fractions muſt be firſt reduced to like ſirnames by confirming reduction, which being perfourmed after the order of the 133. verſe they appeare thus: that is ⅧⅨ. and ſo now by this verſe, I adde the Integers together, to wit, 4 + 3. and they make 7. whereunto addioyning the reſult of the fractions, the totall of this addition is 7. 17/12.

4	+	5.
2		1
3		2
		10

Euma. Then likewiſe in Subtraction, if I would ſubtract 3. 1/12.

$3\frac{1}{2}$. from $4\frac{5}{6}$. First I reduce the fractions to like surname by the 133. verse, & so they appeare thus, as heere you see. Which in one fraction is but $\frac{5}{6}$. And now for any thing that I see, I may begin the subtraction, eyther with the Integers first, or with the fractions, at mine owne libertie, wherein because I like better to begin alwayes with the Integers first, I take 3. from 4 & the remainer is 1. whereto I annere the $\frac{5}{6}$. remaining of the fractions, and so the whole remainer of this subtraction is $1\frac{5}{6}$.

6 — 5
3 — 1
5 — 2
10

Philo. It is so, and you did very well (as the case of your worke required) to begin with the Integers first. But now if you should adde $6\frac{1}{4}$. $10\frac{1}{2}$. and $14\frac{3}{4}$. all together, how would you do it?

Euma. Truly I would first adde all the Integers together, to wit, 6 + 10 + 14. which make 30. and then would I both reduce and adde all the fractions by the 134. rule, and so would they stand thus. All which brought into one fraction is $\frac{11}{12}$. and that being annexed to the summe of the Integers, maketh the totall of this addition, $30\frac{11}{12}$.

9 + 6 + 10
3 1 3
4 2 6
12

Philo. But doe you not see now, that the $\frac{11}{12}$. by you annexed to the Integers instead of a fraction, is not simple a true fraction, but is indeed more then an whole unit or integer.

Eu. Yes, I know by the 21. verse, that it is an improper fraction and is more then an unit or Integer.

Phi. Then ought the unit or units therein conteyned whether it be one or many (by ỹ meaning of this present verse) to be severally added among the Integers, as true Integers, and not to be confounded thus in this improper fraction.

Wherfore here when you see that the fractions will amount to an unit or more, you should haue vsed that discretion which the verse requireth, which is by foresight thereof, to haue begun with the fractions, adding them first, and then seperating such Integers as should haue risen by their addition from them, you should haue added them together with the other Integers, so would the fraction (made of the remainer) be a true fraction.

Euma. Indeede that seemeth reasonable: But how should I sever or distinct the Integers conteyned in such fractions improper, sithe I haue had no rule therefore hetherto.

Philo. I would haue shewed you the rule of the reduction thereof, if you had required me, but you went forward so fast with

addition after your owne phantasie, that I could not, except I should haue interrupted you.

Euma. I pray you holde me excused therein, and shew me yet the rule of this reduction whereof you talke, and also the name of it.

Philo. I name this reduction especially Distinct reduction, which being incident to these mixt operations, distinguisheth the integers from the fractions wherewith they are commixt, and the rule of it is thus.

Distinct reduction. 4.

163. Distinct reduction diuideth the topps,
of improper fractions, by their own basses,
Whereby in the quotient, forthwith their dropps,
the Integers sole, which straight way it places,
Distinctly alone, and as apperteyneth,
it maketh a fraction of that which remayneth. 138

Euma. I vnderstand this verse well euen at the first sight, wherefore I diuide 25. the top of my improper fraction by 12. his owne basse, and so the quotient (as the rule saith) dropps out 2. Integers more, and there is a remayner of 1. besides, wherefore I perceiue these 2. Integers resulting of the fractions must be added together with 6. 10. and 14. the particulers of the giuen Integers, and so they will make 32. & now making a fraction of the 1. remayning, and adioyning it to the same Integers, the totall of this addition willbe $32.\frac{1}{12}$.

Phi. Yea marry, now is this totall truly and orderly deliuered in deed, and thus you may see that when the result of the fractions to be added, amounteth to an vnit or more, then it is requisite and best, to beginne the operation first with the fractions.

The like happeth in subtraction, when the fraction of the abatement, is greater then the fraction of the totall, as for example, I would subtract $4\frac{3}{4}$. from $6\frac{2}{3}$. which fractions being first reduced to like surname by conforming reduction and subtracted onely with a lyne, stand thus.

8 — 9
2 3
3 4
12.

Where now you see, that $\frac{3}{4}$. cannot be subtracted, from $\frac{8}{12}$. by the 37. verse, wherefore I take an entire vnit from the 6. reducing it and the $\frac{8}{12}$. into one improper fraction and so is it $\frac{20}{12}$. and now may $\frac{3}{12}$. be subtracted from $\frac{20}{12}$. and the remayner wilbe $\frac{11}{12}$. and then 4. being subtracted from the 5. left of the 6. the remayner wilbe 1. which with the former fraction, maketh the whole remayner of this subtraction $1\frac{11}{12}$.

Euma.

Euma. Indeed, now I see that the reduction of Integers and fractions together, into one improper fraction, is sometimes both reasonable and very necessary, which is contrary to the opinion I held in our talke vppon the 18. and 21. verses, wherefore I must needs confesse my ignorance, and craue pardon for my follye there vttered.

Philo. Oh sir, are you remembred of that? Indeed you call it to minde, in good time. For now by these workes you may see how impossible it is to performe the operations requisite in these mixt numbers, without such transformations and disguisings, as is there mentioned, and that not onely of mixt numbers, but also of pure Integers into improper fractions, and yet the necessitie thereof will appeare more plainely anon, when you shall come to the multiplication and diuision of mixt numbers, but in the meane season your pardon is graunted, and the rule of this reduction insueth, which for that it serueth to reduce mixt numbers to improper fractions, I thinke best to name improper reduction, whereof beholde the rule.

164. Reducing mixt numbers to improper fractions,
subscribe the Integers first with an ace,
That done, reduce their vnlike affections,
conforming them all to one common basse,
Then adde all to one, so will they appeare,
an Improper fraction, most plainly and cleare. 139

Improper Reduction. 5.

Euma. So then (if I vnderstand this verse rightly) when you euen now tooke the one vnit or Integer from the 6. & made it one improper fraction with the $\frac{8}{12}$. you subscribed the same Integer first with an ace, thus, $\frac{1}{1}$, which being done, you reduced the same $\frac{1}{1}$ and $\frac{8}{12}$ to like basses, by conforming reduction, thus.

$$\begin{array}{r} 12 + 8 \\ \hline 1 \quad 8 \\ 1 \quad 12 \\ \hline 12 \end{array}$$

And finally, you added them both into one fraction, whereby they appeared in the forme of this improper fraction $\frac{20}{12}$.

Philo. I did euen so indeede, and yet is there an other rule for this improper Reduction, which is somewhat briefer, and it is thus.

165. Of whole and broken, one fraction to make,
infold the fractions denominator,
In that whole number, whose product then take,
and adde it vnto the olde numerator. 140

Improper reduction after another fashion. 5.

Whose totall for new numerator accept,
subscribing th'old basse which still must be kept.

And for example hereof I will reduce the selfe same vnit and $\frac{12}{15}$ (which you did,) to one improper fraction by this rule, wherefore as it biddeth me, I vnfold 12. the fractions denominator, in the whole number (which here is but 1.) and the product is 12. wherfore I take the same product 12. and adde it vnto 8. the old numerator of the fraction, and the totall thereof is 20. which totall the verse biddeth me to accept for new numerator subscribing thereto 12. the old denominator which must be still kept, and so in steade of $1\frac{8}{12}$. I haue this improper fraction $\frac{20}{12}$. equall thereto, euen as before by the other rule.

Eu. Then I perceyue that if I would subtract $\frac{12}{15}$. from 7. I may reduce 1. of the 7. into $\frac{15}{15}$. that is into fifteenths (by the 139 rule, subscribing 1. thereto, and working the reduction thus.

And so abating 1. from 7. and $\frac{12}{15}$ from $\frac{15}{15}$. the remainer will be $6\frac{3}{15}$.

15 —— 12
x xz
x xs
15

Or els, secondly, I may reduce 1. of the same 7. after the order of this present verse, and so the remainer of the Integers will be 6. and the improper fraction wil be $\frac{15}{15}$. from which abating $\frac{12}{15}$. there will remaine $\frac{3}{15}$. which with the remainer of the Integers, maketh iust $6\frac{3}{15}$. as before.

Or thirdly, I thinke I may (if I will) reduce the whole 7. first into $\frac{105}{15}$. after the former verse thus.

And now abating $\frac{12}{15}$ from $\frac{105}{15}$, the remainer will be $\frac{93}{15}$. which being secondly reduced by the 63. verse of distinct reduction, maketh also $6\frac{3}{15}$. And thus the remainers euery way agree.

105 — 12
7 xz
x xs
15

Phi. It is very true that you may doe as you thinke, and as you haue done, in breaking the whole 7. into $\frac{105}{15}$. for the sequell of the worke and agreement with the others, is a sufficient proofe of the certaintie and truth of it, wherfore I am [illegible] so far into it. And heere you shall vnderstand, that both [illegible] and also subtraction, is sometimes wrought euen in mixt numbers or things of diuers sirnames without any apparant reduction, or any once mention of reduction, and yet in both operations, there is a conforming reduction secretly wrought notwithstanding.

Concurring reduction.

But because it concurreth with the operations, so that it is not generally perceiued, therefore it is accompted no seuerall worke, nor

nor no mention at all made of it, and these mixt operations are vsed in diuers things, but specially and most commonly in the addition and subtractions of signes, degrees, minutes, seconds, & such like astronomicall conjunctions, which is called Arithmetica circuler, whereof I minde not to say any thing, and also in coines & the known parts of money, wher of I will say somwhat, but not much.

For when pounds, shillings, pence, farthings and such like, are to be added together or subtracted, euery of the lesser kindes of money, are as it were a fraction of ỳ next grosser. As a farthing, which is ¼ of a peny, and a peny is 1/12 of a shilling, and a shilling is 1/20 of a pound: of which addition, see this is the rule.

166. Addition of subtils with grosser had leuer,
sometimes to want reduction then haue it.
For placing all sirnames of one kinde together,
each vnder other as order doth craue it.
Then adde the least first, and as they grow able,
to make 1. or moe of next groser kinde.
So many transport vnto the next rable,
there to be added, and what rests behinde.
Vnable to make vp any one so,
subscribe directly, and forward then go.

Addition of subtils to grosser denomination.

As for example of the practise of this rule: suppose that A. B. C. and D. owe me each of them a seuerall sum of money, to wit, A. oweth me 3. l. 6. s. 4. d. q. B. oweth 3. l. 4. s. 5. d. q. C. oweth 8 l. 8. s. 10. d. q. & D. oweth 6. l. 9. s. 8. d. q. Now if I would know the totall of all these mens debts by addition. First as the rule intendeth, I place all the sirnames of one kinde together, each vnder other ouer-ir, with a line vnder them all, as (in addition of Integers) the 31. verse requireth, thus.

4. A question of debt.

8. l.	6. s.	4. d.	q.
3.	4.	5.	q.
8.	8.	10.	q.
6.	9.	8.	q.

Then beginning to adde the least sirname first, which are the farthings, I finde them but 4. in all, which suffice iust to make 1. peny, that is 1. of the next grosser sirname, and 0. remaineth, wherfore I transport ỳ 1. peny in my minde to bee added to the next rable among the pence, subscribing the 0. remaining directlie vnder the farthings, thus.

8. l.	6. s.	4. d.	q.
3.	4.	5.	q.
8.	8.	10.	q.
6.	9.	8.	q.
			0.

Then going forward in like ouer with the numbers of the next sirname, which is pence, I adde them together

thus, ſaying 1. in my minde and 4. is 5. and 5. is 10. and 10. is 20. and 8. is 28. whereof, becauſe 24. ſerue able to make 2. of the nexte groſſer kinde, that is to ſay, 2. ſhillings, I tranſpoꝛte the ſame 2. ſ. in my minde to be added to the nexte table oꝛ ranke among the ſhillings, ſubſcribing the 4. d. onely, which reſteth behinde, vnable to make a ſhilling directly vnder the pence, thus.

8. l.	6. ſ.	4. d.	q.
3.	4.	5.	q.
8.	8.	10.	q.
6.	9.	8.	q.
		4	0

Thirdly comming to adde the numbers of the nexte groſſer ſirname which is ſhillings, I ſay 2 in minde and 6. is 8. and 4 is 12. which with the other 8. maketh 20. and 9. maketh 29. whereof becauſe 20. make 1. pound, I caꝛy that 1. in minde to be added to the nexte table among the pounds there, ſubſcribing directly vnder the ſhillings the 9. onely, which reſteth behinde vnable to make 1. pound, and ſo the woꝛke ſtandeth thus.

8. l.	6. ſ.	4. d.	q.
3.	4.	5.	q.
8.	8.	10.	q.
6.	9.	8.	q.
1	9.	4.	0.

Finally, I come to the laſt & groſſeſt ſirname, which is pounds, and adde them together thus, ſaying, 1. in minde and 8. is 9. and 3. is 12. and 8. is 20. and 6. is 26. which 26. I ſubſcribe wholy vnder the pounds, foꝛ that it is the groſſeſt ſirname, beyond whom there is none groſſer, noꝛ further place to tranſpoꝛt any thing. And ſo the totall of theſe 4. mens debts, as appeareth vnder the line is 26 l. 9. ſ. and 4 d.

8. l.	6. ſ.	4. d.	q.
3.	4.	5.	q.
8.	8.	10.	q.
6.	9.	8.	q.
26.	9.	4.	0.

Euma. I vnderſtand the oꝛder, reaſon and practiſe, of this addition very well, and the better foꝛ that which I learned befoꝛe in the gloſſe of the 153. and 154. verſes touching reduction of Integers.

Phi. And the ſubtraction is as eaſie both to vnderſtand and alſo to practiſe, foꝛ as in addition the ſubtills amounting to any groſſer ſirname, were alwayes tranſpoꝛted to the ſame groſſer, there to be added, ſo in ſubtraction, the want of euery leſſer ſirname is ſupplied, by boꝛrowing 1. of the nexte groſſer and conueying it to the ſame ſubtills there to be ſubtracted.

Whereof after I haue rehearſt the rule, I will ſet one example onely and ſo paſſe ouer to multiplication, the rule is thus.

Subtraction of ſubtils from groſſer.

167. Subtraction of ſubtills, and groſſe together, doth boꝛrow a groſſe where ſubtills are ſcant,

To

To mend their number, whereby you may euer ſubtract ſuch ſubtils without any want,
Which done, you muſt ſtill repay agayne one in next place beneath for euery ſuch loane.

Now for example hereof, ſuppoſe that I beeing in your debt 25.l. 16.s. 4.d. had paide you thereof 9.l. 18.s. 6.d. would know what is the remainer of my debt. 5. Another queſtion of debt.

For anſwer hereof, after I haue placed the ſums and ſirnames in like order as before, with their line vnder them according to the 38. verſe, thus.

25.l.	16.s.	4.d
9.	18.	6.

I begin firſt with the ſubtils or leaſt ſirname which heere is pence, where firſt I finde 6.d. to be ſubtracted from 4.d. but becauſe 4.d. is too ſcantie to ſpare 6.d. to be abated from it, I borrow (as this rule intendeth) one of the next groſſer ſirname, that is 1. of the ſhillings to mend or augment this number 4. and ſo putting the ſame ſhilling to this 4.d. it is made as you know 12.d. from which I may now ſubtract the 6. pence without any want, and there will be 10. remaining, which I ſet vnder the line, as the remainer onely of that ſirname pence, thus.

25.l.	16.s.	4.d
9	18	6
		10

This being done, the rule ſaith, I muſt repay 1. in the next place beneath, that is, in the abatement or lower number, (euen as I did in ſubtraction of Integers) for the loane of euery ſuch groſſer ſo borrowed: wherefore now comming to the next groſſer ſirname, which is ſhillings, I ſay, 8. and 1. which I owe is 9. which I take from the 16. (euen as in ſubtraction of Integers) and there reſteth 7. which 7. I ſubſcribe directly, as heere you ſee it.

25.l.	16.s.	4.d
9.	18.	6.
	7.	10.

But now heere, for the 6. made 16. I muſt repay 1. againe to the next place beneath, by the 29. rule, ſaying 1. and 1. which I owe, are 2: which 2. cannot be taken from the 1. aboue it, wherefore I barrow yet 1. more from the next groſſer ſirname, that is 1. pound, therewith to amend the 1. in the ſecond place of ſhillings, ſo the ſame 1. pound conueyed vnto the ſame ſecond place, maketh ẏ 1. there to be 3. (that is to ſay 30 s.) for euery vnit in ẏ place is 10.s by the 4 verſe: now ſubtracting the ſame 2. from this 3. there remaineth 1. which I ſubſcribe vnder the line directly in the due place thus: for by the 20. rule, it is better to ſay 2. out of 3. leaueth 1. then to ſay, 20. out of 30. leaueth 10.

25.l.	16.s.	4.d
9.	18.	6.
	17.	10.

Thus hauing ouerpast two of thole surnames, I come to the third which are pounds, and are the last and grossest: where first I must repay 1. for the lone of the pound I boroued lately from thence.

Wherfore I say, 9. and 1. that I owe make 10. which I subtract from 15. and there resteth 5. which 5. I place vnder the line directly, with the first place of the pounds thus.

25.	16.	4.
9.	18.	6.
15.	17.	10.

Finally, comming to the second place of this surname pounds I say by the 42. verse, 8. in my minde abated from 2. leaueth 1. and so subscribing that 1. remaining directly in his proper place the remainer of that surname onely appeareth to bee 15. and so the whole remainer of my debt is 15. pounds, 17. shillings, and 10. pence, as the fabrique of this worke expresseth.

Eu. I vnderstand this worke so perfectly already by this your example, that I thinke I will not misse to do the like, either in addition or subtraction at any time, wherfore if there be nothing els, I pray you passe ouer to Multiplication as you determined.

How addition and subtractiō are prooued.

Phi. There is nothing els to be done herein, but when you doubt of your worke, to examine it by the due triall: that is, to proue addition by subtraction, and subtraction by addition, according to the glosse of the 30. maxim, the experience wherof I commit to your selfe to practise at more leasure, and so begynne the matters of Multiplication, thus.

Of the Multiplication of simple fractions. Chap. 16.

Simple Multiplication.

168. Multiplie fractions of the vulgar sort,
by folding each numerator in other
And likewise their bases, and then to make short,
place the first product ouer the tother,
Now interliuing them in their first fashion,
and so haue you done your multiplication.

Eu. What? if this verse containe all that is to be done in the multiplication of fractions, I perswade my selfe that I can do it already: for here is no mention of any reduction proceeding.

Phi. Neither needeth there any mention to be made thereof here in the Multiplication of simple fractions, for they may be either multiplyed or diuided, how vnlike by different soeuer their denominators are: wherefore seeing you conceiue the verse so readily

as

as you say, let me see you (for a proofe) multiply together these 2. fractions $\frac{2}{3}$ and $\frac{3}{4}$, and tell me their true product.

Eu. To doe that, first I fold 2. in 3. that is each numerator in other, and the product thereof is 6. then I likewise fold their basses each in other, that is 3. in 4. and the product thereof is 12. and finally I place 6. the first product ouer 12. the other product with a new line betweene them thus, $\frac{6}{12}$ wherefore the same $\frac{6}{12}$ is the product of this Multiplication, but this product seemeth to me to be lesser then any of the fractions of whom it was produced.

Philo. It is so, for seeing the multiplier here is lesse then an vnit, the product by the 33. maxim shalbe lesse then the multiplicand, or els it cannot be a true product, for sith by that rule the very intent, indeauour, and finall scope of multiplication, is alwayes by 2. giuen numbers to finde out a third, which may be in such proportion to any one of those giuen numbers, as the other is to an vnit, therefore by the same rule, when these $\frac{2}{3}$ and $\frac{3}{4}$ (being the giuen fractions) multiplied together produce $\frac{6}{12}$ the same product (if it be true) shalbe to $\frac{2}{3}$ one of the giuen fractions in such proportion as $\frac{3}{4}$. the other of ỳ same giuen fractions, is to an ace or vnit, & againe as ỳ same product $\frac{6}{12}$ is in proportion to $\frac{3}{4}$. so shal ỳ same $\frac{2}{3}$ be to an vnit.

So that the same product, with the 2. fractions multiplied together and 1. shall make 4. true proportionals as you learned in the glosse of the sayd 33. rule.

Eu. But how may I be assured that these fractions are in such proportion together, for it seemeth to me a very obscure matter to trie it, sith I perceiue not that it may be done by any rule that I haue learned hetherto.

Phi. Yet will I proue it to you, by a very playne and familier example which is this, let 6. pence which you know to be $\frac{1}{2}$ of 1. shilling stand in stead of this product, and let 8. pence, which is $\frac{2}{3}$ of a shilling & 9. pence which is $\frac{3}{4}$ of a shilling, be taken in stead of the fractions giuen to be multiplied together, and let 12. pence be the whole entire shilling or vnit diuided.

Now I say as before that as 6. d. is to 8. d. so is 9. d. to 12. d. for as 6. d. is $\frac{1}{2}$. of 8. d, so is 9. d. $\frac{1}{2}$. of 12. d. and agayne I say as the same 6. d. is to 9. d. so is 8. d. to 12. d. or to 1. entire shilling, for as 6. d is $\frac{1}{2}$ of 9. d. so is 8. d. $\frac{2}{3}$ of 12. pence or of 1. entire shilling.

Euma. I see now very playnly that it is so as you say, and can finde by diuers rules that the numbers of pence representing the fractions are true proportionals, whereby I perceiue that as Integers

gers alwayes encreaſe by Multiplication together, ſo contrarily fractions decreaſe almuch by the ſame operation, & yet both wayes the rule is alwayes true, that the product multiplicand, multiplier, and 1. in all multiplications are euer 4. true proportionals.

The reaſon of Multiplication demonſtrated.

Phi. That fractions decreaſe and war leſſe by Multiplication muſt needes be ſo by reaſon, yea I ſay if a number of true Integers be multiplied by a fraction, it muſt neceſſarily be decreaſed therby.

For if you ſhould multiplie this number 4. by 3. the product will be 12 and if you multiplie the ſame 4. by 2. the product will be but 8. but if you multiplie it but by 1. it will continue 4. onely, as it was before, and thus you ſee, that as you abate the multiplier, ſo the product ſtill wareth leſſer and leſſer.

Eu. That muſt needes be ſo by common reaſon, for once 4. cannot produce ſo much as twice 4. or as thrice 4.

Philo. Very well ſayd, then if you abate your multiplier to leſſe then 1. ſhall not the product of that multiplication by that common reaſon, be leſſer then when your multiplier was an entire vnit.

Eu. It ſeemeth that it ſhalbe leſſe by this induction.

Phi. And euery fraction (as you know by the 17. verſe) is leſſe then 1. wherefore now if the ſame 4. be multiplied by $\frac{1}{2}$ or $\frac{1}{3}$ or any other like fraction, the product ſhalbe leſſe then when it was multiplied by an entire vnit, and ſo conſequently it ſhalbe leſſer then 4. and then a multo fortiore, when 2. fractions are multiplied togethers (where neyther of them is either any number, nor yet ſo much as an vnit) it muſt neceſſarily follow that their product ſhalbe leſſer then eyther the multiplicand or multiplier.

Eu. Very good, now I vnderſtand the reaſon of this product, I pray you proceede with the reſt, for I am fully ſatiſfied herein.

Phi. Although I doubt not but you are perfect inough in this multiplication of ſimple fractions, yet before we leaue it, I would haue you multiplie theſe 2. fractions together $\frac{3}{5}$ and $\frac{5}{7}$.

Euma. That ſhall you ſee me do quickly, wherefore folding firſt their numerators together, I ſay 3. times 5. is 15. which I ſet downe in ſome by place as here at the end of this line thus. $\frac{15}{}$ The nert I fold their denominators each in other ſaying, 5. times 7. is 35. which I ſubſcribe vnder the former 15. and then interline them, and ſo the product, as here you ſee, is $\frac{15}{35}$.

Phi. You haue done truly inough, but yet when ſuch fractions as theſe laſt were, are to be multiplied together, there is a very commodious

modious rule to abridge your labor therin, which besides ꝑ shortnes of the worke bringeth the product also in lesser tearmes. & it is thus.

169. Multiplie fractions whose bias tearmes fall,
Equall togeather omitting them all,
And take but the 2. vneqnall tearmes soly,
For they make the product fully and wholy,
But if they be equall both wayes a crosse,
Their product doth make one vnit in grosse. 144

An abridgement of labour in multiplication.

So that by this rule when the bias tearmes of any of the fractions giuen to be multiplied together be equall, as you see that 5. & 5. the bias tearmes of $\frac{5}{7}$ and $\frac{5}{9}$ the giuen fractions in your last example are, then I say omitting both those bias tearmes, as this verse biddeth, you shall neede to do no more, but to take the vnequall termes onely and make them a fraction not changing their places, and so haue you the whole product of that Multiplication.

Euma. Then here if I take 3. and 7. the vnequall tearmes, and make them a fraction not changing their places, but 3. numerator and 7. denominator, stil as before, the same $\frac{3}{7}$ is the whole product of the giuen fractions.

Philo. It is so, & is equall in valure to $\frac{35}{63}$ your former product, as you may trie eyther by the 56. or 102. verse, and so $\frac{1}{4}$ and $\frac{3}{7}$ multiplied together produce $\frac{3}{28}$.

And agayne, if I would multiplie $\frac{1}{2}$ $\frac{4}{5}$ and $\frac{6}{7}$ together, I omit 4. and 4. the 2. bias equall tearmes, & take the product of the rest of the numerators onely for the numerator: and likewise the product of the rest of the bases onely for denominator to the product, and so the whole product of these 3. fractions (by this rule) is $\frac{12}{70}$ and is equall to $\frac{72}{140}$. the product which would haue come of the worke, after the common forme of Multiplication.

Euma. This rule indeede is passing briefe where it serueth, but if I had all these fractions to multiplie together, viz. $\frac{2}{3}$ $\frac{4}{5}$ $\frac{5}{6}$ $\frac{6}{7}$ and $\frac{7}{8}$ might I say forthwith that their whole product is $\frac{2}{3}$.

Philo. Yea, you might boldly say so, and so may you likewise by the same rule say presently that the product of all these improper fractions, $\frac{3}{2}$ $\frac{5}{4}$ $\frac{6}{5}$ $\frac{7}{6}$ $\frac{8}{7}$ is but $\frac{3}{2}$ which by distinct reduction is 4. true Integers.

Euma. Very good, Now touching the last part of this maxim, I vnderstand it thus, that if the tearmes of any 2. fractions be equall, not onely byasly one way, but both wayes, crossewise, as are these, here $\frac{4}{7}$ $\frac{4}{7}$ & such like, thē the product of such 2. fractions maketh alwayes

wayes iustly an vnit, and the reason thereof is apparant, for sithe the products of their numerators is equall to the product of their denominators, they must needes make iustly an vnit by the 21. verse.

Phi. You vnderstand the last part very well, which indeede extendeth but to 2. fractions at once, and you haue aswell alledged the authoritie that confirmeth it, wherefore it resteth to make you partaker of one rule more, which teacheth how to worke a kinde of abridging reduction, proceeding the multiplication, by reason whereof the product succeeding, appeareth presently in least termes, not needing any further reduction afterwards, which rule is thus.

Preuenting Reductiõ. 6.

170. Infolding fractions be sure to consider,
That all bias termes aswell as downeright,
Be first reduced to primes together,
So shall the product in like numbers light. } 145

The reason and proofe of this verse appeareth by the 146. verse, wherefore reduction is soonest accomplished, when the compound termes of euery fraction, aswell byassy as downe right, are reduced to primes together before the multiplication: as for example, I will multiplie these 2. fractions together $\frac{12}{15}$ and $\frac{5}{8}$, now here I may perceiue presently that 12. & 15. the downright termes of the first fraction are compounds together, & are commensurable by 3. wherefore (before I begin my multiplication I reduce them forthwith to primes together, euen as they stand in this maner, first I begin with 12. the lesser termes of that fraction, saying how many times 3. in 12. & it answereth 4. times, wherfore I cancell the same 12. and place 4. the quotient thereof right ouer it in stead of 12. thus.

4 / xx 5 / 15 8

Then come I to the greater termes, which is 15. saying likewise how many times 3. in 15. and it answereth 5. times, wherefore I cancel the 15. and subscribe the quotient 5. instead thereof thus.

4 / xx 5 / 15 8

And now if I respect the termes of both these fractions 5
down right onely, that is to say, each numerator to his own denominator, as 4. to 5. & 5. to 8. they appeare to be primes together as you see.

But now by this verse the bias termes aswell as the downe right termes must be likewise reduced to primes together also, wherefore now I beholde them bias-ly as here they stand.

4 / xx 5 / 15 8

5

Conferring 4. to 8. and 5. to 5. and I finde the first couple to be commensurable by 4. (who by the 87. verse is greatest cõmon measure

to

ea them both, & the second couple are commensurable by 5. where-
fore I reduce the sayd first couple as I did the other before, begin-
ning euer with the least first, which here is 4. saying how
many times 4. in 4. and it answereth once, wherefore I
deface the same 4. and put the vnit of the quotient ouer
it, instead thereof thus.

Then come I to 8. the greater tearme, asking likewise
how many times 4. in 8. and it answereth me twice, wherfore I can-
cell the 8. and subscribe 2. the quotient thereof vnder it
in stead thereof as here you see it.

And now although the other bias tearmes are also
commensurable by themselues, yet neede I not to pro-
ceede to any further reduction of them, but omitting
them wholy because they are equal (as the last verse before taught) I
take the other 2. vnequal termes only for the whole product of this
multiplication, & so haue I $\frac{1}{2}$. instead of $\frac{6}{12}$. which would haue come
otherwise by the common order of multiplication.

Eu. This is a very propper rule, and as easie as it is briefe, and
the reduction therein vsed, me thinkes may well be named preuen-
ting reduction, because it doth as it were preuent the product before
hand, forcing it to appeare in primes together at the first choppe.

Philo. You may name it so then, if you thinke good, but now let
vs goe in hand with the Diuision of simple fractions, whose rules
are readie to come in place, and are such as insueth.

Of the Diuision of simple fractions. Chap. 17.

Diuision simply.

171. Diuision of fractions biddeth describe,
a bias crosse first, and forthwith to place,
The diuident at that crosses left side,
but let the diuisor, the right side embrace.
Then multiplie both waies as the crosse lies,
beginning still with the first numerator,
From whome the new numerator doth rise,
and contrarily the denominator,
So is the fraction new made which insueth,
the quotient of such diuision in truth.

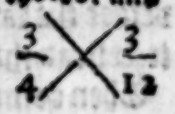
Here is the whole order of diuision of fractions, aswell impropper as propper, contayned in this rule, whether the lesser diuide the greater, or the greater the lesse, and yet to make the thing more

cleare:

cleare by example, I will diuide $\frac{3}{4}$ by $\frac{3}{12}$ wherefore first describing a byas crosse, as the rule of this operation biddeth thus.

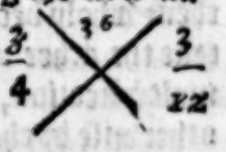


I forthwith place $\frac{3}{4}$ being the diuidend, at the left side thereof and $\frac{3}{12}$ the diuisor at the right side as here you see it.



And then I multiplie both wayes euen as the crosse lieth poynting, that is 3. by 12. and 3. by 4. but I beginne alwayes with the first numerator, that is to say, with the numerator of the diuidend, because the product rysing of that Multiplication must euer be the Numerator to the quotient, and contrarily, the product of the other bias tearmes, shall euer be the denominator thereto as the rule it selfe affirmeth.

Now the product of 3. by 12. is 36. which being the new numerator, I place in the vpper angle or space of the crosse, as the ordinarie vse is, and then forthwith cancell those 2. tearmes multiplied, as here you see.



And the product of 3. by 4. is 12. whose place is directly vnder this Numerator in the lower space as due denominator thereto, where therefore I set it, cancelling the multipliers thus.



And so this new fraction $\frac{36}{12}$ is the quotient of this Diuision, which by the 163. verse of distinct reduction, maketh 3. whole vnits or Integers.

Eu. This operation is easie inough to performe, but me thinketh your quotient is greater then the diuidend, which seemeth very strange.

The reason of this diuision. Demonstration.

Phi. It is greater indeede, and the reason thereof is, that as an vnit is greater then the diuisor in this Diuision, so therefore the quotient thereof shalbe greater then the diuidend, els were it not a true quotient, which is contrarie to the order in the diuision of integers, for there the quotient is alwayes lesse then the diuidend.

But in the diuision of true fractions it must necessarily fall out otherwise by the 45. maxim, for sith the intent and effect of diuision, is by 2. numbers giuen, to search out and finde a third in such rate and proportion to the diuidend, as an vnit is to the diuisor, as in the sayd maxim and glosse thereof is largely declared.

Therefore by that maxim, when $\frac{3}{4}$ and $\frac{3}{12}$ (being the 2. giuen fractions by whom the third is sought) are diuided the one by the other and yeeld in the quotient $\frac{36}{12}$ or 3. vnits, the same quotient (if it be a

true

true quotient must be in such proportion to $\frac{1}{4}$ the diuidend, as 1. is to $\frac{1}{12}$ the diuisor, so that the same diuidend, diuisor, quotient and vnit, shall make 4. true proportionals, as the glosse before mentioned doth truly affirme.

Eu. I pray you make the reason of this quotient yet somewhat playner by an example, as you did the reason of the product in the glosse of the 168. verse.

Phi. Then let 9. pence which you know is $\frac{3}{4}$ of a shilling, stand in steade of this diuidend, and take 3. pence which is $\frac{1}{12}$ of a shilling in instead of the diuisor, and now if you diuide 9. by 3. will not the quotient be 3.

Eu. Yes, that is certayne, but then the same quotient is 3. pence, which is but $\frac{1}{4}$ of a shilling, and not 3. entire shillings, as the quotient of your former diuision seemed to be.

Phi. Although the quotient there, may implicatiuely seeme to signifie 3. shillings, yet in truth, it was neither 3. shillings nor 3. pence, but 3. whose proper sirname is neyther shillings nor pence, but it is times, arguing thereby þ $\frac{1}{12}$ is contayned in $\frac{3}{4}$ three times, for by þ 61. verse, the very office & dutie of euery diuision, is to search how often or how many times the diuisor may be quoted or found in the diuidend, whereof the quotient being the desciser, sheweth how often the same doth redound &c. so then when $\frac{3}{4}$ is diuided by $\frac{1}{12}$ thereby is sought how often or how many times $\frac{1}{12}$ is contained or found in the same $\frac{3}{4}$ and then the quotient who is the desciser of the question answereth directly, how often the same is contayned therein, saying, 3. times: for your selfe confessing before that the quotient would be 3. affirmeth that the same 3. are pence & not shillings, but I say agayne, they are properly neyther pence, nor shillings, but times.

For if I should aske how often or how many times you had beene at Venice, it were no good answere for you to say 1000. miles, and euen so when I seeke, and search, and aske how often or how many times 3. pence is contayned in 9. pence, it were no good answere for the quotient to say 3. pence or 3. shillings, for it might aswell say 3. miles or 3. weekes, but the quotient who answereth directly, to the question asked, sayth, three times, or thrice.

So then, as 3. pence is contayned 3. times in 9. pence, so is $\frac{1}{12}$ contayned 3. times in $\frac{3}{4}$ and as an vnit is in proportion to 3. the giuen diuisor, so is the quotient 3. in like proportion to 9. the giuen diuidend, so making 4. true proportionals as the glosse of the 61. verse requireth, and here is expressed, $\frac{1}{12}$ $\frac{3}{4}$.

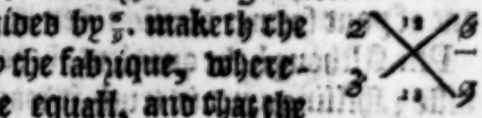
Eum.

Eu. And so if I alowe the quotient to be 3. shillings, I may notwithstanding say, that as 1. shilling is in proportion to 3. pence this diuisor, so is the quotient 3. shillings to 9. pence the diuidend. for as 1. shilling is 4. times 3. pence, so is 3 shillings 4. times 9. pence.

Phi. You may so and say truly, and the reason thereof is this, when one fraction is diuided by another, whether they be true fractions or improper, the quotient alwayes expresseth the proportion between the diuidend and the diuisor wherfore if both the fractions be equall, then both the tearmes of the quotient shall also be equall.

But if the diuidend be the greater, then the numerator (who alwayes expressly representeth the diuidend) shalbe the greater, and finally if the diuisor be the greater, then shall the denominator (who euermore expressly representeth the diuisor) be the greater.

As for example $\frac{2}{3}$. being diuided by $\frac{6}{9}$. maketh the quotient $\frac{18}{18}$. as here appeareth by the fabrique, wherefore I conclude that $\frac{2}{3}$. and $\frac{6}{9}$. are equall, and that the quotient of their diuision therfore, is constantly 1. by the 77. verse.



Eu. Their equalitie is manifest also by the 40. maxim.

Philo. But before when $\frac{3}{4}$. was diuided by $\frac{3}{12}$. the quotient (as you sawe) was $\frac{36}{12}$. where the numerator is the greater tearme, wherefore I conclude that the diuidend $\frac{3}{4}$. (who is representeth by the same numerator) is the greater fraction, and that as 36. is to 12. so is $\frac{3}{4}$. to $\frac{3}{12}$.

Euma. That hath beene sufficiently proued already.

Phi. Well then let $\frac{2}{3}$. be diuided by $\frac{3}{4}$. and the quotient wilbe $\frac{8}{9}$. as here the fabrique sheweth. Where now because the denominator is the greater tearme, therefore I conclude that $\frac{3}{4}$. the diuisor (who is represented by the denominator) is the greater fraction, & that as 8. is to 9. in proportion, so is $\frac{2}{3}$. to $\frac{3}{4}$.



Note well hereby how to know the greater fraction from the lesser.

Eu. But how may I trie their proportion to be such.

Philo. The triall is very soone seene by the partes of a shilling. whose $\frac{2}{3}$. you know is 8. pence, and the $\frac{3}{4}$. is 9. pence, and so you see their proportions agree iustly.

Eu. I perceiue it very well now, and thus I haue not onely learned Diuision, with the reason thereof, but also thereby I haue gotten a readie way to know which is the greater fraction, of any 2. fractions propounded.

Philo. You haue so, but yet there be certaine compendious formes of diuision which I will also impart vnto you, whereof the first is thus.

Abridgement of diuision of fractions.

172. Diuiding fractions whose numerators
are equall together in no wise forget,
To take but onely their denominators,
for if the diuisor haue but his basse set,
Aboue tother basse in forme of a fraction,
the quotient appeares and makes no detraction. 147

Eu. Then by your fauor I will trie this rule by the selfe same fractions and example of diuision, which you propounded first, to wit, I will diuide $\frac{3}{4}$ by $\frac{3}{12}$, whose numerators are equall, wherefore they being setled in a readines, thus: $\frac{3}{4} \times \frac{3}{12}$ I now meddle not with their numerators, nor with any crosse multiplication, but take onely 12. the basse of the diuisor, and set it for new numerator ouer 4. the basse of the diuidend, thus: $\frac{3}{4} \times \frac{12}{12}$ (4)

And so haue I done, and the quotient now is $\frac{12}{4}$, which by distinct reduction, I finde to be 3. Integers agreeable to the former worke.

Philo. You haue done well, and made a good triall. Now another forme is thus.

An other abridgement of diuision in fractions.

173. Diuision of fractions whose basses are equall,
reiecteth the basses and workes the diuision,
Betweene the tops onely, which then in especiall,
disdayning their basses haue them in derision,
Accompting themselues alwayes in such cases,
meere Integers hauing no need of such basses. 148

The sence of this verse is so plaine, that it needeth no exposition, onely I will lighten it by an example, which is this: I would diuide $\frac{15}{16}$ by $\frac{3}{16}$. Heere (the basses being equall) I reiect them, & taking their tops onely for true integers, I diuide 15 by 3. and the quotient is 5. which is the selfe same quotient, that would haue issued by ordinary diuision, after the forme of fractions, as here appeareth by the fabrique, for $\frac{240}{48}$ by distinct reduction, commeth to 5. Integers. $\frac{15}{16} \times \frac{240}{48} \times \frac{3}{16}$ (48)

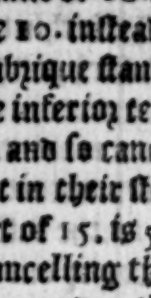
Euma. I perceiue the agreement of the workes very well, and hereby it should seeme that any fractions reduced to like surnames, are subiect and lyable to this kinde of diuision.

Phi. That is without question: but now the last rule is thus,

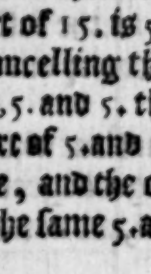
A third abridgement of diuision in fractions.

174. Diuiding fractions, reduce euery way,
all the termes giuen to primes together,
Saue biasly onely, which done as I say,
compells the quotient like termes to delyuer. 149

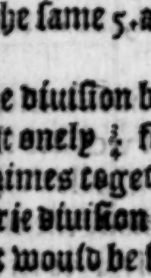
As for example I will diuide $\frac{6}{12}$ by $\frac{10}{15}$, wherfore after that I haue made a crosse, & placed the termes thus: I reduce them euery way fauing byasly, beginning with the 2. superior termes, first saying thus: the halfe of 6. is 3. wherfore cancelling the same 6. I note the 3. ouer it instead thereof: then say I likewise, the halfe of 10. is 5. which I foorthwith write ouer the 10. instead thereof, & so cancelling that 10. the fabrique standeth thus.



Then comming to the inferior termes (12. and 15) I say first, the third part of 12. is 4. and so cancelling the 12. I subscribe the 4. vnder it in their stead, and then I say likewise, the third part of 15. is 5. which I also subscribe vnder the 15. cancelling that 15. thus.



Now finally I reduce, 5. and 5. the equall termes of the diuisor to vnits, saying, the fift part of 5. and 5. is 1. and 1. which I place in stead of each 5. one aboue, and the other vnderneath and so cancelling the same 5. and 5. the fabrique appeareth thus.



So that now working the diuision by byas multiplication, there resteth but onely $\frac{4}{4}$ for the quotient, whose termes are primes together, whereas the quotient by ordinarie diuision would haue beene $\frac{20}{15}$.

Eu. I know very wel it would be so, and perceiue that this rule, is but such a kinde of preuenting reduction in diuision, as the 170. verse taught, touching multiplication.

Phi. It is euen so, and thus hauing finished the diuision of simple fractions, now the multiplication & diuision of mixt numbers, are mixt in ouer as followeth.

Of the Multiplication and Diuision of Secondary mixt numbers. Chap. 18.

175. Multiplication and diuision eke
of any fraction with an whole number,
Will haue the whole number first made fraction like
by drawing a line and putting 1. vnder,
VVhich done, as with fractions thē may you proceed,
to multiplie or to diuide as you need.

Mixed multiplication and mixt diuision,

As for example, I would multiplie 7. by $\frac{3}{4}$. or I would diuide 7. by $\frac{3}{4}$. now this verse saith, that both these operations, will haue the

Integers

Integers or whole numbers first made fractionlike, as you learned by the 164. verse, that is to say, by drawing a line & putting 1 vnder it thus $\frac{7}{1}$. which is a thing soone done, and being once done, then may you proceede with them as with meere fractions, either to multiplication or diuision, as occasion serueth.

Fum. Why, if this be all that is to be done heerein, I can tell already, that the product of $\frac{7}{1}$. by $\frac{4}{9}$. shalbe $\frac{28}{9}$. by the 143. maxim, as heere appeareth by the fabrique, which by distinct reduction is $3\frac{1}{9}$.

$$\frac{7}{1} \times \frac{4}{9} \qquad 28 \qquad 9$$

And I can likewise say, that the quotient of $\frac{7}{1}$ diuided by $\frac{4}{9}$ is $\frac{63}{4}$, by the 146. maxim, as here the fabrique sheweth, which by distinct reduction is $15\frac{3}{4}$.

$$\frac{7}{1} \div \frac{4}{9} \qquad 63 \qquad 4$$

Phi. What you are wax very cunning vppon the suddaine, but in truth, to him that hath passed the multiplication and diuision of simple fractions, and hath learned reduction as you haue done, the mixt opperations cannot be difficult, wherefore I will passe to the next rule, which concerneth the like opperations, when both the multiplicand and multiplier, or the diuidend and diuisor, are al mixt numbers, whereof the rule is thus.

Mixt numbers must be reduced, before they be multiplied or diuided.

176. Mixt numbers with mixt, nor vnmixt will offer,
to yeeld to diuision or multiplication,
Till both be reduced to fractions improper,
or to whole numbers to their contentation, 151
VVherefore reduce them first seuerally,
and then they submit themselues by and by.

This verse certifieth you plainely in effect, that mixt numbers cannot by any possibilitie be multiplied nor diuided by other numbers, neyther mixt nor vnmixt, till both be reduced to improper fractions, or els to whole numbers after, as the same mixt numbers are most aptly disposed.

But when they are once seuerally reduced so, then it affirmeth that they submit themselues by & by, and become subiect forthwith to both opperations, as for example, let $6.\frac{1}{2}$. & $8.\frac{1}{3}$. be multiplied together: now sith this rule telleth me that such mixt numbers will not in any wise yeeld to this opperation, till they be both seuerally reduced to improper fractions, therefore I indeuor first to reduce them accordingly by the 164. or 165. verses of improper reduction, and so the same reduction beeing performed, these subborne vnruly and vntractable mixt numbers, $6.\frac{1}{2}$. and $8.\frac{1}{3}$. are

thereby chaunged and transformed into the shape of these simple improper fractions $\frac{27}{4}$ and $\frac{44}{5}$. which are coequall to them, and very tractable aswell to diuision as to Multiplication, so that now, I may eyther multiplye them together, or diuide the one by the other, as meere fractions, at my pleasure.

Euma. Indeed now I my selfe can say by the 168. verse, that the product of their multiplication is $\frac{1188}{20}$. as heere appeareth by the ordinarie fabrique. which by distinct reduction maketh $59\frac{8}{20}$.

1188
27 44
4 5
20

And likewise by the 171. verse, that if the $\frac{44}{5}$, be diuided by the $\frac{27}{4}$, the quotient of that diuision wilbe $\frac{176}{135}$, as heere appeareth also by the fabrique, which by Distinct Reduction I finde to be $1\frac{41}{135}$.

44 176 27
5 135 4

And so touching these operations, I thinke my selfe sufficiently instructed, sauing that where this verse saith, that mixt numbers wil neuer yeeld to these operations, til they be reduced to improper fractions, or to whole numbers to their contentation, and you expounding it say, that they must be reduced to improper fractions or to whole numbers, (after as they are most aptly disposed) the truth is, I do not vnderstand that last kinde of reduction, neither know I how to finde their apt disposition therein.

Phi. I will make all that matter plaine to you, by an example by and by, which is this, I would diuide $16.\frac{12}{5}$. by $3.\frac{6}{5}$. wherein before I beginne, I consider whether these seeming fractions, be truly fractions, or mixt numbers, or true Integers disguised onely in ỹ shape of improper fractions, which is no hard matter to iudge by the 21. verse now if I finde them, true Integers, disguised onely in the shape of improper fractions, then you know in reason, it is more naturall to reduce them to Integers with their fellowes, by distinct reduction, then to reduce true Integers into counterfet fractions by improper reduction, and more easie to worke them as whole numbers, then as fractions: and therefore when such seeming fractions, are neyther pure fractions nor mixt numbers, but true Integers exactly, then are they best contented and disposed to returne to Integers, and to be wrought as Integers.

But otherwise, they must needes be reduced alwayes to improper Fractions, as is afore saide. Doe you vnderstand the matter now?

Euma. Yea very well I thinke, for in this case you meane that the

the $\frac{12}{3}$. (which indeede are 4. true Integers) should be reduced to their proper shape, by distinct reduction, and so ioyned with the 16. other of their fellowes, whereby the whole diuidend wilbe 20.

And then that the $\frac{6}{2}$. (which are also true Integers) should likewise be reduced to their proper shape, and so ioyned also to the other 3 of their fellowes, and so should the whole diuisor be 6.

And then finally diuiding 20. by 6 the quotient would be $3\frac{2}{6}$. Is not this your meaning.

Phi. Yes truly, you vnderstand me very perfectly.

Eu. Then surely I must needes confesse, that all this is but very reasonable, and as for the reduction, it is no new thing, but such as I learned before, wherefore touching my doubt herein I thanke you, I am now fully satisfied.

Phi. Wel, yet there resteth behind, one other very necessarie kinde of reduction, which (according to the propertie thereof) I name exchanging reduction, and the rule thereof is thus.

177. To change the sirname or basse of a fraction,
multiplie onely the top of the same,
By such new sirname, as likes your affection,
diuiding the offcome that so thereof came,
By his proper basse whose quotient allowe,
for top to the basse new chosen by you. 152

Exchãging reduction. 7.

This reduction, as the verse it selfe importeth, serueth to alter or change the sirname or basse of a fraction, to such other sirname as you would desire, and also by it, whole numbers may be reduced into the shape of fractions, of what sirname you will, so that it hath many necessarie vses: and now to expound it by an example.

Admit I haue $\frac{12}{15}$. which I would haue changed into $\frac{16}{20}$. wherefore as this verse willeth me, I multiply 12. the top only, of my fraction by that new sirname, which I like or would haue, that is by 20, and so the product or ofcum is 240. which product diuided by 15. the proper basse of my said fraction (as the rule requireth) bringeth 16. in the quotient, wherefore now alowing or assigning this quotient for topp or numerator to $\frac{16}{20}$. the basse new chosen by me (as the rule biddeth) thereof commeth this fraction $\frac{16}{20}$. and so haue I changed my $\frac{12}{15}$, for $\frac{16}{20}$, both which fractions are equall together.

And thus if you had $\frac{12}{15}$. of a pound in mony, which you would haue exchanged into some certeine known coyne, as into shillings, testernes, groats or pence, &c. by this reduction you may easily do it, for as a shilling is $\frac{1}{20}$. of a pound, so these $\frac{12}{15}$. being chang-

ged into $\frac{16}{20}$. declareth manifestly that $\frac{12}{15}$. of a pound is 16. shillings.

Eu. Truly I imagin that this reduction must needes be very necessarie, and by this your example, I vnderstand both the rule and also the maner of working it very well, wherefore now I pray you shew me how you apply it to the reduction of whole numbers into improper fractions of any sirname.

Phi. Then do you name some number which you would haue so reduced, and appoint what sirname you would haue it to beare.

Euma. I would haue 40. reduced into $\frac{1}{7}$.

Phi. To do that, make your number fraction-like, by subscribing 1. to it, as the 175. verse taught you, and then obserue the rule as I did.

Eu. Then I set my number so fraction-like, with the sirname or denomination which I would it should haue, in this order. $\frac{40}{1}$ 7

And now vsing it as if it were a fraction, I multiplie 40. the top thereof by 7, and it produceth 240. which diuided by 1. the basse of my fraction-like number, continueth 240. wherefore I take the same 240. it selfe, allowing it for top to 7. the basse so chosen by me, and so it is $\frac{240}{7}$. and is equall to $\frac{40}{1}$. or 40.

Phi. It is euen so, and thus hauing finished the explication and necessarie doctrine of all the members, braunches, and diuisions, of the first part of Vulgar Arithmeticke, according to the order prescribed in my Arithmeticall table thereof, I will now go in hand with the second part, in order following.

Finis Libri primi.



THE SECOND BOOKE,

named *Portus Proportionum* : containing the second part of Vulgar Arithmeticke, touching the Relation or Comparison of Numbers, aswell in quantitie as qualitie.

Of Numbers Relative or compared in Quantitie.
The first Chapter.

Philomathes.

178. Numbers Relatiue, are numbers compar'd,
either in quantitie or els in qualitie,
For when each to other haue any regard,
then are they relatiues in generalitie,
As when it is said, though 10. exceede 4.
it is but twice 5. or once halfe a score. } 153

Boetius 21 cha 1 book.

The 108. verse of my former booke, and the glose thereof tolde you, that numbers absolute, were numbers abstract, absolutely set loose from all comparison or relation to number or figure.

Now this verse sheweth you, that numbers relatiue (although they be also abstract from sirname or denomination (as numbers absolute are) yet they are, or may be, compared together 2. wayes. That is to say, they may be compared together in their quantities, or in their qualities: for by this rule, when any one number hath any regarde (that is to say, any respect or relation) to another, then are they both generally to be accompted Relatiues, whereof the last part of this verse conteyneth in it selfe 3. most manifest examples.

As first, when it is sayde that 10. exceedeth 4. heere 10. and 4. appeare presently to be relatiues together, for 10. exceedeth not it selfe, nor hath any excesse but in respect of 4. to whom it is now compared or referred. Secondlie, when 10. is sayde to be twice 5. heere 10. is referred to 5. as double to it, and so they are relatiues, for 10. is not twice any other number iustly, nor double to any other then onely to 5. Thirdlye, when it is sayde to be once halfe a score, nowe it is referred to 20. as the halfe thereof: for 10. of it selfe is an whole number, and no halfe, but in respect or regarde of 20. to whome it hath relation

and whose correlatiue it is, and so forth of all others.

Eu. This is very euident.

Phi. Thē marke ꝑ definitiō of relation in quantity which is thus.

Eucl.3.d.5 This antecedēt is by Boetius named Dux, and the consequent, Comes.

179. Relation in quantitie alwayes is meant,
when 2. termes onely are compared together,
Whereof the first is nam'd th' antecedent,
but tother is called the consequent euer. } 154

180. And quantitie is of 2. kindes or sortes,
for either it is the difference or excesse,
Betweene 2. numbers, or els it importes,
the rate or proportion such numbers expresse.

Boetius ca.14.li.1. Scheubeli. tract.2. cap.1.

This rule first teacheth you very plainely, that by relation in quantitie, is meant nothing els but the conference or comparison of 2. numbers onely together, shewing you further, that euery 2. termes or numbers so compared together, haue proper names according to their places, so that the first is named the Antecedent, and the latter is called the Consequent.

Antecedēt and consequent.

As for example, if I compare 6. to 4. heere 6 is the Antecedent, and 4. is the Consequent, but if I compare 4. to 6. now 4. is the antecedent and 6. is the consequent, for in all such like comparisons and conferences, we esteeme that place or number to be first which is next the left hand, as we vse to write or read, and not that next the right hand, as we doe in the notation and operations of numbers.

Saligracus lib.2.ca.1. verse.5.

Secondly it declareth, that relation in quantitie is diuided into 2. kindes, for either such relation consisteth and is referred to the quantitie of the difference, or excesse betweene 2. numbers, or els to the quantitie of the rate or proportion betweene them.

Eu. I vnderstand you in all this very well, sauing I vnderstand not what you meane, by the quantitie of difference or excesse betweene numbers, nor by the quantitie of their proportions.

181. *Phi.* The difference or oddes, cald also th' excesse,
is the remainer left of the greater,
After subtraction from thence of the lesse,
as by example will appeare better. } 155.

Saligracus lib.2.ca.1. verse 6.

Eu. I pray you propound that example.

Phi. Then for example, I take 6. and 4. the 2. termes last before rehearsed, betweene whome, if I would knowe the difference, oddes, or excesse, I must (by this verse) subtract 4. which is the lesser, from 6. which is the greater, and so will remayne 2.

where-

wherefore I conclude that this Remayner 2. is the quantitie of the difference, odds, or excesse between 6. and 4. And so as you see, the difference, odds, and excesse, is but one selfe same thing diuersly named, for looke how much any one number exceedeth another, so much it eddeth or differeth from it. The difference, the excesse & the odds, betweene 2. nūbers, is all one thing.

And therfore the operations of the differences of numbers, is but the operations of their remaynders. So that if you would adde the difference of 2. from 5. to the difference of 3. frō 7. you must adde 3. to 4. And if they were to be multiplied together, then 3. must be multiplied by 4. The like is to be vnderstoode of the subtraction and diuision of differences.

Euma. Very well, now I vnderstand fully what the difference or excesse is, and consider also that their operations must necessarilie be such, and so done as you say, and therfore if the difference of 2. from 5. should be subtracted from the difference of 3. from 7. the remaynder shalbe but 1. But if the same difference of 2. from 5. should be deuided by the same difference of 3. from 7. the quotient by your saying shalbe $\frac{1}{4}$.

Philo. It must needes be so.

Euma. But I pray you what is the proper definition of that which you call the rate or proportion betweene numbers.

For although you did tell me somewhat thereof in your speache vppon the 47. verse, yet I remember you saide there, that the full explycation thereof belonged not to that first part, neither could I as then fully vnderstand it, wherfore you left it vnperfectly expounded.

Philo. You remember my saying very well, and indeede I did there make you somewhat acquainted, not with the definition of proportion but with numbers proportionall, because as I told you in the glosse of the 61. verse, the very rules, lawes, and proofes of Multiplication and Diuision, are grounded vppon proportionalitie, so that without giuing you some taste, what numbers were proportionall in that kinde of proportionalitie, which then I ment to speake of, you could neither vnderstand the very first verse of multiplication, nor yet of Diuision, nor many other besides, but now in this second part we shall haue fit place and due time to expound both proportion and proportionallitie more fully, wherefore first to answere your question I say that,

182. Proportion or rate is nothing but the quotient, Of th'antecedent diuided by the Consequent. } 156 Salignac. liber. 2. cap. 1. verse. 7.

So

So that if you aske me what proportion 8. hath to 4. I answer double proportion, for in diuiding 8. the antecedent by 4. the consequent I finde the quotient is 2. which is as it were the proper index or token of double proportion, thereby shewing that 8. conteyneth 4. twice.

Agayne, if you aske me what proportion 4. hath to 8. I diuide 4. the antecedent by 8. the consequent, and the quotient by the 78. verse and the notes thereof is $\frac{1}{2}$, which reduced & expresst in least termes (as proportions alwayes ought to be) is $\frac{1}{2}$, signifieng that 4. is $\frac{1}{2}$ of 8. which proportion is called Subdupla, as you shall knowe better anon.

And although aswell the differences, as the proportions betweene any tearmes, may be considered both together in the selfe same tearmes, yet the consideration of the differences or excesses is one thing, and the consideration of the rates or proportion is another, for proportion includeth both equalitie and inequalitie, but it includeth not the difference, and therefore it was necessarie that Relation in quantitie should be diuided into two kindes, as it is both here and in my table.

For although generally the greater inequalitie and lesser also, may be applied aswell to differences as to proportions, yet the speciall kindes of inequalitie, as Multiplex, Superparticular, and Superpartiens, & likwise, Submultiplex, Subsuperparticular, and Subsuperpartiens, w all other subdiuisions of their kindes, belong only to proportiõs & not to differẽces, as you shal learn very shortly.

And for asmuch as by this present verse proportion is defined to be nothing els but the quotient of the antecedent, deuided by his consequent, therefore like as euen now I called the operations of the differences betweene the tearmes, the operations of their remaynders: so now here I name the operations of the proportions betweene the tearmes, the operations of their quotients for if the proportion of 3. to 2. should be added to the proportion of 4. to 2. you must adde $\frac{1}{2}$ to $\frac{2}{1}$, as if they were fractions improper, so will the totall be the proportion of $\frac{3}{2}$.

The form of adding proportions and likewise of subtracting proportions.

Likewise if you would multiplie the proportion of 3. to 2. by the proportion of 4. to 2. you must multiplie $\frac{3}{2}$ by $\frac{2}{1}$ as if they were the tearmes of improper fractions, & so (working by the 170. verse) the product shalbe the proportion of $\frac{3}{1}$, the like is to be iudged of subtraction and diuision, whereby you may see that the operations of proportions, do not onely imitate the like operations of fracti-

The forme of Multiplication and diuisiõ of proportions.

ons

ons in forme of worke, but also that their antecedentes are vsed as numerators, and their consequents as denominators.

Eu. It seemeth by consequence of reason, that the operations of proportions must needs be in such sort, if the definition of proportion contayned in this present verse be true: but how many kindes of these proportions are there.

Phi. The kindes of proportion are as infinite as numbers are in difference, for there can be no 2. numbers named or giuen, betweene whom there is not some kinde of proportion, but the first and generall diuision thereof is thus.

Boetius cap. 21. pri. l. b. Arithmetices Tonstall pag. 190.

183. Proportion is parted in generalitie,
Into equalitie, and inequalitie. } 157

Eu. Which call you the proportion of equalitie.

184. *Phi.* Proportion of equalitie is wher,
Two tearmes compar'd are equall together,
As six vnto six, or ten vnto ten,
VVhich are both equall as you may consider, } 158
And yet it hath more this marke in the face,
To wit, that the quotient is: I wayes an ace.

Boetius ibidem. et Tonstallus ib. d. Scheubelius tract 2. chap. 1.

This verse doth not onely expound it selfe by a most playne example, but also it sheweth you that in proportion of equalitie, the quotient of the antecedent diuided by the consequent is alwayes iustly an ace or vnit, and neyther more nor lesse, as when 6. is diuided by 6. or 10 by 10. the quotient wilbe iust 1.

Eu. That must needes be so by the 77. verse, but I pray you what proportion is that now which you call proportion of inequalitie.

Boetius cap. 21. l.b. 1. arithmetices, Tonstall. pag. 190. Scheubelius tract. 2. cap. 1.

185. *Phi.* Proportion of inequalitie is,
VVhere tearmes vnequall are confer'd together,
As 6. vnto 3. or els here in this,
Of 3. vnto 6. chuse which you haue leuer, } 159
For this proportion is the professor,
Both of the greater and eke of the lesser.

This verse sheweth you playnely inough, that proportion of inequalitie is the conference of 2. vnequall numbers together, as 6. to 3. or 3. to 6. 4. to 2. or 2. to 4. and so likewise of any other 2. tearmes that are not iustly equall: and further it sheweth you that this proportion of inequalitie contayneth 2. kindes, namely the greater inequalitie and the lesser.

Euma. I perceiue well by the 183. verse, that if there be a proportion

proportion or rate betweene any 2. termes, the same must eyther be a proportion of equalitie or of inequalitie, and therefore when the tearmes are not equall, the proportion of necessitie must be a proportion of inequalitie, but what you meane by the greater inequalitie or lesser, I yet vnderstand not.

Tonstall. pag 190. Records whetst. f 5.

186. *Phi.* Proportion of the greater inequallitie,
Is when the greater tearme is Antecedent,
Which is diuided into a dualitie, } 160
Of simple and mixt, as shall appeare euident.

Euma. Then if I compare 6. to 3. or 5. to 4. doe you call these comparisons, proportions of the greater inequalitie.

Philo. Yea truly, for in both your examples, the greater tearme is antecedent, and it is all one, to say proportion of the greater inequalitie, or to say proportion inequalitie the greater, and in this proportion, the quotient of the antecedent diuided by the consequēt, is alwayes greater then an vnit, as if you diuide 6. by 3. the quotitient wilbe 2. and likewise if you diuide 5. by 4. the quotient will be 1 $\frac{1}{4}$. each of which Quotientes (as you see) exceede an vnit.

Euma. That is a good note whereby to know it.

Phi. Then harken what the proportion of the lesser inequalitie is.

Tonst. pag 190. Recordes whetst. fo. 5.

187. Proportion of the lesser inequalitie,
Is when the lesser tearme is Antecedent,
VVhich is diuided by like authoritie, } 161
In simple and Mixt as th'other precedent.

Eu. So then if I compare 3. to. 6. or 2 to 3. or any such lesser number to a greater in that sort, then I perceiue that the proportion betweene such 2. numbers is proportion of the lesser inequalitie.

Phi. It is so, and in this proportion the quotient of the antecedent diuided by the consequent, is alwayes lesse then an vnit, as if you diuide 3. by 6. the quotient will be $\frac{1}{2}$, which in least termes you know is $\frac{1}{2}$.

And likewise if you diuide 2. by 3. the quotient will be $\frac{2}{3}$, each of which quotientes as you see, are lesse then 1.

Eu. So then I perceiue that proportion of equalitie, maketh the quotient to be alwayes one vnit exactly, the proportion of the greater inequalitie, maketh the quotient alwayes to be greater then an vnit, and the proportion of the lesser inequalitie, maketh a quotient alwayes lesser then an vnit, which diuersities I trust to remember, but I trow the verses say, that both sortes of these proportions of inequalitie are further diuided into simple and mixed proportions.

Philo.

Philo. They are so and eche of them are subdiuided againe, as both the table and also the next verse teacheth, which is thus.

188. Simple proportion, hath 3. sorts in all,
whereof the first is multiplex by name,
The second we superparticuler call,
the third, Superpartiens, and each of the same,
Haue infinit kindes comprised vnder them,
from whence they shoot vp as twigs from the stem. 162

Tonstall pag. 190: Scheube. tract. 2. cap. 1.

Whereby you are to vnderstand that there are 3. generall kindes of simple proportions of the greater inequalitie, whereof the first is called multiplex, the second superparticular, & the third superpartiens, euery of which 3. generall kindes conteyne infinite particuler proportions vnder them.

Eu. So it seemeth indeed by this verse, but is it possible to know them all, or haue they all seuerall names.

Philo. Yea it is possible to know and name all proportions, giuen betweene any 2. numbers appointed, aswell as to knowe & name any number assigned, (as I will shew you by & by) because proportions geuen, are as determinate and certeyne as numbers geuen or assignd are: but to name either ỳ greatest proportion or the least, is as impossible, as to name the greatest number, or the least fraction, because proportions both increase infinitely as numbers doe, and decrease infinitely as fractions do, but now to define the 3. generall kindes of proportions, whereof this verse speaketh, let vs begin first with multiplex, whose definition is thus.

189. Multiplex is when the Antecedent,
Contaynes the consequent of times iust,
Be it twice or thrice or els more frequent,
As by 3. to 1. is playnely discust. 163

Boetius ca. 23. lib. pri. et Xylander super Diophantũ pag. 12.

Hereby you may perceiue easily, that when 2. numbers are conferred together, whose Antecedent contayneth the consequent oftentimes iustly or exactly, without any ouerplus, that then the proportion betweene such 2. numbers, is generally called multiplex, which is as much to say in english as manifold, such are the rates or proportions of 2. to 1. 3. to 1. 4. to 2. 6. to 2. and such other, and in this proportion the quotient of the Antecedent, diuided by the consequent, is alwayes a number of true Integers onely, according to the which number, the same proportion taketh a particuler name, as if the number in the quotient be 2. it is called Dupla, or double, if the quotient be 3. it is called Tripla, if 4. then Quadrupla, if 5. then

Quintupla

Quintupla, and so forth infinitely, for the which we haue no other English wordes, but twofold, threefold, fourefold, fiuefold, &c.

Euma. Very good if this be all touching multipler, I pray you proceede to the definition of Superparticuler proportion.

Boetius ca. 24. lib. pri. et Xylander super Diophantum pa. 12.

Phi. 190. Superparticular proportion is that,
VVhose consequent once abated or taken,
From the Antecedent, yet leaueth thereat,
An aliquot part remayning forsaken. } 154

As thus, let 3. be the Antecedent, and 2. the Consequent, now from 3. abate 2. and there will remayne 1. which is an aliquot part of 2. viz. $\frac{1}{2}$ thereof, or (which is better) let the Antecedent 3. be diuided by the consequent 2. and the quotient wilbe $1.\frac{1}{2}$.

So likewise take 4. to 3. and diuide 4. the antecedent by 3. the consequent, and the quotient wilbe $1\frac{1}{3}$, for in this Superparticular proportion, the quotient of the antecedent, diuided by the consequent, is alwayes an vnit and a fraction, whose numerator is also an vnit, of which fraction the proportion taketh his particuler name, according to the quantitie of the denominator thereof, for if the quotient be $1\frac{1}{2}$, then it is named Sesquialtera, if $1.\frac{1}{3}$, then Sesquitertia, if $1\frac{1}{4}$. then Sesquiquarta, if $1\frac{1}{5}$, then Sesquiquinta, and so forth infinitely, which names cannot be englished otherwise, but thus, once and an halfe, once & a third, once and a quarter, once and a fift, &c. But now I will shew you what Superpartient proportion is, and how it is knowen, which is thus.

Boetius li 1. ca. 28. et Xylander pa. 12.

191. Superpartient proportion is knowen,
By th'antecedent which still doth contayne,
The consequent once and mo partes then one,
VVhich after Diuision do shew them selues playne. } 165

As for example, let 5. be Antecedent and 3. the Consequent, now in diuiding 5. by 3. the quotient is $1.\frac{2}{3}$ arguing that 5. the antecedent containeth 3. the consequent once, and also $\frac{2}{3}$ thereof.

Likewise take 7. to 4. and diuide 7. the Antecedent by 4. the Consequent, and the quotient will be $1\frac{3}{4}$, declaring thereby that the Antecedent containeth the Consequent once, & $\frac{3}{4}$ thereof besides.

So that in both these examples, you see the Antecedent containeth the Consequent once and not one single part thereof onely, as it did in proportion Superparticular, but once with many partes, (for by the 13. verse, mo then one is many) such are the proportions of 9. to 5. 10. to 6. 12. to 7. and such other.

And therefore in this Superpartient proportion generally, the Antecedent diuided by the Consequent, bringeth in the quotient an Integer

Integer and a fraction whose numerator is a number, and according to the quantitie of both the tearmes of that fraction, the sayde proportion taketh his particular name, as in the first example here giuen of 5. to 3. where the fraction was $\frac{2}{3}$, wherefore the same proportion is generally Superpartiens, but the particuler name therof according to the tearmes of the fraction is Superbipartiens tertias, or els shorter Superbitertias, both which names in english is as much to say, as once with two thirds ouerplus. Agayne if the fraction be $\frac{3}{4}$ as it was in the other comparison of 7. to 4 then is the same proportion generally Superpartiens also, but is particulerly called after the termes of that fraction Supertripartiens quartas or Supertriquartas, which is asmuch to say, as once with $\frac{3}{4}$ ouerplus, and so foorth of all others infinitely.

Wherfore, as soone as you haue by the 182. verse diuided the antecedent of any proportion by the consequent, the quotient sheweth you the proportion betweene them, for if the quotient be many vnits exactly, then by the 189. verse, the proportion is Multipler, if it be but one vnit, and a fraction of an aliquot or single part, then it sheweth the proportion to be Superparticular, by the 190. verse, but if it be an Integer and a fraction of many parts, then it is Superpartiens by this verse.

And thus after as you see the part or parts of the fractions in any such quotient, you may readely giue each proportion found, the proper name due accordinglie, but if you will, you need not allwayes precisely expresse the peculiar name of euery proportion, but may signifie the same by the termes thereof onely, as we do when we say the square made of a circle diamater, is to ỹ same circle in such proportion as 14. is to 11. which if we should expresse by proper name, we should say were in proportiõ Supertripartiens vndecimas, & likewise when we say an ell is in such proportion to a yarde as 5. is to 4. if we should expresse the speciall name of the proportion, we should say it were in Sesquiquarta proportion. Archimides rules illustrated by M. Ihon Dee, and published by M. H. Billingsley in the 12. booke of Eu. fo. 357.

Eu. I trust after a litle exercise herein, & without recourse to your table, & diligẽt perusing therof, to be better acquainted w̃ the names of these proportiõs, wherfore now I pray ye define mixed proportiõ.

192. *Phi.* Mixed proportion is no more in sence,
But Multiplex ioyn'd with superparticuler,
Or els when Multiplex and superpartiens,
Make one proportion ioyntly together. } 166 Boetius lib. 1. cap. 29, 30. Scheub. tract. 2. cap. 1.

Example of 5. to 2. here 5. is not onely double to 2. which is in Multipler

Multiplex proportion, but it is $\frac{1}{3}$ of 2. ouer and besides which falleth in superparticuler proportion, wherefore this being a proportion mixed of Multiplex and Superparticuler together, is by generall name called Multiplex Superpartiens proportion, but by speciall name it is called Duplasesquialtera, and so 10. to 3. is not onely Multiplex because 10. containeth 3. oftentime, viz. thrice, but it is also Superparticular because it containeth $\frac{1}{3}$ of 3. ouerplus by reason whereof it obtayneth the name of Triplasesquitertia.

The like commixion or composition, is when Multiplex and Superpartiens ioyne together, as 8. to 3. where if the Antecedent be diuided by the consequent, the quotient wilbe $2\frac{2}{3}$ now the 2. Integers argue the proportion to be Dupla, by the 189. verse, and the fraction argueth it to be Superpartiens by the 191. verse, so both put together make the whole proportion so mixt, to be generally named Multiplex Superpartiens, but by especiall name according to the number of Integers, and tearmes of the fraction, it is called Duplasuperbitertias or Duplasuperbipertiens tertias.

And so if the Antecedent diuided by the consequent, should containe the consequent 3. or 4. times or more, and also some partes thereof ouerplus, then should such proportion be named according to the number of times and partes so contayned, as 15. to 4. which is Triplasupertriquartus, and 24. to 5. which is Quadrupla superquadrapartiens quintas, and so foorth of other like.

Eu. So then I perceiue that you call that proportion simple & vnmixt or vncompound, which is Multiplex onely, or Superparticular onely, or Superpartiens only, but such as take part of Multiplex and superparticular, or part of Multiplex, and Superpartiens together, those you accompt mixt proportions, and so I see you make 5. generall kindes of these proportions, whereof 3. are simple, and 2. are mixt.

Philo. You say truth, and all these are comprehended vnder proportion of the greater inequalitie, which as the 186. verse affirmeth is when the greater number is Antecedent and now you shal vnderstand that proportion of the lesser inequalitie (that is by the 187. verse, when the lesser number is Antecedent) hath as many kinds both simple and mixt, & as many seuerall names in all things as hath the other.

Boetius lib. primo cap. 22. Scheub. tract. 2. cap. 1.

And their names differ in nothing, but that this proportion of the lesser inequalitie, hath alwayes the proportion Sub. annexed to it, as for example, the proportion of 6. to 2. is generally Multiplex

tipler, and because 6. conteyneth 2. thrice, therefore it is by speciall name Tripla.

But the proportion of 2. to 6. is generally submultipler and specially subtripla, and likewise 2. to 3. is generally subsuperparticular, but specially subsesquialtera, and so 3. to 17. is generally submultipler superpartiens, but by particuler name it is subquintupla superbitertias.

Eu. Me thinkes these words Sub. and Super. in one proportion cannot agree together in any sence, for the one signifieth vnder, and the other ouer, or vppon, which in mine opinion are quite contrarie.

Philo. I tolde you euen now, that the particle Sub. serueth onely to make the proportion following it, knowne to be a proportion of the lesser inequalitie, for there is no other difference betweene the proportion of 7. to 4. and 4. to 7. but the fore setting of Sub. to the latter proportion, and so the first is Supertripartiens quartus, or Supertriquartas, and the latter is Subsupertripartiens quartas or Subsupertriquartas.

The annexion of which proposition, was very well and wisely inuented by Arithmeticians, for the eschuing & auoiding of fractions, and heedeth no absurditie at all, if you consider it well.

And as for all the particuler kindes, names, & diuisions of these proportions, sithe they are agreeable in all things to the other 5. kindes of the greater inequalitie, and haue none other difference then I haue tolde you, I thinke it needlesse to rehearse them: onely it shall not be amisse to put you in remembrance, that proportions ought alwayes to be reduced to their least termes, and so to be expressed either in the forme of improper fractions, if they be of the greater inequalitie, or of true fractions if they be of the lesser inequalitie, sauing that both wayes they must haue 2. lines betweene their termes, for a difference from fractions, as I haue tolde you heretofore, and as you sawe me describe them in the notes of the 59. verse, for by such description they are most apt and readye to be applyed to any kinde of operation.

Wherefore if I would expresse Sesquialtera proportion, it is better exprest thus $\frac{3}{2}$. then thus $1\frac{1}{2}$. and likewise dupla superbitertias is better described thus $\frac{8}{3}$. then thus $2\frac{2}{3}$. In like manner, tripla is better represented thus $\frac{3}{1}$. then thus 3. And so of all others.

But now, halting away from relation in quantitie, to relation in qualitie, I conclude with this verse insuing.

Viz. to the whetſtone of wit, fol. 5. and ſo to fol. 10.

193. The names of proportions becauſe they are many
and very forward to frame in a verſe,
Seeke in my table, where if there miſſe any,
reſort to Record, who doth them reherſe,
Yet thinke neither him nor any els able,
to count all their names which are vncountable. 167

Euma. Well then I truſt by conference with your table, to finde the names of proportion, growing or happening betweene any 2. numbers, wherefore when you will, I pray you proceede to relation in qualitie.

Of Relation or compariſon of numbers in qualitie, but chiefely and generally of proportionalitie. Chap. 2.

Eue. 3. d. 5. Salig. lib. 2 ca. 2. ver. 1.

194. Relation in qualitie, is when you finde,
ſundry compariſons conferred together,
Whoſe quantities ſtill are all of one kinde,
for qualitie is not where their kindes differ. 168

This verſe conteyneth the definition of relation in qualitie, whereby you may ſee the diuerſitie betweene it, & relation in quantitie.

For relation in quantitie, by the 179. verſe, is when but 2. terms onely are compared together, that is, the Antecedent to the conſequent. But relation in qualitie, by this definition, is ſundry compariſons conferd together, whoſe quantities are euer all of one kinde, for if the quantities differ in kinde, then is ſuch conference no relation in qualitie, by the plaine words of this verſe.

Now you know by the 180. verſe, that quantitie generally is but of 2. kindes, that is, firſt quantitie of difference or exceſſe, and ſecondly, quantitie of rate or proportion.

Wherefore, when ſundry comparisons are made onely of difference to difference, or onely of proportion to proportion, then ſuch conparisons are properly ſaid to be all of one kinde, or of like qualitie.

As for example, when the difference betweene 6. and 3. is compared to the difference betweene 12. and 9. ſuch conference is called a relation in qualitie, becauſe both the comparisons are of one kinde of quantitie, to wit, onely of differences.

And ſo likewiſe, when the proportion of 12. to 4. is compared to the proportion of 3. to 1. ſuch comparison or conference is a relation in qualitie alſo; becauſe both the comparisons are of one kinde of quantitie, that is to ſay, onely of the quantitie of the rates

or

of proportions betweene the termes or numbers.

Suma. You haue made this verse so plaine by your exposition and examples, that I cannot choose but vnderstand it, and further I gather by these examples, that as euery comparison in quantitie consisteth of an Antecedent and a consequent: so euery relation or comparison in qualitie, consisteth of 2. antecedents and 2. consequents at the least, because otherwise there cannot be sundry comparisons made, for therein the comparison is not of one terme to another, but of the difference or proportion betweene one couple of termes, compared to the difference or proportion betweene an other couple of termes.

Phi. It is very true as you say, but yet such 2. antecedents and 2. consequents, may be in some cases comprehended wholy in 3. termes onely, as I will shew you more plainer heereafter, and in the meane season, I will proceed to the diuision of relation in qualitie, which as appeareth by my tables, hath 2. branches, which are these.

195. Numbers haue 2. wayes relation in qualitie,
whereof the first is euer in generall,
Comparisons made in proportionalitie,
the second is when they are disproportionall. } 169

Sacrob. li. 2. ca. 2. ver. 2. Vrsticius li 2. ca. 13.

This verse following the order of my table, sheweth you, y numbers relatiue in qualitie, are of 2. sorts, or haue relation 2. wayes, that is, first when the comparisons or relations make a proportionalitie, and secondly when they are disproportionall: wherefore now as order requireth, that I first declare what proportionalitie is, and then how it is deuided. So beholde both the definition and part of the deuisions thereof also comprised in the rule insuing.

Of this second kinde of comparison (viz. in disproportionalitie) there is no Arithmetike, as saith Vrsticius.

196. Proportionalitie, in generalitie,
call'd but proportion for shortnes of name,
Is when comparisons, all of like qualitie,
in quantitie are but one and the same

Euc. 4. d. 5 Sche. tract 2. cap. 3. Boetius li. 2. ca. 40 41. & 43. Xylander lib. de minutiis, fol. 34. 35.

197. And it is either first Arithmeticall,
consisting in comparing of difference,
Or els secondly 'tis Geometricall,
which of proportions is the like conference. } 170

198. Both kindes of which proportions are double,
to wit, continuall, and discontinuall,
That is continuall, whose termes without troble,
are each vnto other still proportionall,

So that euery meane therein alwayes goeth, for antecedent and conſequent both.

Heere is much matter contayned in this maxim, for the firſt verſe thereof ſheweth you, how this word (proportion) for the briefenes thereof in vtterance, is generally taken and vſurped, not onely in common ſpeach, but alſo of the beſt writers, yea and often times in this preſent treatiſe in the ſtead of Proportionality.

And next, that it diſineth proportionalitie to be the conference of diuers compariſons all of like qualitie, and likewiſe of one ſelfe ſame quantitie.

And then the ſecond verſe declareth, that ſuch proportionalitie is diuided into 2. kindes, that is, firſt Arithmeticall, which conſiſteth in comparing the quantity of differences: and ſecondly, Geometricall, which conſiſteth in the like conference of proportions.

Eun. I pray you ſhew me ſome example hereof to make all the matter more cleare.

Phi. Then for example (touching the firſt kinde of proportionality) take theſe 4. numbers 2.5.8.11. betweene whome I may make eyther but 2. compariſons thus, ſaying, as 2. differeth from 5. ſo doth 8. from 11. or if I will, I may make 3. compariſons betweene them thus, ſaying, as 2. differeth from 5. ſo doth 5. from 8. and 8. from 11. where (as you ſee) euery of theſe compariſons, conferences, or relations, are all of like quality, that is to ſay, they are all of differences only, and becauſe the ſame differences are all equall, or of one ſelfe ſame quantitie (for the quantitie of the difference in euery compariſon by the 18. verſe is iuſt 3.) therefore the ſame compariſons by the firſt verſe of this maxim are named a Proportionalitie.

Likenes in quantitie and qualitie.

And agayne becauſe the ſame proportionalitie conſiſteth ſo of the likenes, equalitie, and identitie of differences, therfore by the ſecond verſe of this Maxim, the ſame is an Arithmeticall proportionalitie.

Eun. This I vnderſtand very well, and therefore by that which you haue ſaid, I imagine that theſe numbers, 3.6.9.12. make a proportionalitie Arithmeticall, for in vſing the like conference, their difference or excelle, appeareth euery where equall in quantitie, that is to ſay 3.

Philo. You haue ſaid very truly. Now therefore touching the ſecond kinde of proportionalitie, take example by theſe numbers, 3.9.27.81. which I compare now, not in their differẽces (as before) but

but in their quotients or proportions together, saying thus, as 3. is to 9. so is 9 to 27. and so is 27. to 81. whereby is ment that as 3. is $\frac{1}{3}$. of 9. so 9 is $\frac{1}{3}$. of 27. and so 27. is $\frac{1}{3}$. of 81.

Eu. Inveede I playnely perceiue, that euery of the comparisons here made by you, are all of one kinde or like qualitie, as the first verse of this present maxim requireth, that is to say, all of proportions onely, & not partly of differences and partly of proportions.

Phi. You say wel, and now because the same proportions in quantitie, are all equall, and but one and the same proportion, that is to say Subtripla, therfore I say, such comparisons, conferences, or relations, by the first verse of this maxim are a proportionalitie. Euc. 5. d. 5

And further, because this proportionalitie consisteth of the similitude, equalitie and Iwemtitie, of the quotients or proportions betweene numbers (and not of their differences, as the other did) therefore I say (by the second verse of this maxim) that the same is a Geometricall Proportionalitie.

Eu. So here I learn by your speach, that when such comparisons or relations of numbers, consist onely altogether of differences, or onely altogether of proportions, then your meaning is, that the same comparisons, are all of one kind or of like qualitie, and when such differences or proportions are all one, or equall together, then I am to vnderstand, that such comparisons are of one selfe same quantitie whereby they are a proportionalitie.

So that I perceiue proportionalitie is no more in effect, but 3. or moe equall differences compared together, or els 2. or more equall proportions conferred together, and when such equalitie is onely of differences, then it is a proportionalitie Arithmeticall, but when it is of proportions onely, then it is a proportionalitie Geometricall.

Phi. It is euen as you say, but now the last verse of this maxim saith further, that both these kindes of proportionalities, are double or twofold, that is to wit, Continuall, and Discontinuall: and it teacheth you that, that proportionalitie is continuall, whose termes without any let, trouble, or interruption, are each to other still proportionall, so that all the meanes or middle termes therein, alwayes go both for antecedents & consequents also. As for example, take the Arithmeticall proportionalitie lastly propounded by your selfe, which was this, 3.6.9.12. whose termes as you see, are all proportionall each to other continually, from the first to ỹ last, for that their differences are all equall, and 6. & 9. the 2. meanes or

midle termes serue & go both for antecedents & also for cõsequents. For when in comparing them we say thus. As 3 differeth from 6. so doth 6. from 9. and 9. from 12. You heere and see, that 6. is twice repeated and twise compared, that is, once as consequent to 3. & next as antecedent to 9. and so likewise is 9. twice repeated, & twice compared, once as consequent to 6. and next as antecedent to 12. wherfore such a proportionalitie is sayd to be continuall.

The like is to be said if the proportionalitie be Geometricall, as here 3.9.27.81. wherein euery terme continually from the first to the last, is still proportionall, for that now their quotients are all equall, and here 9. and 27. the 2. meanes, serue also both for antecedents and consequents, for when in comparing them we say thus. As 3. is to 9. so is 9. to 27. and 27. to 81. you see as before, that 9. and 27. are twice expressed and twice compared, wherfore this proportionalitie is also continuall.

And this Proportionaltie continuall may consist onely in 3. termes actually, but not in fewer, as heere 2. 4. 6. or heere 3. 9. 27. and yet euery such 3. termes are esteemed and reputed to be 4. potencially, because euery such midle terme, hath alwayes a certeyne power and authoritie, to supplie the roome, and serue in the stead of 2. termes, so that if these proportionalities were fully and perfectly extended, and described accordingly, they would appeare in 4. termes, whereof the 2. meanes shall alwayes be equall or both one, as for example Arithmetically thus, 2. 4. 4. 6. and Geometrically thus. 3.9.9.27. or thus $\frac{3}{9}$ and $\frac{9}{27}$.

Eu. Indeed by this description it is apparant, that in the one proportionalitie 4. serueth for 2. termes, and in the other 9. doth the like, which (as I said but late) argueth that there are 2. proportions at the least in euery Proportionalitie, because there are 2. antecedents and 2. consequents at the least to make a proportionalitie, although the same 2. antecedents and 2. consequents, may be comprehended in such 3. termes only, according to your former saying.

Phi. That is very true, and heerewithall note, that as the antecedent and consequent of any proportion, are said to be the termes of that proportion, so the 2. proportions of any proportionalitie, (whether they be both differences, or both quotients) are said to be proportionalls and the termes of that proportionalitie, whereby you may see a great diuersitie betweene proportion and proportionalitie, and yet as I said before, we most commonly call the whole 4. termes abusiuely, but a proportion.

Eumia.

Eu. I perceiue it well, for I see that you your selfe do not vnely ost times in your speach, vse the one insted of the other, but also sometimes I finde it so vsed euen in the very Maxims and rules.

Phil. You may doe so in many of them, and no great maruaile, sithe this maxim it selfe affirmeth, that it is more generally called proportion, then proportionalitie, and yet I cannot denye it to be a fault, though excusable to him that vnderstandeth himselfe therein. But now let vs returne to the definition of proportionalitie discontinuall, which is thus.

199. Porportion disioyn'd or discontinuall,
is any 4. termes by couples so plac'd,
That the first 2. in relation are equall,
To the relation betweene the 2. last,
But yet the 2. meanes are not in like fashion,
iust equall with them in like relation. } 171 Boetius libr 2. cap. 40.

As for example, first take these 4. tearmes, 11. 9. 5. 3. which I compare Arithmetically thus, saying as 11. exceedeth 9. so doth 5. exceede 3. wherein you see the excesse or difference betweene 11. & 9. the first couple, is equall to the excesse or difference betweene 5. and 3. the latter couple, wherefore such comparisons or conferences being of like qualitie, and of equall quantitie, are a proportionalitie by the 196. verse, but because the excesse or difference betweene 9. and 5. the 2. meanes, is not iust, equall, and of the same quantitie that the other differences are, therefore by this verse, the same proportionalitie is called disioyned or discontinuall.

Euma. Indeed it is manifest, that the difference betweene 11. & 9. the first couple is 2. and so likewise is the difference betweene 5. and 3. the latter couple also, but the difference betweene 9. & 5. the 2. meanes or middle tearmes, is not 2. but 4. and so not equall nor like to the other differences.

Phil. You say truth. and for further elucidation hereof, I secondlie propound an example in these 4. termes, 8 4. 12. 6. which I confer Geometrically thus, saying as 8. is to 4. so is 12. to 6. wherein euen as before, so now you see the comparison or relation of the 2. first termes to be equall to the comparison of the 2. last, for ech antecedent is to his proper consequent in double proportion, but the comparison of the 2. meanes, that is of 4. to 12. is in Subtripla proportion, which is nothing equal or like to Dupla, wherfore this kinde of proportion is also disioyned or discontinuall, and is so called, because it holdeth not continually from the first terme to

the laſt, neither is any terme thereof compared vnto repeated more then once, and it cannot conſiſt in fewer termes then 4. but it may conſiſt of any euen number of termes aboue 4. whereas Proportionalitie continuall, may conſiſt in any number of termes euen or odde, aboue 3. but not in fewer, as I told you before.

Eu. In al this that you haue ſaid I vnderſtand you very well, and hereby I perceiue now, that all the mention ſo often made in the 20. 47. 58. 59. 61. 91. 92. and 93. verſes, and their gloſes, and almoſt generally in all the verſes of the whole 8. Chapter of your former booke, touching 4. numbers proportionall, is onely meant and to be vnderſtood of numbers proportionall in Geometricall Proportionalitie diſcontinuall.

Phi. It is ſo indeed, for of this proportionalitie eſpecially, dependeth all the chiefe rules, principall concluſions, and moſt cunning ſecrets of Arithmetike. And although (as I ſaid) it may conſiſt in any euen number of termes aboue 4. and not in fewer, yet generally and ordinarilie, when mention is made of proportionalitie or proportionals, therby is onely meant 4. termes or numbers in Geometricall proportionalitie diſcontinuall.

Euma. Very good, this I hope I will remember.

Philo. Then now next followeth the diuiſion of Proportionalitie diſcontinuall into 2. kindes, with the names, definitions, and expoſitions of them both, which is ſuch as inſueth:

Eu. 21. d. 7

200. Proportion diſioynd or diſcontinuall,
is eyther direct or reciprocall,
That is direct, whoſe conference growth,
from firſt to ſecond, and from third to fourth. } 172
201. And that is conuerſed or reciprocall,
whoſe termes are conferd in order diſlocall,
For that the firſt to the third is compard,
and then the fourth to the ſecond backwarde.

Here is declared vnto you, that proportion diſcontinual, is either direct or conuerſed, which otherwiſe is alſo named reciprocall.

Touching direct proportionalitie, ſomewhat hath beene ſayd already in the gloſſe of the 91. verſe, but now this verſe expreſſye defineth that proportionalitie to be direct, whoſe conference or relation of termes, groweth directlie from the firſt terme to the ſecond, and from the third to the fourth, euen in ſuch order as they are locallie deſcribed or ſituated, as theſe heere, 8. 6. 4. 2. which Arithmeticallye, are directly compared thus: as 8. exceedeth

exceedeth 6. so doth 4. exceed 2. where you may see the conference or comparison of the tearmes is such, that the excesse of the first terme aboue the second, is the excesse of the third aboue the fourth, and so the antecedent of each couple is compared to the next consequent rightfoorth directly and orderly as they stand, and as their places naturally require.

But proportionalitie conuersed or reciprocall, is defined by this maxim, to be that proportion, which beginning with the first tearme, compareth it forwardly or directly, yet not to the next tearme but to the third, & then secondly beginning with the fourth or last tearme, cõpareth it to the second conuersedly, in a backward order dislocally, that is as you would say, contrarie to their places. As when in comparing these 4. tearmes. 8. 2. 4. 6. I say thus, as 8. exceedeth 4. or differeth frõ 4. so doth 6. exceed 2. or differ from 2. in which comparisons or conferences you see a very perfect proportionalitie by the 196. verse, but if the same tearmes should be compared directly as they stand, then were they vtterly disproportionals, for the excesse of 8. from 2. is thrice the excesse of 6. from 4. wherfore such proportionalities are called conuersed or reciprocall.

The like is to be sayd of Geometricall proportionalitie, as if these tearmes 6. 3. 4. 2 being compared first directly, that is to say 6. to 3. and 4. to 2. be afterwards at some other time or vppon some occasion, displaced from their ordinarie places and set in this maner, 6. 2. 3. 4. or thus $\frac{6}{2}$ & $\frac{3}{4}$: now you see if they should be compared together directly in such order as they now stand, their comparisons would be vtterly disproportionall, for 6. to 2. is Tripla and 3. to 4. is Subsesquitertia, which are nothing like, but if the first be compared to the third, and the fourth to the second, that is, 6. to 3. and 4. to 2. then will the comparisons be equall, and the 4. tearmes will make a perfect proportionalitie, which for the cause aforesayd is called and named proportionalitie conuersed or reciprocall.

Eu. I vnderstand both your diffinition & proportionalitie discontinuall, & also the 2. kinds therof, viz. as it is either direct or conuersed, very well hetherto: and to my iudgement, direct proportionalitie varieth from conuersed proportionalitie, but only in the location or situation of the tearmes, for so I gather by the glosse of the 98. verse.

Phi. Your Iudgement is not amisse therin, but now before I passe any further, I thinke good to giue you one note by the way, which

is this:

Euclid. 12. d. 5. Termini homologi et heterologi.

202. Note that in euery proportionalitie,
We properly call the 2. Antecedents,
Gender like tearmes, for likenesse in qualitie,
Which name also serues for the 2. Consequents.
But th'antecedent of th'one proportion,
And consequent taken sole of the tother,
Are called tearmes dislike, because in relation,
They nothing agree in rate like the other. 173

Eu. So then in this proportionalitie $\frac{4}{8}$ and $\frac{3}{6}$ 8. and 6. are genderlike tearmes, and so likewise are 4. and 3. but 8. and 3. and likewise 6. and 4. (which are as it were the byas tearmes in fractions) are those which you call tearmes dislike.

Phi You conceiue the meaning of the note well, wherefore now to proceede, you shall vnderstand that there are certaine properties common to both kindes of proportionalitie, I meane aswel to proportion Arithmeticall as Geometricall and aswel to proportionalitie continuall as discontinuall, whereof this is one.

Proportionalitie ordinate. Eue. 17. & 18. d. 5. where, the one definition calleth this kinde of proportinalitie, a proportiõ by equalitie, and the other an ordinate proportionalitie, but in truth the effect of both, is but one thing.

203. If 2. rankes of tearmes haue such equalitie,
Aswell in number as proportionalitie,
That as the tearme set in the first row,
Compared to his fellowes directly doth show.
So euery tearme of the second ranke eke,
Compared to his fellowes directly doth like, 174
Then all the middle tearmes being forsaken,
The 4. extreames alone being taken,
Keepe like proportion together directly,
And are proportionals also exactly.

As for example, first Arithmetically I take these 2. rankes or rowes of numbers here described, where in the first ranke is as you see, 4. 2. 6. 10. and in the second is 7. 5. 9. 13. now I say, that as 4. exceedeth 2. so doth 7. exceede 5 and as 2. differeth from 6. so doth 5. from 9. and agayne as 6. differeth from 10. so doth 9. from 13. and because these numbers are proportionals thus directly, therefore by the 92. verse they shalbe also alternately, wherefore as 4. differeth from 7. so doth 2. from 5. and 6. from 9. and 10. from 13. and so now by this present maxim, all the middle tearmes being forsaken or omitted, the 4. extreames onely being taken, shalbe proportionals directly also, wherefore I conclude that as 4. differeth

4	7
2	5
6	9
10	13

reth from 10. so doth 7. from 13. and alternately, as 4. differeth from 7. so doth 10. from 13. The like demonstration is to be made also geometrically, as in these 2. rankes of termes adioyning may appeare.

3 — 6
2 — 4
4 — 8
5 — 10

Euma. Indeede I see that as 3. is to 2. in the first ranke, so is 6. to 4. in the second ranke, and as 2. is to 4. in the first ranke, so is 4. to 8. in the second, and agayne, as 4. is to 5. in the first ranke, so is 8. to 10. in the second, wherefore alternately as 3. is to 6. so is 2. to 4. and 4. to 8. and 5. to 10. and finally as 3. the first extreme of the first ranke, is to 5. the last of that ranke, so is 6. the first extreame of the second ranke, to 10. the last of that ranke, but why you put that indented figure betweene the 2. rankes of numbers, as yet I see not the cause nor reason.

Phi. You shall vnderstand the reason thereof hereafter when we come to the golden rule, and in the meane season, I cannot but say, that you haue demonstrated this kinde of proportionalitie very well, which by Euclid is sometimes called an equalitie ordinate, & sometimes proportion by equalitie, but now I will proceede to the next propertie which is somewhat vnlike to this, and is thus.

204. VVhen 2. ranks of tearmes whose nũbers are equal
In conference so together do fall,
That as the first tearme of one of the rowes,
Vnto the second tearme thereof goes,
So doth the second tearme of the tother,
Shew to the first tearme his owne elder brother,
And so euery terme directly reherst,
Like his collaterall, taken conuerst,
Then shall their extreames (all meanes ouerpast)
Conuersedly be proportionals last.

Euclid. 19. d. 5. Proportionality perturbate

175

As for example Arithmetically, I take these 2. rankes of tearmes here described, both which rankes are equall in number as you see. And now in comparing their differences, I say as 5. the first tearme of the first ranke differeth from 7. the second tearme of that rowe directly, so doth 4. the second tearme of the second rowe differ from 6. his elder brother, I meane from ỳ first terme of that row conuersedly: and as 7. in the first ranke exceedeth 2. there directly, so doth 9. the collaterall of 2. in the second ranke, exceede 4. in

15 — 6
7 — 4
2 — 9
8 — 3

that

that ranke conuersedly. And finally as 2. differeth from 8. in the first rowe directly, so doth 3. differ from 9. in the second ranck conuersedly: wherefore by this rule I conclude, that ouerpassing al the meane tearmes, the 4. extreames taken onely at the last, shalbe proportionals conuersedly, and therefore I affirme, that as 5. differeth from 8. directly, so doth 3 from 6. conuersedly.

Demonstrator. For here by supposition 7 —— 5. is 6 —— 4 and 7 —— 2. is 9 —— 4 wherefore by the 46. verse, as 2. differeth from 5. so doth 7 —— 5. differ from 7 —— 2. that is by supposition, as 6 —— 4. differeth from 9 —— 4. which by the sayde 46. verse is as 6. differeth from 9.

So that as 5. exceedeth 2. so doth 9. exceed 6. wherefore nowe from the extreames of the one ranke to the extreames of the other (ouerpassing all the meanes) I say as 5. differeth from 8. directly, so doth 3. differ from 6. conuersedlye.

The demonstration of this kinde of proportion geometrically differeth little from the former, as for example in these 2. rankes of tearmes here described, wherein as 3. is to 4. directly, so is 15. to 20. conuersedly, and as 4. is to 5. directly so is 12. to 15. conuersedly, and as 5. is to 6. directly, so is 10. to 12. conuersedly, for here the products of 3. by 20. of 4. by 15 of 5. by 12. and of 6. by 10. are equall together, aswell by supposition as by the 100. verse, wherefore sith the products of 3. by 20. & of 6. by 10. are equall, the same 3. shalbe to 6. directly as 10. is to 20. conuersedly, by the glosse of the 98. verse.

3 — 20
4 — 15
5 — 12
6 — 10

Eu. I vnderstand the meaning of this maxim very well, and the better for your playne examples and demonstrations, but I pray you hath this kinde of proportionalitie any speciall name.

Phi. Yea, it is called by Euclid and others, sometime an equalitie perturbate, and somtimes a proportion by equalitie in proportionalitie perturbate: and thus much touching proportionalitie in generall, wherefore now I will entreate of both kinds more particulerly, and first of Arithmeticall proportion continuall in order following.

Of Arithmeticall Proportionalitie continuall in 3. termes onely, with the doctrine and properties thereof. Chap. 3.

Tonstall pag. 104.

205. To finde the middle terme betweene,
two extreames Arithmeticall:
Adde both th'extreames, so halfe I weene,
shall make the meane proportionall. } 176

There

There is no other learning that I know (consisting but in 3. tearmes of this propotionalitie) but only the finding of the meane propottionall, to any 2. extreames assignd, and the declaration of 2. properties naturally incident thereto, now touching the finding of the meane, this verse teacheth it so playnely that it needeth no further explication.

Eu. So then if 5. and 15. were 2. extreames, vnto whom I were appointed to finde the meane propottionall, arithmeticallie I must ad 5, and 15. together, and their totall will be 20. whereof the one halfe, to wit, 10. shalbe the meane propottionall to be placed betweene the former extreames thus 5. 10. 15.

Phi. It is iustly so, for you may easily see, that as 5. differeth from 10. so doth 10. from 15. and so their compartions are propottionall in Arithmaticall propottionalitie continuall, by the 198. verse, but now I will rehearse the 2. properties adherent to this propottionalitie, and so will bring you to the last point of finall bounds and issue of it. The first propertie is thus.

206. Euery propottionalitie, if it be Arithmeticall,
Succeeding like in qualitie, but in 3. tearmes continuall,
Hath euermore this propertie, insuing by that meanes,
The middle tearme doth equifie, just half the 2. extremes. 177

Tonstall. pag. 201. Salig. lib. 2. ca. 3. verse 4.

As for example, let 9. 12. 15. be 3. tearmes giuen in Arithmaticall propottion continuall, and let the difference betweene 9. and 12. be 3. as 9. differeth from 12. so let 12. differ from 15. now I say by this verse, that 12. the middle tearme, doth equifie, (or is iust equall to) halfe the totall of 9 + 15. which are the 2. extreames.

For here by supposition, and by the gloss of the 44. verse, 9 + 3 and 12. are equall, and likewise 12 + 3 and 15. are equall, wherfore by the 30. verse 12 + 12. and 9 + 15 are equall, and so consequentlye 12. the middle tearme is equall to halfe 9 + 15. which was to be proued. The other propertie is thus.

207. If any middle tearme of 3, be halfe his 2. extreames,
Then the propotionalitie, between them by that meanes
Is onely Arithmatical, consisting in the difference,
And also is continuall, as will appeare by conference. 178

Tonst. pa. 201. Salignac. lib. 2. cap. 3. verse 5. Boetius lib 2. ca. 43.

This verse being the conuerse of the next former, needeth no great proofe, yet for example, let 3. 7. and 11. be three tearmes giuen, whose 2. extreames, to wit, 3. and 11. make 14. whereof let 7. the middle tearme be halfe. Hereupon I say by this verse that 3. 7. and 11. are in Arithmaticall propottion continuall, for by supposi-

tion

tion 3 + 4 is 7. and 7 + 4. is 11. wherefore as 3. differeth from 7 so doth 7. from 11. wherefore by the 197. verse the same 3. 7. 11. are in Arithmeticall proportionalitie continuall.

Eu. All this is very manifest, and here I perceiue also that you often times vse the woord proportion instead of proportionalitie.

Phi. You haue sayd truly, but now to bring you to the last point, and finall scope, whereunto this kinde of proportionalitie tendeth as I promised, marke well the verse following.

Salig. lib. 2. ca. 2 verse 9. Boetius. lib. 2. cap. 43.

208. Proportion Arithmeticall,
Continued in mo tearmes then three,
VVith like excesse continuall,
Encreasing still as oft we see, } 179
Is thereby nam'd immediately.
Progression Arithmeticall,
VVhose doctrine craues espetially,
To place the least tearme first of all.

Lo sir, hereby you may perceiue that euen as a brooke within the bonds of his owne bankes, is knowen and called by his proper name, which it retayneth, vntill it fall into some greater riuer or into the sea, whereby it doth not onely then end, but also loseth his former name, and yet naturally cannot refrayne, but still endeauoureth his course to that ende, euen so this Arithmeticall proportion continuall whilest it exceedeth not 3. tearmes, is knowne by that name, and retayneth the same, but if it be continued in a greater multitude of tearmes, as very many times it is, then doth it both take his end, and also loose the name thereof, for progression Arithmeticall as it were a greater riuer or sea, swallowing it vp, changeth it into his owne name, in so much that all the whole multitude of such tearmes, is thencefoorth knowen and most properly called by the name of progression arithmeticall, in which progression, the first tearme ought alwaies to be the least as this verse affirmeth, & yet as the course of the brooke alwayes inclineth towards the greater riuer or sea, so the conclusion of this kind of proportionalitie is at the length to fall & conuert into such a progression whose doctrine for asmuch as it is both wittie & pleasant seruing to many good vses, I wil here vnfold at large before I goe any further, and then will returne againe to Arithmeticall proportionalitie discontinuall.

Eu. It seemeth by this verse & your speach that these numbers 3. 6. 9. 12. 15. 18. & likewise these 1. 2. 3. 4. 5. 6. 7. 8. are seuerall progressions

ſions arithmeticall, ſith their proportionality is not onely continued in mo tearmes then 3. But alſo the exceſſe or difference betweene euery tearme is equall, and the tearmes encreaſe all alike from the firſt to the laſt, & moreouer the leaſt tearme is firſt in place and order as this verſe requireth.

Phi. So are they indeed, but this laſt progreſſion 1,2.3.4.5.6,7,8. becauſe it beginneth with 1 & hath but 1 alſo for ỳ cõmon difference or exceſſe, is ſomtimes named alſo natural progreſſion, for ỳ al nations naturally nũber in that order: now what doctrine conſiſteth in this arithmaticall progreſſion, ỳ next chapter ſhall fully declare. Progreſſiõ naturall.

Of Progreſſion Arithmeticall. Chap. 4.

209. When any Numbers mo then three,
Do ſeeme to grow and ſtill encreaſe,
And in their growth do all agree,
To differ ſtill by like exceſſe,
Such ordered tearmes this art doth call,
Progreſſion Arithmeticall. 280

210. Whoſe doctrine doth no leſſe depend,
Vppon th'exceſſes then the tearmes,
Yea, euen the ſpaces helpe doth lend,
To further that which it concernes.

Eu. The firſt part of this rule cõtaineth the definition of arithmetical progreſſiõ, is plaine inough, & agreeth with ỳ laſt rule before, but ỳ laſt I vnderſtand not, for I know not what is ment by ỳ ſpaces.

Phi. Do you know that theſe termes 3.6.9.12.15.18. are a progreſſion Arithmeticall.

Eu. Yea, I am aſſured thereof aſwell by the diffinition of it in this preſent verſe, as by the laſt verſe before. The numbers of any progreſſion are properly called the tearmes of that progreſſion.

Phi. Well, then how many numbers or tearmes be there of them.

Eu. There be 6. nũbers, which I remẽber in the 18. verſe & gloſſe thereof, you ſayd are as vſually called 6. tearmes as 6. numbers.

Phi. I did ſo, you remember it wel, but now know you what the cõmon difference or exceſſe betweẽ ỳ termes of this progreſſiõ is.

Eu. Yea very well, for by the 181. verſe, I know it to be 3.

P. And know you not how many ſpaces is betweẽ theſe 6. termes.

Eu. No truly, I cannot tell what you meane by a ſpace, except that therby you meane the voyd roome or diſtance which of neceſſitie is and muſt needes be, betweene tearme and tearme, to diſtinguiſh the one from the other, and if you meane ſo, then is their 5. ſpaces in this progreſſion.

Philo.

Phi. My meaning is none otherwise, but euen so as you say, and therefore there are 5. spaces onely as you also sayd.

For there must needes be one space lesse then there are tearmes alwayes, except some ouerthwart person would frowardly describe a number of tearmes in a circuler forme, for the nonce so purposing to satisfie my saying, but now sith ye know what is meant by the spaces, you shall forthwith see how the conclusions of Progression consist aswell in the excesses, as in the tearmes, and also how the very spaces giue great ayde thereunto.

1.2. To finde how many excesses are in the progression and to sum them,

211. For first, how many tearmes once knowen,
Be they in number mo or lesse,
So many spaces are saue one,
And euery space hath his accesse,
212. Of which excesses true to cast, 181
The totall summe without mistrust,
Abate the first tearme from the last.
The rest doth shew their totall iust.

Euma. If I vnderstand this rule rightly, the first thing that is to be sought, is the number of the tearmes, which being once knowen, then it followeth that there are so many spaces sauing one as there are tearmes, the truth whereof appeareth by the progression last propounded, where there were these 6. termes 3. 6. 9. 12. 15. 18. and 5. spaces: then secondly if I would cast the totall summe of all the excesses betweene these tearmes, I must abate 3. the first tearme, from 18. the last tearme, and so remayneth 15. which by the second verse here, is the totall summe of all the excesses or differences.

Phi. You vnderstand the verse very well, and that it is so, appeareth of it selfe manifestly, for your selfe affirmed before, that the common difference or excesse betweene the termes is 3. and the number of spaces by your owne confession also is 5. now three times 5. is iust 15. agreeable with the remaynder of the last tearme, after the abatement thence of the first.

Eu. That cannot be denied, wherefore the number of the tearmes being first knowen, now secondly I know readily how to summe or find the totall of all the excesses or differences.

Phi. Then harken to the next verse, which teacheth you a third poynt.

3. To find the common difference or excesse.

213. And thirdly if you would estate,
To know th'excesse betweene the places,

The

The firſt terme from the laſt ſubtract,
the reſt diuided by the ſpaces,
The quotient playnely doth expreſſe,
the common difference or exceſſe. } 182

Loe heere you ſee now, how the very number of ſpaces ſeruety to ſome vſe in the worke of progreſſion: As for example, if you know the number of the termes, and alſo the extreames of a progreſſion, that is to ſay, the firſt and laſt termes, but are ignorant of the common exceſſe or difference, then this preſent verſe ſheweth you how to finde it.

Euma. I will trie it by your fauour, euen with the former progreſſion 3.6.9.12.15.18. from whoſe laſt terme I ſubtract 3. the firſt terme, ſo reſteth 15. which I muſt diuide by 5. the number of the ſpaces, and ſo the quotient bringeth 3. for the common difference, which is iuſt as I found it before, by the 181. verſe, when all the termes were ſuppoſed to be knowne.

Phi. So then you ſee, how both the rules agree to worke one true effect, though not both in one manner, wherefore I proceede thus.

214. And fourthly if you wiſh to know,
any meane terme in the rowe.
Then from the number of his place,
if you abate a ſingle ace,
And fold the reſt with diligence,
but by the common difference,
That offcome added to the firſt,
doth yeeld the terme for which you thirſt. } 183

4 To finde any midle terme of the progreſſion.

As for example, let the former progreſſion, viz. 3.6.9.12.15.18. proceede and be continued foorth ſtill, vntill it haue 40.50. or 100. termes, and let the terme that I deſire to know be the 24. terme of the ſame progreſſion. Now as the firſt terme occuppeth the firſt place, ſo you know the 24. terme occupieth the 24. place, wherfore ſithe the number of his place is 24. I ſay by this verſe. 24—1. is 23. which I folde or multiply by 3. the cõmon difference, & the product is 69. vnto which I adde 3. the firſt terme, and ſo the totall yeeldeth 72. for the 24. terme of this progreſſion, which is the terme I wiſhed to know.

Euma. Very good, this I vnderſtand well, wherefore I pray you proceede.

215. And fiftly if you liſt to trie,
what terme is either firſt or laſt,

To finde the extreames, that is, the firſt & laſt termes of the progreſſion.

5 The common exceſſe multiplie,
by number of the ſpaces paſt,
Whoſe offcome from the laſt terme cut,
the reſt remaynes firſt number cleare,
Or els the firſt terme thereto put,
doth make the latter terme appeare.

216. But if th'extreames be both vnknowne,
diuide the totall ſum of all,
By number of the termes alone,
whoſe quotient doubled keepe you ſhall,
And from it toll the product made,
of all the ſpaces by the exceſſe,
So halfe the remnant by that trade,
ſhalbe the firſt extreme doubtles,

217. But if the exceſſe and both th'extremes,
be all conceald for ſome pretence,
The leaſt extreme, then by that meanes,
ſhalbe the common difference,
Which for to find, you muſt augment,
the number of the termes 1. more.
Infolding that incontinent,
in halfe the ſum they were before,
And let that offcome next diuide,
the totall of the whole progreſſion,
So ſhall the quotient be your guide,
to yeeld your want to your poſſeſſion.

184

The varitie of chances that happen, ſometimes by ignorance of the extreames, and ſometimes by concealing the common exceſſe, & ſometimes by both, cauſeth this rule to be longer then any of the reſt.

Yet hath it no obſcuritie at all, for if any of the extreames, with the common exceſſe and the number of the ſpaces be knowne, then the firſt part thereof ſheweth you, how you may thereby finde the other extreame wanting alſo.

But if you know neither of the extreames, yet if you know the number of the termes, and their totall ſum with the common exceſſe, then the ſecond part teacheth you, how you may thereby come to the knowledge of the firſt extreame, and ſo conſequently to the knowledge alſo of the laſt.

But if neither the common exceſſe, nor any of the extreames be

expreſ-

expressed, yet if you know the number of the termes & their totall, then the third part teacheth you, how by such knowledge to finde both the first extreame, and also the common difference, both which as the same part of the rule affirmeth, are but one self same nũber.

Euma. Indeed so the rule affirmeth, whereof I purpose to make a triall.

Philo. So may you do, using still this former progression, 3.6.9.12.15.18. for example of your practise.

Eu. Then to begin, first seeing the common excesse in this progression is 3. & the nũber of the spaces 5. according to the prescript of the first part of this maxim, I multiply the same 3. & 5. together, & the offcum is 15. which I cut off or abate from 18. the last terme, and so the rest being 3. sheweth the first terme cleerely. Againe, if to the same offcum 15. I put 3. the first terme, so doth 18. appeare for the last terme, according to the propoze of the rule.

Secondly, supposing that I know neither of the extremes, but know the number of the termes to be 6. their totall to be 63. & their common excesse to be 3. Yet seeking by the second part of this rule to finde the extreames as there is prescribed. I diuide (as that rule biddeth me) 63. the totall sum of all the terms, by 6. the number of the terms, & ỳ quotient is $10\frac{1}{2}$. which doubled is 22. ỳ 22. I keep. Then do I multiply 5. the number of the spaces, by 3. the cõmon excesse, and the product is 15. which I toll or take away from the 22. before kept, and so remaineth 6. halfe which remainder being 3. is the first terme, and so now hauing the fyrst terme, I may consequently finde the last, by the first part of this rule.

Phi. You say very true, wherfore proceede to the third part.

Euma. Then thirdly supposing that I neither know any of the extremes, nor yet the common excesse of the giuen progression, but that I know the number of the termes to be 6. and their totall to be 63. I seeke by the last part of this rule, to finde the first terme or common difference, which by the words of the rule is all one: wherfore as the rule willeth me, I augment 6. the number of the terms, 1. more then it is, and so it is made 7. which I folde incontinently in 3. the halfe of 6. which was the number of the termes before, and the product is 21. whereby finally, I diuide 63. the totall sum of the progression, and so the quotient yeeldeth 3. which is not onelie the first terme of this progression, but also the common difference, euen very truly and iustly, as the rule teacheth and affirmeth, wherefore now I pray you go forward with the rest.

6 To finde the number of the termes.

218. *Phi.* Sixtly, the last terme bated lesse,
by foremost terme of all the rowe,
And then diuided by th'excesse,
the quotient which dooth thereof growe.
Augmented but one vnitie,
doth tell how many termes there be,
219. Or adde the first terme to the last,
whereby diuide the totall sum,
VVhose quotient doubled, loe in hast,
the number of the termes is come. 185

As for example, suppose that 3. and 18. the 2. extreame termes onely, with 3 the common excesse or difference of the former progression is knowne, but the number of the termes is not expresst. Then to finde the same, let 3. the first extreame be abated from 18. the last extreame, so is the remainder 15. which being diuided by 3. the common excesse, quoteth 5. Now 5 + 1. is 6. the true number of the termes, by the first part of this Maxim.

Or els thus, if 3. and 18. be knowne to be the first & last termes, and the totall of all the termes be also knowne to be 63. (as it is in the former progression) but the number of the termes & also the cõmon excesse is vnknowne: then shall I adde the same 3. & 18. together, which make 21. whereby I diuide 63. the totall summe, and so the quotient is 3. which doubled maketh 6. the true number also of the termes, as the rule prescribeth.

Euma. All this is plaine inough, and I vnderstand it well.

7 To sum the whole progression, or to finde the totall of the terms.

220. *Phi.* The termes once told by 6. rule past,
at length their totall sum to find,
Note downe how many termes thou hast,
in some place by which likes thy minde,
Then adde the first terme to the last,
and note likewise their totall giuen,
Of these 2. noted numbers plac'd,
if th'one be odde, tother will be euen.
Then let halfe that which euen is,
But multiplie the other whole,
The product will not faile nor misse,
to deale the totall sum for doale. 186

The last point of progression generally, is to find the totall sum of all the termes, which is commonly called the summing of the progression, which thing this rule hath taught very playnely and

hereby

hereby you may perceiue, that the knowledge of the 2. extremes and the number of the termes of any progression, is sufficient to attayne to the totall sum of all the termes, as for example.

There is an Arithmeticall progression consisting of 200. termes whereof the first is 5. and the last is 1000. now if you vnderstand this rule well, let me see how you can thereby geue me the totall sum of the same progression.

Eu. You say there is 200. termes, which number sith I know I note downe in a by place, as the rule biddeth me thus. 200

Now by this rule I must add the first terme which you say is 5. to the last, which you affirme to be 1000. so is their totall 1005. which number I likewise note downe nere to the former noted number as here you see it in the voide roome, & now as the rule saith I see that one of these noted numbers is euen, & the other is odde, wherefore I must let 100. which is one halfe of the euen number, multiplie, 1005. the whole odd number and so amounteth 100500. the totall sum of those 200. termes. 200 1005

Phi. This was well done of you, wherefore now sith you are so prompt in conceiuing the rules, I will for your recreation and further encouragement heerein, propound some pleasant questions to you answerable by these rules, whereof this is the first.

6. A question of the age of children.

There were 8. brethren wise and sage,
The yongest (as it came to passe)
Was seauen and twentie yeeres of age,
And 50. yeeres the eldest was.
And each of them from first to last,
Did differ like in age from other,
Their oddes in yeeres I would haue cast,
And eke the age of euery brother.

Eu. This is a prettie question, but I promise you I know not how to answer it, by any thing that I haue yet learned.

Phi. No do? why do you not perceiue, that the 8. brethren are as it were the number of the termes in a progression, whereof the yeeres of the yongest brother being 27. is the first (because it is the least) and 50. being the yeeres of the eldest, is the last. For seeing the differences betweene their ages, are all alike and equall, as the question importeth, their ages must needes proceede by an Arithmeticall progression, whose common excesse dooth represent the difference of their ages.

Eum. Indeede it seemeth so, but yet I pray you for my learning

answer this first question your selfe, that so I may the more redily conceyue the order thereof, and of other like questions hereafter.

Phi. I am well contented, wherefore sith the extreames of this progression is knowne to be 27. and 50. and likewise the number of the termes is known to be 8. I am now to seeke but the common difference or excesse, which representeth their seuerall age or ods in yeeres, and that by the third rule of this chapter I find thus.

First I abate 27. the first terme, from 50. the last terme, so resteth 23. which I diuide by 8 — 1. That is by 7. the number of the spaces and so the quotient yeeldeth 3 2/7. for the common excesse or number of yeares betweene brother and brother, wherefore I conclude that the difference or odds in yeers, whereby euery of the brethren was elder then the last borne before him, is 3 2/7 yeeres, and so the youngest brother being 27. yeeres olde, the rest must needes be of the ages following.

The youngest which heere is first, 27. yeeres.
The next, 27 + 3 2/7, that is 30 2/7.
The third, 30 2/7 + 3 2/7, that is 33 4/7.
The fourth, 33 4/7 + 3 2/7, that is 36 6/7.
The fift, 36 6/7 + 3 2/7, that is 40 1/7.
The sixt, 40 1/7 + 3 2/7, that is 43 3/7.
The seuenth, 43 3/7 + 3 2/7, that is 46 5/7.
The eldest 46 5/7 + 3 2/7, that is 50.

Eum. This question is very truly answered, and as plainly described in al things concerning the proofe thereof as may be, wherfore I trust it shall be an instruction for me, in all other like.

Phi. Then heere is another question not much vnlike to it, which I would now see you answere, and it is thus.

7. Another question touching the age & number of children.

A certayne man had children store,
whose youngest, was but 6. yeeres olde,
The eldest 40. and no more,
and each of them in order told,
The one did still in age exceede,
his yonger borne by 2. yeres iust,
How many now came of his seede,
I wish to see by you discust.

Eu. I will do mine endeuour herein, as you shall see, and I perceiue by the forme and processe of the former question, that heerein is an Arithmeticall progression, whereof 6. the yeeres of the youngest childe, and 40. the age of the eldest, are the 2. extreames.

And

And I may gather by reaſon that the common exceſſe or difference is 2. for ſo the very expreſſe wordes of the queſtion importe, but the number of the termes, is not expreſt, wherefore I muſt ſeeke to know it by the firſt rule of this chapiter. And therefore (as that rule willeth me) I abate 6. the firſt extreame from 40. the laſt, and the remainer is 34. which I diuide by 2. the common exceſſe, and there iſſueth in the quotient 17. wherefore by the ſame rule 17 + 1. that is to ſay, 18. is the number of the termes, or of the mans children. But how may I trie that to be true?

Philo. That may you do by Addition, which is the onely proofe of all Arithmeticall progreſſions, for if you adde the yeeres of euery childes age, as I did in the former queſtion, if then the number of the yeares, and the number of the children agree according to the propoſt and ſuppoſition of the queſtion. The anſwer muſt needes be true. Addition the proofe of all progreſſions.

Eu. Then foraſmuch as the youngeſt (which here is to be accompted the firſt) by the propoſt of the queſtion is 6. yeeres olde, and that the ſecond or next borne before him, muſt needs exceede him by 2. yeeres iuſt: then ſhall both his age & all the reſt be as followeth.

The firſt or youngeſt, 6. yeeres.	The tenth, 22 + 2, that is 24.
The next. 6 + 2, that is 8.	The eleuenth, 24 + 2, that is 26.
The third. 8 + 2, that is 10	The twelfe, 26 + 2. that is 28.
The fourth, 10 + 2, that is 12	The thirteenth, 28 + 2, that is 30
The fift, 12 + 2, that is 14	The fourteenth, 30 + 2, that is 32
The ſixt, 14 + 2, that is 16	The fifteenth, 32 + 2, that is 34
The ſeuenth, 16 + 2 that is 18	The ſixteenth, 34 + 2, that is 36
The eyght, 18 + 2, that is 20	The ſeuenteenth, 36 + 2, that is 38
The ninth, 20 + 2. that is 22	The eighteenth. 38 + 2, that is 40.

Which number of yeeres agree truly with the number of children, and in all other things accordeth to the ſuppoſition of the queſtion.

Philo. I cannot but commend your diligence, and confeſſe that you haue done very well, both in working the queſtion, and alſo in deſcribing an occularie proofe thereof: and therefore to the intent you may yet perceiue the better, how to apply the termes of a progreſſion, to the things which repreſent them in any queſtion propounded, I will put forth another queſtion, and anſwere it alſo my ſelfe, whereunto I would haue you giue good heede, and my queſtion is thus. 8. A queſtion touching a iourney.

A certaine footman trauailing frō Chicheſter to London (which

2. Cities are 50. miles distant) encreased his Journey euery day by 2. miles more, then he went the day next before, whereby at 5. dayes end, he came to London: now the question is how many miles his first dayes iourney was.

Here the 5. dayes doe represent the number of the termes, and therefore the number of the termes, must be applied to the 5. dayes the 2. miles dayly encreased, expresse the common excesse or difference, and therefore the excesse is onely to be applied to the 2. miles and the 50. miles distance, betoken the totall sum of the termes, wherfore the totall sum of the termes is to be appropriate onely to the 50. miles distance, now heere is neither the first terme nor the last expresst nor represented by any number geuen, but if the first tearme were once knowne, the sum of congruence must necessarily represent the miles of the first dayes iourney.

Therefore I seeke to finde the first terme by the 5. rule of this Chapiter, diuiding 50. the totall sum of the termes by 5. the number of the same termes, and the quotient is 10. which doubled make 20. Then take I 8. the product of 4. (the number of the spaces) made by 2. the common excesse, and abate it from the same 20, and the remaynder is 12. whose halfe being 6. I take (by that rule) for the first terme, and so affirme, that this trauailers iourney the first day was 6. myles.

Eum. Then must his second dayes iourney be 8 miles, the third 10. miles, the fourth 12. miles, and the fift 14. miles, wherfore now if 6. 8. 10. 12. and 14. make iust 50. then must the answere needes be true, & I see it is so, for the halfe of 6 + 14. multiplyed by 5. (the number of the termes) produceth iust 50. the sum of all the termes, as the last rule of this Chapter teacheth.

And I thanke you much for this application of the termes, to the order of the question, for hereby I finde greater light then I perceiued before, and yet me thinkes this question was harder to answer, then any of the other 2. before.

Phi. You thinke so, because herein, both the extremes of the progression were suppresst and not knowne, otherwise all things were plaine and easie enough to him that vnderstandeth the 7. principall rules of progression conteined in this chapiter, and knoweth skilfully how to apply them. Wherefore for your further exercise, you shall haue some moe questions, whereof take this for one.

4. A question of Kerseys sold.

A Clothier hauing 20. fine Deuonshire kerseis, well worth 5. pound the peece ready money, offered them to sale at that price

to

to diuers marchants, but could not get so for them, at the last he met with a marchant, that offered to giue him a crowne for the first kersey, and 3. crownes for the second, and 5. crownes for the third, and so to augment his price 2. crownes more for euery kersey then before, which offer the Clothier tooke very gladly.

Now therefore I demaund of you, what the marchant payde for these 20. kerseies, and what the Clothier lost by this kinde of sale of the iust valure of 5. pounds whereat he esteemed euery of the kerseies.

Euma. In this question I perceiue that the 20. kerseies represent the number of tearmes, and that 1. crowne signifieth the first tearme, and the 2. crownes increased in the payment of euery kersey expressly representeth the common excesse, wherefore sith the question is, what the totall sum of the payment is, (which sum is represented by the totall summe of the tearmes) I haue no more to doe but to finde the same totall, and so the chiefe part of your question is answered.

Now for the finding thereof, I should by the last rule of this Chapter, adde the first tearme to the last, but because the last is yet vnknowen, I must first seeke it out by the first part of the fift rule of this chapter, wherefore according to the same, I multiplie 2. the common excesse, by 20 — 1. the number of the spaces, and the product is 38. and then 38 + 1. that is 39. shalbe the last tearme, now therefore the number of the tearmes, and also both the extreames being knowen, I proceed to summe the progression by the last rule of this chapter, noting downe first the number of the tearmes in a by place thus. 20

Then adding 1. the first tearme to 39. the last, they make 40. which number I also note down neere my former noted number thus. 20 40

Philo. And now sith both your noted numbers be euen, you may take halfe of which of them you will, and multiplie by the other whole, for whether you multiplie halfe 20. by whole 40. or halfe 40. by whole 20. the product both wayes wilbe 400.

Euma. That is very true, but then why doth the last rule say that if the one be odde, the other wilbe euen, sith here is none odde, but both euen.

Phi. That rule doth not denie that both of them may be euen, but rather affirmeth, that they will not be both odde at once, for if they were, then could you not take the moytie or halfe of any

of

of them without a fraction.

Eu. So. I am satisfied therein, and therefore now to answer your question, I say the marchant paid 400. crownes for the 20. Kerseys. and so the clothier lost nothing of the price whereat he esteemed his Kerseys. for 400. crowns, by the 154. verse maketh 100. pound, and so much doth the 20. Kerseys iustly peeld after the rate of 5. pound a peece.

Philo. Well done, Now what say you to this question.

10. A question of running for a wager.

There are 100. egges placed directly in a rank, euery one a yard distant from other, and a yard distant from the first egge there standeth a basket, now a certayne yong man hath vndertaken for a wager, to gather vp these 100. eggs one after another, and at the taking vp of euery seuerall egge, to returne to the basket, & therein to put them without breaking any, before another young man his companion, shall run to a certaine place 5000. yardes distant from the basket, and shall returne thether agayne, wherefore the question is, which of them shall winne the wager.

Euma. First I see the 100. egges represent the number of the termes the 1. yard distance betokeneth the common excesse, and the.

Phi. Nay, stay, you are deceiued, for here you must consider, that the setting of the fyrst egge requireth the going of 2. yardes, and the setting of the second requireth the going of 4. yardes, and so the third of 6. yardes, &c. for when he goeth to set the fyrst egge he goeth forward 1. yarde, and then in returning agayne to the basket he goeth backe another yard, and likewise when he goeth to set the second egge, he goeth 2. yardes from the basket, and comming backe he goeth other 2. yardes, and so he goeth continually 2. yards more for euery egge, then he did for the next before, wherfore 2. yards being the iourney of the fyrst egge, is the first terme.

Eu. It must of necessitie be so by reason as you say, and therfore I perceiue now the common excesse or difference in this progression is 2 also and not 1. as I mistaking supposed it to be, wherfore now to proceede I seeke the last tearme, by the fyrst part of the 5. rule of this chapter, as I did in the question last before, that is by multiplying 2. the common excesse by 99. the number of the spaces, and so I produce 198. and then by that rule 198 + 2. is the last terme: wherfore now by the last rule of this chapter adding 2. the first terme to 200. the last, the total is 202. which multiplied by 50. (halfe the number of the termes) maketh 10100. being halfe that total multiplyed by 100. the whole number of the termes produceth

reth both wayes 10100. the totall number of the tearmes, that is the true totall of all the yards which he passeth in going to gather these egges.

But his companion running 5000. yardes right forth runneth asmuch also in returning, all which make but 10000. yardes wherefore (if he haue no mischance by the way) he is most like to win the wager, sith he runneth not so much by 100. yards, as doth the other that gathereth the egges, as may easily appeare by subtracting his 10000. yards from the others. 10100. yardes.

Philo. You haue answered very well and truly, but now how do you answer this question. A certayne man oweth me a sum of money which shalbe nameless, which he is bound to pay me at 12. seuerall dayes by progression arithmeticall, paying at the first day 10. r. and encreasing euery payment afterward by 5. pounds, till the dates be expired. now I demand of you what his whole debt is. 11 A question of debt.

Eu. Here the 12. dates represent the number of the tearmes, the 10. r. for the first payment expresseth the first terme, & the 5. pound of encrease, sheweth the common excesse, so that if I knew the last terme, then would the last rule cleare all the rest of the doubt.

Therefore first seeking the last tearme by the first part of the 5. rule, I finde it to be 65. & then by the last rule of this chapter, the product of 65 + 10. by 6. the moitie of the termis, shalbe the totall of all the tearmes, and so consequently the whole debt of this debter, which totall is 450. r. as here appeareth.

	75
	6
	450

Phi. It is very true, but if the question had bin, what this debters. 8. payment should be, then by the 4. rule of this Chapter 8 —— 1. that is say 7. multiplied by 5. the common difference produceth 35. and then 35 + 10. maketh 45. the number of pounds due at his 8. payment.

Eu. But is there no triall of this, for I would see some proofe to assure me hereof.

Philo. The obseruation of the rules is sufficient assurance for you, but if you will not credit those rules which you your selfe haue heretofore confessed to be very true, yet remember that I told you before, how Addition is the ordinarie proofe of all progressions Arithmeticall, and for example thereof, I haue here set out the addition of al the tearmes of this last progression, whereby it appeareth euidently that 450. pounds is the true totall of the debt, and that the eight payment was iust 45. r. The triall of the worke.

1	10
2	15
3	20
4	25
5	30
6	35
7	40
8	45
9	50
10	55
11	60
12	65
	450

Eurus.

Euma. Indeede this demonstration is very manifest, and although I mistrust not any thing you say, yet I like such proofes very well.

12. Another question of debt.

Philo. Well sayd, but now a certaine man is to receiue 260. pounds at 12. seuerall feast dayes by progression arithmetical, and because he knoweth not what to demaund for this first payment, he would gladly giue the same to some man that could ascertaine him what he hath to receiue at euery seuerall feast, wherefore if you can doe this feate for him, you may hap to haue the first payment for your labour.

Euma. O sir, such haps happen seldom to men of our profession but indeed som such rewards (though they came but seldom) would make a man the willinger to call his wits about him. and although I am perswaded, the man you speake of hath no name, whereby I am out of hope of his gift, yet neuertheles I wil trie what I can do to the question, and first I see that the 12. feast dayes represent the number of the termes, and the 260. pounds represents their totall, but heere is neither the first terme nor the last, nor yet the excesse between the termes expresst or represented by any number, but are all suppresst and concealed, wherfore I thinke I must flee to the last part of the 5. rule, for counsell in this behalfe.

Phi. It is the very onely rule you ought to vse in this case.

Eu. Then according to that rule, I augment the number of the tearmes 1. more then they are saying, 12 + 1. is 13. which multiplied by 6. the moytie of their former number produceth 78. by which product I diuide 260. the totall of this progression, and the quotient in least tearmes is $3\frac{1}{3}$.

Wherefore by the same rule $3\frac{1}{3}$. is not onely the first terme, but also the common excesse in this progression, and so consequently, your mans first receipt ought to be $3\frac{1}{3}$ r. that is 5. markes or 10 nobles, and by so much ought euery of his receipts afterward to exceede other, as appeareth here by the proofe thereof, set foorth in such order as you did the last question, wherin (as you see the totall made by the addition of the 12. seuerall payments maketh iust 260. pound.

1	$3\frac{1}{3}$
2	$6\frac{2}{3}$
3	10
4	$13\frac{1}{3}$
5	$16\frac{2}{3}$
6	20.
7	$23\frac{1}{3}$
8	$26\frac{2}{3}$
9	30.
10	$33\frac{1}{3}$
11	$36\frac{2}{3}$
12	40.
	260

Phi. This was very well done of you, & sith you are so running I will put onely one question more which M. Record hath in his ground of artes, and so will end this arithmeticall progression, and it is thus.

I haue

13. A question of money disbursed to diuers persons.

I haue disbursed 685. pounds to diuers persons, how many I know not, neyther know I what one had moze then the other, but yet I remember that the first had 19.r. and the last had 118.r. and that euery one from first to last, had moze then his fellow paide next before him, by an equall portion of monie, now would I know of you, how many persons receiued this money of me, and how much euery of them had.

Euma. This is a very proper question and as wittily inuented, and such a one, as were it not for that little experience which I haue already tried of numbers poore in the like, I should thinke it were as indissoluble as Gordians knots, but now by that little knowledge that I haue in these progressional rules, I dare undertake to vnfold euery wrinckle of it, and to lay the answere nakedly before you, & that without long time as you God-willing shal see.

For first the 19.r. payd to the first person, and the 118.r paid to the last, doe expressly represent the 2. extreames of the progression, and the whole 685.r. by you disbursed, doth as plainely expresse the totall sum of the tearmes: but here, neyther the number of tearmes (which representeth the number of persons) nor yet the common excesse, which representeth the equall portion of monie that euery of them had moze then other, is reuealed or expresst by any number.

Wherefore first I seeke the number of the tearmes by the last part of the 6. rule of this Chapter, adding the extreames as here is taught thus: saying, 19 + 118. make 137. whereby I diuide 685. the totall summe, and the quotient is 5. which doubled as that rule also willeth, yeeldeth 10. for the true number of the tearmes, so consequently of the persons, that receiues the money.

Now resteth to finde the common excesse, which I seeke by the third rule saying thus, 118. —— 19. is 99. which diuided by 10 —— 1. the number of the spaces, quoteth 11. for the common excesse, or difference, that is to say, the number of pounds which euery person receiued moze then other, and so to answer this question I say, that there were 10. persons which receiued this money of you, and that.

The First person receiued, 19. r.
The Second 19 + 11.r. that is 30.r.
The Third 30 + 11 r. that is 41.r.
The fourth 41 + 11.r. that is, 52.r.
The Fift 52 + 11. r. that is, 63.r.

The

The Sixt, 63 + 11.r. that is 74.r.
The Seuenth, 74 + 11.r. that is 85.r.
The Eight, 85 + 11.r. that is 96.r.
The Ninth, 96 + 11.r. that is 107.r.
The Tenth, 107 + 11.r. that is 118.r.

And now it is easie to finde by the last rule, that the totall sum of 19.30.41.52.63.74.85.96.107.118. beeing the 10. tearmes of this progression, make iust 685.r. the money by you disbursed.

Phi. You haue wrought euery part of this question so well and orderly, applying both the numbers proposed in the question, to the due tearmes of the progression, & also the rules to their right vse, that although many mo questions might be propounded touching progression arithmeticall, yet supposing aswell by that you haue alreadie learned and practised, as by your owne wordes, that you are sufficiently able to wade through any other, I will here conclude progression Arithmeticall, and returne agayne as I promised to explicate the properties of the other kinde of arithmeticall proportionalitie, called proportion disioyned or discontinuall.

Of the properties of arithmeticall proportionalitie discontinuall, Chap. 5.

221. If in Proportionalitie,
Direct and Arithmeticall,
The tearmes agree in qualitie,
Disioynd or discontinuall,
Then shall the summe of both the meanes,
Collected in one totall,
Iust counteruaile the 2. extreames,
And to them both be equall. 187

Tonstall fol. 201. Salig. pag. 66. 2.

As for example let 8.6.4.2. be 4 tearmes assigned in arithmeticall proportionalitie discontinual, directly. now I say by this verse that the totall of 6. and 4. the 2. meanes or middle tearmes is iustly equall to the totall of 8. and 2. the extreames, for both by supposition & by the glosse of the 44. verse, 6 + 2. are equall to 8. and likewise 2 + 2 is equall to 4. and therefore by the 36. verse. 6 + 2 + 4. shalbe equall to 8 + 2 + 2. wherefore by the 45. verse, 6 + 4. & 8 + 2. are equall together. Another propertie is thus.

Salig. p. g. 66. 3.

222. If 4. vnequall numbers wade.
So that their 2. extreames,

Be

Be equall to the totall made,
By adding both their meanes.
Then are those tearmes to build vppon,
Foure numbers iust proportionall,
In numerall Proportion,
Disioynd or discontinuall. } 188

As for example, let 3.5.7.9. be 4. vnequall numbers, whose extreames being 3. and 9. make their totall equall to the totall of 5. & 7. ỹ 2. meanes, then I affirme by this verse, that the same 3.5.7.9 are 4. proportionals in numerall or Arithmeticall proportion discontinuall, & that as 3. differeth from 5. so doth 7. from 9 for let the totall of 5. and 7. be 12. and then by supposition the totall of 3. and 9. shall also be 12. wherefore as 3. differeth from 5. so by the 46. verse 12 —— 5. differeth from 12 —— 9. which by order of the worke is, as 7. differeth from 9.

Fumus. This verse seemeth to be the conuerse of that next before it, by the doctrine of which 2. verses, a man may easily and quickly iudge whether any 4. numbers propounded be in arithmeticall proportion discontinuall or not.

Philo. You say truth, and that is all that I can find worthy the noting in this kinde of proportionalitie: wherefore here I will finish Arithmeticall proportion, both continuall and discontinuall, directly and indirectly, and will now entreate of Geometricall proportionalitie particularly.

Of Geometricall Proportionalitie continuall in 3. termes onely, with the doctrine and properties. Chap. 6.

223. To finde out the meane Geometrically,
to any couple of extremes assigned,
Fold those extremes together by and by,
and from the offcome which thereof you find,
Extract the square roote, so haue you the meane,
to place in the middest betweene each extreame. } 189

Euclid. 20. prop. 7. Tonstall pag. 204. Sahg. pag. 91. ver. 2. Boetius lib. 2. cha. 44.

Proportionalitie geometricall, doth so farre excell proportionalitie arithmeticall, that for the excellencie & dignitie thereof it worthilie deserueth the name of proportionalitie, or proportion onely without any further addition, for that it truly consisteth of the proportions of numbers indeed, whereas proportion arithmetical consisteth

sisteth onely in the differences of numbers as you learned by the 180. and 197. verses, and therefore generally from henceforth whensoeuer I name proportion, proportionalitie, or proportionals without any addition, of other wordes it is to be vnderstoode of proportionalitie Geometricall.

Now when this proportionalitie is continuall, and consisteth but in 3. termes onely, the learning that resteth in it, standeth either in finding the meane proportionall, to 2. extreames giuen, or els in finding the third proportionall to the first and second assignd.

But as the finding of the meane is first in order, and therefore ought first to be taught, so doth this first verse of this Chapter, teach the maner of finding it, yet so as you cannot well vnderstand it till you haue learned the extraction of rootes, neuertheless I will explaine the verse in the meane season, by such example as I can, and that is thus: let 4. and 9. be 2. extreames assignd, to whom I should finde a meane proportionall, that is to say, such a number as shalbe consequent to the first, and antecedent to the last, in continual proportionalitie, as hath beene shewed heretofore, now as this verse biddeth, I fold these 2. extreames 4. and 9. each in other, and the product is 36. whose square roote is 6. and so the same 6. is the meane proportionall to place betweene 4. and 9. the 2. extreames thus 4. 6. 9.

Euma. I know very well, and it is manifest that the same 3. termes 4. 6. 9. are continually proportionals, as you affirme, but how may I know, that 6. is the square roote of 36.

Philo. You must credit my word for that matter, till you be better able to iudge thereof, because here is neither place to talke any further of rootes, sith you know them not, nor time to teach how to drawe them before they be ripe and readie to appeare aboue ground.

Euma. Well I am content to beleeue your merry speech, but what say you then to the finding of the third proportionall.

Philo. Touching that, I say thus.

Euclid. 18. prop 9. Ramus. ca. 15. li. 2. Salig. pag. 92. verse 3. 4.

224. To finde a third number proportionally,
infold the second in it selfe onely,
Diuiding the ofcum by first of the twaine,
The quotient yeeldeth the third very playne,
225. Or els let the first, the second diuide,
causing that quotient to be multiplied,

That

By the same second, diuided but late,
the offcome doth offer the third in like rate.

Heere are (as you see) 2. wayes how to finde the third number, as for example of the first way, I haue these 2. numbers 4. and 6. to whom I would finde a third in such proportion to 6. as 6. is to 4. wherfore (by the first of these verses) I multiplie 6. the second number in it selfe, & it produceth 36. which offcome or product I diuide by 4. the first number, and the quotient is 9. which is the third number proportionall thus, 4. 6. 9. euen as it was before in your former example

Eum. Then likewise if I would finde the third proportionall to the selfe same numbers, after the order of the latter verse, I must cause 4. the first number to diuide 6. the second, & so the quotient wilbe $\frac{6}{4}$. which multiplied by the same 6. produceth $\frac{36}{4}$, which by distinct reduction is 9. also agreeable with the former worke.

Phi. I perceiue that you can by these verses quickly finde the third proportionall to any 2. numbers assigned, wherfore to the end that you shall not be vtterly destitute of a rule to serue your turne, touching the finding of the meane proportionall, till you haue learned the extraction of Roots, I will shew you one rule, whereby you may finde not onely the meane proportionall, but also both the extreames, in any kinde of proportion that you will require, and it is thus.

226. To finde 3. numbers continually,
in such proportion as you would request,
First set the tearmes of that proportion by,
in primes together, for they are the least,
Which done, if the antecedent so set,
multiplie both it selfe and his consequent,
And that consequent, without any let,
multiplie it selfe also incontinent,
Then the 3. products thereof proceeding,
among all numbers shalbe the 3. least,
In the giuen proportion succeeding,
continually to serue your request. } 191

Tonstall pag 125. Pelcrarius c2. 17. li. 3. Ground of Artes. quayre. C 6. 7.

As for example, I would finde 3. numbers continually proportionall, in Sesquialtera Proportion, wherefore as this rule exhorteth me, first I set the termes of this proportion in some by place in primes together thus, 3. to 2. or thus. $\frac{3}{2}$.

Which done, I cause 3. the antecedent so set, to multiplie,

both it selfe and his consequent, whereby I produce 9. and 6. and then I cause 2. the consequent, to multiplie it selfe also, and it produceth 4. so haue I these 3. numbers 9. 6. 4. being not onely both th'extremes and middle number in such proportion as I requested, but also in the least tearmes.

Eu. This is a very good rule and easie, wherefore as occasion serueth I will vse it, in the meane season, till I haue learned to extract the roots of numbers.

Phi. So may you do, and now I will shew you some notable properties incident to such 3. termes continually proportionall, whereof all saiuing the very last are playne inough, but that last shalbe expounded to you hereafter when you shalbe more capable thereof, The first propertie is thus.

Boetius lib. 1. cha. 44.

227. Proportion Geometricall,
whose termes are onely three,
Proportionlike continuall,
doth order by decree,
The offcome of the two extreames,
iust equall and all one,
VVith th'offcome of the middle tearme,
made by it selfe alone. } 192

As Arithmeticall proportionalitie continuall, hath such a propertie, that halfe the totall of any 2. extremes, is alwayes iust equal to their middle terme. So this verse affirmeth, that this Geometricall proportionalitie continuall, by especiall propertie thereof, alwayes maketh the product of any 2. extreames, equall to the product of their middle terme made by it selfe, which you may easilye proue, euen by the former 3. proportionals 9. 6. 4. for if you multiply together 9. & 4. the 2. extreames, you shall finde their product 36. & so much iust commeth of 6. times 6. the product of the middle terme. The like whereof is to be said generally of all other.

Euma. This is very apparant, wherefore I pray you goe to the next propertie.

This is the conuerse of the next forme,

228. *Phi.* VVhen any 3. such termes are found,
whose middle terme alone,
Infolded in it selfe doth found,
coequall and all one.
To th'offcome made as it befalls,
of both the 2. extremes,
Then are the 3. Propoationalls,
continuall by that meanes. } 193

Euma.

Euma. This verse is in effect but the conuerse of the last before, and is so manifest, that it needeth neither demonstration nor example.

Philo. It is so indeed, wherefore I proceede to another thus.

229. The middle terme proportionall, of any 3. true Integers, And each extreame collaterall, are compounds both together, For no 2. primes together shall, haue any third proportionall. 194 — Euclid. 17. prop. 9.

As for example, let 9. 6. 4. be true numbers continually proportionall. Now 6. the middle terme, and 9. the first extreame, are compounds together, and so are 6. and 4 the latter extreame, by the 133. verse, for if they were primes together, they could haue no third proportionall by this present maxim. But 9. and 4. the 2. extreames are primes together, wherefore I may conclude by this verse, that there can no third number be assigned vnto them in such like proportion as they are together, that is to say, in proportion Dupla sesquiquarta.

Euma. All this I vnderstand very perfectly, for though (as you say) there is no third proportionall to be found to 9. and 4 in true numbers, yet I suppose you meane not, but that in fractions or mixt numbers, there may be a third proportionall found, eyther greater then 9. or lesser then 4.

Philo. You haue declared my meaning truly indeed: wherfore now I will rehearse the last propertie, but I will but rehearse it onely. And it is thus.

230. The two extremes proportionall, of any three assigned, Are like flats all in generall, by very course of kinde, VVherefore betweene all like flats is, one number meane proportionall, VVhich rule by reason cannot misse, it doth conclude so naturall. 195 — Euel. 11. 18 & 20. pr. 8

And thus haue we done with Geometricall proportionalitie continuall in 3. termes onely. And now will we go in hand with the same proportion, when the termes exceede the number of 3. whereby it obteyneth the name of Geometricall progression, as the first verse of the next chapiter following declareth.

231. When any numbers moe then three,
encreaſe all like proportionall,
Then call we their whole company,
Progreſſion Geometricall. } 196

Eu. So then I perceiue, that Geometricall proportion continual at the length by abounding in termes aboue 3. in number, turneth into a Geometricall progreſſion, & is generally ſo named & called.

And ſo it ſeemeth that the vttermoſt ſcope and finall end of Geometricall proportionalitie continuall, is at the laſt to turne into a progreſſion of his one nature and name, euen as Arithmeticall proportionalitie continuall, turned into a progreſſion of his owne nature and name before in the 208. verſe.

Phi. Your iudgement heerein is good, and therfore as you had a certayne doctrine there, riſing and proceeding from progreſſion Arithmeticall, ſo is there another like doctrine heere riſing and proceeding from this progreſſion Geometricall, which commonly conſiſteth in 4. principall rules, whereof the firſt teacheth the proportionall finding of all the ſeuerall termes wanting in any giuen progreſſion vnfiniſhed, ſo that thereby the ſame progreſſion is amplified and encreaſed to the full number of termes required. And the rule thereof is thus.

232. VVhich company to amplifie,
with moe termes ſtill proportionall,
And as they growe and multiply,
to know each ſeuerall terme withall.
Diuide the ſecond by the firſt,
and by that quotient ſtill infolde,
The laſt terme there produced erſt,
till all your termes you may beholde. } 197

Amplification of termes.

As for example, let 3. 9. 27. 81. be an vnfiniſhed progreſſion giuen me to amplifie with moe termes continually proportionall vnto the full number of 20. termes in all, whereof euery ſeuerall terme is required to be expreſſed and knowne. Now therefore to finde the 16. termes heere wanting, proportionally to the former 4. I diuide 9. the ſecond terme of this progreſſion, by 3. the firſt terme (as this rule biddeth me) and the quotient is 3. by which quotient I multiplye 81. the laſt terme of the giuen progreſſion,

and

and it produceth 243. for the fift terme.

Which being done, I eftsoones multiplie the same 243. (which is now the last terme earst produced) by 3. also, whereby I produce 729. for the sixt terme, and now sith the same 729. is the last terme produced, I multiply it by 3. also, and it produceth 2187. for the seuenth terme.

And so I continue, still tripling or multiplyeng euery terme last before produced by 3. vntill I may beholde all my termes, that is to say, till I haue the full number of 20. termes to finish the giuen progression, as was required.

And so euery terme is manifestly expreſt, in such forme as the fabrique of the worke heere sheweth.

The termes.	Their order.
3	1
9	2
27	3
81	4
243	5
729	6
2187	7
6561	8
19683	9
59049	10
177147	11
531441	12
1594323	13
4782969	14
14348907	15
43046721	16
129140163	17
387420489	18
1162261467	19
3486784401	20

Luma. This is no harde matter, for the worke is more wearysome to continue, then difficult, eyther to vnderstand or to doe, though a man had learned but euen the multiplication of Integers onely.

Philo. It is so, yet is there no waye to abridge the labour therein, when the knowledge of euery seuerall terme is required, as heere it was. But when the knowledge of some one of the meane tearmes, or of the last extreame onely, is specially required, and the knowledge of all the meanes is not needefull, as many times it is not. Then to continue such multiplycation from tearme to tearme, still vntill the last be knowne, would be but a long tedious worke and labour in vaine, sith the same may be found and knowne much sooner, by the rule following which is thus.

233. But if you seek the last extreme,
Or but some middle terme to know,
Then writing right as you can ayme,
the first 6. termes all in a row,
A cypher to the first subscribe,
and so 1. 2. 3. 4. and 5.

In order to the rest beside,
and then your purpose to atchieue,
234. Infold now which 2. termes you will,
of those 6. termes progressionall,
And then diuide the offcome still,
by very first terme of them all,
Which done, the quotient shall possesse,
such roome and place in that progression,
As their 2. vndersums expresse,
when they are ioyned by addition.
235. VVhich roome or place neuertheless, 198
is more by 1. then you expect,
For if the tenth tearme you would gesse
those ouer 4. and 5. elect,
Though 4. and 5 do make but 9.
and ouer 9. do point his place,
Yet they in truth to you assigne,
the tenth in order of their race.
239. And as you thus haue found one terme,
so seeke the next you wish to learne.

En. This is a very long matter, I pray you shew me an example of the effect and practise, for so I thinke I shall better conceiue the meaning of the rule then by many words.

Phi. The matter cannot well be comprehended in prose in fewer words, nor yet in plainer sentence, then it is heere in meeter. But for an example, I take 3.9.27.81. the vnfinished progression before giuen, supposing that I were demaunded onely, what number shalbe the tenth terme of this progression now therefore to finde the same, I needs not to multiplie euery terme, last produced continually as I did before, till I come to the tenth, but as this rule willeth me, after I haue (by the former rule) gotten onely the first 6. termes of the progression, I write them downe right all in a row subscribing a cipher to the first, and then 1.2.3.4.5. in order vnder the rest, beside as here you see. 3.9.27.81.243.729.
Which being done, now to atchieue 0 1 2 3 4 5
my purpose in finding the tenth terme, I choose such 2. of these 6. termes to multiplie together, as serue most meet to make the same, which by the 235. verse of this rule, is 243. and 729. because 4 + 5. their 2. inferior numbers or vnder sums making 9. by addition, shall cause the terme succeeding by their opperation, to be the

tenth in order, and so multiplying the same 243. and 729. together they produce 177147. which diuided by 3. the first terme, as the 234. verse of this maxim requireth, there issueth in the quotient 59049. for the tenth terme of this progression, wherefore filling the 7. 8. and 9. places, (whose tearmes are omitted) with ciphers, for your plainer instruction, I place the same 59049. in the tenth place of the rowe amongst his fellowes, as the verye true tenth terme in order although the vnder number thereof be but 9. as the 235. verse of this rule affirmeth, and as heere appeareth.

3 . 9 . 27 . 81 . 243 . 729 . 0 . 0 . 0 . 59049.
0 1 2 3 4 5 6 7 8 9

Euma. I suppose I vnderstand all the whole matter now, by this example: and for a triall thereof, as you haue found the tenth terme, so will I (as the last verse of the rule biddeth me) seeke the next I wish to learne, and that is the eleuenth: for performance whereof, I perceiue I must multiply those two termes together, whose vndersums make but 10. by addition, and that heere is 9. and 59049. (the last terme found by you) whose product is 531441. which diuided by 3. the first terme, yeeldeth in the quotient 177147. for the eleuenth terme of this progression. Which quotient therefore I place amongst his fellowes in his due place, subscribing thereto but 10. as his proper Gradnall number. And so it standeth thus.

Gradnall numbers are those which being subscribed to the termes (as the 233 verse willeth) shew the place of the terme sought, being vied as the 234. verse requireth.

3 . 9 . 27 . 81 . 243 . 729 . 0 . 0 . 0 . 59049 . 177147.
0 1 2 3 4 5 6 7 8 9 10

Philo. You haue found the terme truly, and placed both it and the Gradnall number thereto belonging, rightly and orderly, as you may see by my worke in the rule next before: whereby I perceiue you vnderstand the rule. But whereas you tooke 9. and 59049. to multiply together, because 1 + 9. their vndersums or gradnall numbers make 10. although you did well in doing so, yet you might aswell haue taken 729. whose vndersum is 5. and haue multiplyed it by it selfe, because 5 + 5. maketh 10. aswell as 1 + 9. and so the product and quotient would haue come to like end, and the terme thereby succeeding, would haue been the same it is now, and so of any other.

Eu. That is more then I knew of, for it is not so mentioned in ꝑ rule, but now me thinks I could presently at the next multiplication, find ꝑ 20. terme of this progressiõ, for if these 2. terms last found be folded together, & their product diuided by 3. the ordinery diuisor.

The quotient must neves be the 20. terme by the 235. verse of this rule, because 9. & 10. being their vnversums or gradnall numbers make 19. by addition.

Phi. You haue told the truth euen as you shall finde it, if you list to make any triall thereof by this rule.

Eu. Then for a triall sake I multiplie 59049. and 177147. together, and their product is 10460353203. which I diuide by 3. and the quotient yeelds 3486784401. for the 20. terme.

And so the termes found by this rule interruptly agree iustly to the selfe same termes found by the former rule continually, as by conference with the fabrique of your worke there appeareth, which agreement argueth the worke to be good and true.

Philo. It dooth so, and thus you see how by this rule you may with a few multiplications, finde out the last terme or any middle terme of an vnfinished progression, which you will require, which is a great abridgement of labour, especially in a progression containing 200. or 300. termes.

Euma. Indeed it is so: wherefore I must needs commend this rule greatly, and acknowledge it very necessary and commodious.

Philo. Well, now followeth the third principall rule, which teacheth both the finding of a progression, & also the amplification thereof, with moe termes continually proportionall in any kinde of proportion required, and that also in the least numbers that are of Integers. And it is thus.

237. But if you would a progresse make,
in some proportion that you like,
The primes of that proportion take,
and set together cheeke by cheeke,
238. Which done, let th'antecedent fold,
it selfe and eke his consequent,
And make the consequent as bold,
to fold it selfe incontinent,
Then set the products gotten so, } 199
iust distant-like beneath the rest,
Proceeding forth to finde out mo,
in forme and fashion heere expresst,
239. To wit, let th'antecedent still,
fold euery terme so newly plac'd,
And cause the consequent aswell,
to multiplie the onely last,

So ſhall the products which proceede,
in that proportion be the leaſt,
VVhich as you found, ſo may you breede,
ſtill mo and mo the way is preſt.

The ſence of this rule is thus, if you would haue a progreſſion made, in any proportion you liſt or like, you muſt firſt take the tearmes of that proportion in primes together, and ſet them one by the other, as if you would haue a progreſſion in Subdupla proportion, you muſt take the primes or leaſt tearmes of that proportion which as you know is $\frac{1}{2}$ and ſet them thus, 1. 2.

If you would haue it in Subtripla proportion, then muſt you take the primes of that proportion, which is $\frac{1}{3}$ ſet them thus. 1. 3.

If you would haue it in Subſeſquialtera proportiõ, then muſt the primes of that proportion wich is $\frac{2}{3}$, be ſet in like order.

And likewiſe for Subſeſquitertia the primes therof which are $\frac{3}{4}$. muſt be ſet thus 3. 4 and ſo fourth of all others, wherin as you ſee the antecedent is alwayes to be placed firſt as order requireth.

Euma. Very good, this I vnderſtand well, but I marualle why you put all your examples in proportions of the leſſer inequalitie only, as in Subdupla, Subtripla, Subſeſquialtera, and ſo fooꝛth, and not in Dupla, Tripla, Seſquialtera or ſome other proportion of the greater inequalitie.

Philo. Why doe ſee you not remember that by the 208. verſe the firſt tearme of euery progreſſion, allwayes ought to be the leaſt, and alſo that the 231. verſe defineth progreſſion Geometricall, to be when any numbers mo then 3. increaſe all like proportionall, which is a ſilent argument alſo that the firſt tearme muſt needes be the leaſt, or els they cannot increaſe proportionally.

Euma. All this I know and remember well inough.

Phi. And then you know alſo by the 187. verſe that all proportions wherein the leſſer tearme is antecedent, are of the leſſer inequalitie.

Euma. I muſt needes graunt that to be true.

Phi. But in euery progreſſion directly compared, the leaſt terme being firſt, is Antecedent to the ſecond, which ſith it ought to encreaſe aboue the firſt, muſt neceſſarily be greater then it, and that ſecond is antecedent to the third, which increaſing alſo aboue that, muſt needes be greater then it, and ſo fooꝛth continually vnto the very laſt, which is the greateſt.

Wherefore I affirme, that all progreſſions (taken directly) are in

proportion

proportion of the lesser Inequalitie, and therefore I conclude further, that if any progression be casually propounded in proportion of the greater Inequalitie, then do not the termes or numbers thereof increase proportionally (which is contrary to the foresaid definition of progression) but rather they decrease proportionality from the first to the last.

As in this example 243. 81. 27. 9. 3. wherein you see the first terme is greatest, and the second decreaseth or falleth downe to $\frac{1}{3}$. thereof, and so doth the third fall in like proportion beneath the second, and so fourth euery terme is but $\frac{1}{3}$. of his next antecedent vntill the last which is least.

And therfore say I, that such a progression ought to be altered into the proportion of the lesser Inequalitie, by transposition or inuertion of the termes thus. 3. 9. 27. 81. 243. Or at the least wise, if it be permitted to stand as it did yet in the application thereof to any rule or question of progression, the least terme must euermore be reputed and taken for the first, and the greatest for the last, otherwise the rules thereof cannot hold, but must needes faple and be vncertayne.

Euma. I thanke you heartely for imparting this reason vnto me, whereby you haue not onely cleared me of the matter wherat I mused, but also made me to perceiue how the proportion of the greater inequalitie changeth into the lesser, and the lesser into the greater, onely by transposition or inuertion of the termes, whereof I tooke not so much heede before, wherefore I pray you now goe forward to expound the rest of this Maxim, by finding a progression of halfe a dosen the least tearmes that are in Subsesquialtera proportion, for an example.

Philo. With a good will, and to performe it, I take $\frac{1}{3}$ the primes of that proportion, and set them thus. 2. 3.

Now as these 2. tearmes are primes together, so are they the very originall prime numbers, and as it were the first parentes, of whom all the rest that I seeke are begotten, and from whom they proceede, yet not immediately, but as it were by certayne discents and generations, as you shall see by and by.

For these prime termes being so placed as before, now as the rule biddeth me, I make 2. the antecedent first to multiplie it selfe, saying 2. times 2. is 4. and then I let it multiplie the Consequent, saying, 2. times 3. is 6. and thirdly or lastly, I make the

Consequent

Consequent as bold to multiplie it selfe, saying 3. times 3. is 9. so haue I these 3. products 4.6.9. which I set distantlike in a new ranke beneath their 2. prime tearmes, as their first aspiring, in this order.

2 . 3
4 . 6 . 9

And thus haue I produced 3. the least tearmes that are in the desired proportion.

Euma. Why this hetherto, is nothing els but that which I learned in the 226. verse, touching the finding of 3. tearmes in any proportion required.

Philo. You say very true, but now to proceede further, to the finding of mo termes (which is not taught in that verse) this rule sayth, that I must cause the Antecedent still, (that is to say continually from time to time, ranke after ranke) to fold euery tearme so newly placed, and the Consequent to multiplie onely the last of them, wherupon I cause 2. the Antecedent, to multiplie 4.6.& 9. the tearmes newly placed, and likewise 3. the Consequent, to multiplie 9. onely the very last of them, whereby I produce these seuerall products 8.12.18.27. which I set also in a new ranke, as a second generation, thus: And so now I haue 4. the least tearmes that are in this proportion.

2 . 3
4 . 6 . 9
8 . 12 . 18 . 27

And now the last verse of this Parim sayth, that as I found these, so may I breede still moe and moe, for the way is prest and readie, which is (as I sayd but euen now,) by folding euery of the tearmes of this new ranke, first by 2. the Antecedent, and the last onely by 3. the Consequent, and so to place downe their seuerall products, beneath the rest in a new ranke still from time to time, wherefore to doe accordingly, I multiplie 8.12.18. and 27. the tearmes of the last new ranke, first by 2. the Antecedent, and then doe I iterate the multiplication of 27. which is the last of them by 3. the consequent whereby I produce 16.24.36.54. and 81. which seuerall tearmes I set distantlike vnder the rest in a new ranke, as a third generation thus: and so as you see I haue found 5. the least tearmes that are in this proportion.

2 . 3
4 . 6 . 9
8 . 12 . 18 . 27
16 . 24 . 36 . 54 . 81

And because I yet lack one terme of the number which you required, I begin the worke againe after the former trade, now causing the prime antecedent to multiplie euery terme of this newest or last ranke, and the Consequent to vnfold onely 81. the last

tearme

tearme thereof, whereby I produce these 6. seuerall products 32. 48. 72. 108. 162. and 243. which I subscribe vnder all the rest, as their last generation for this time, because the same contayneth the full number of tearmes of that progression which you required, and so haue I produced a progression of 6. the least tearmes that are to be found in Subsesquialtera proportion, according to your request, and the fabrique resembleth that delectable fourme or order, which Quintilian calleth a Quincunx as here appeareth.

2 . 3
4 . 6 . 9
8 . 12 . 18 . 27
16 . 24 . 36 . 54 . 81
32 . 48 . 72 . 108 . 162 . 243

And as I haue found this progression of 6. tearmes being the least nũbers that are in Subsesquialtera proportion, so may you by continuing the like fourme of worke, produce a progression of mo tearmes either in this, or in any other proportion that you will, whereby you may gather, that there are as many diuers kindes of geometrical progressiõs, as there are sundry kindes of proportions of the lesser inequalitie which are infinite.

Euma. Truly I vnderstand the effect of the rule very well, by this sight of the practise thereof, but I pray you, is it not possible to finde 6. lesser numbers in proportion continually Subsesquialtera, then are these 6. of the last ranke.

Phi. No verely, for first you know by the 132. verse, that the 2. extreames of euery ranke here are primes together, whereby it followeth, that all the termes of euery ranke are primes together, & secondly you know by the 134. verse, that, If numbers giuen be primes together, then haue they no lesse proportionals neuer. And so finally if they haue no lesser proportionals, then of necessitie they themselues must be, and are, the least in that proportion.

Eu. That cannot be denied, whereby I perceiue it is no small busines to amplifie a progression in Subsuperparticular or Subsuperpartient proportion, or in any mixt proportion, to the number of 20. 40. or 100. tearmes.

Philo. Indeede it is not so easie to doe in any proportion as in Submultiplex: but now let me see how you can finde a progression of the like number of tearmes in proportion Subsesquitertia, and then we will goe to the last rule whereof I spake.

Euma. I will doe my indeauour therein, and therefore first to beginne, I take $\frac{1}{4}$ the least tearmes of that proportion, setting them

them downe each by other thus. 3 . 4.

Which done, I fold 3. the antecedent first in it selfe, and next in 4. the consequent, and then do I make the same consequent to multiplie it selfe also, and so haue I these 3. products 9. 12. & 16. which I subscribe distant like vnder the prime, tearmes or first parents of this proportion, in a new ranke as their first generation, in such order as you did before, thus. .3 . 4.

And now to proceed to the finding of mo termes, 9 . 12 . 16

I multiplie euery of these 3. tearmes so newly setled, first by 3. the Antecedent, and then I also iterate the multiplication of the last of them, by 4. the Consequent, whereof insueth these seuerall products 27. 36. 48. and 64. which I subscribe beneath the rest in a third ranke, in like order as before 3 . 4

thus. 9 . 12 . 16

This being done I multiplie euery 27 . 36 . 48 . 64

tearme of the last ranke by the same Antecedent, iterating the multiplication of the last by the Consequent also, euen as before, and so I produce 81. 102. 144. 192. 3 . 4

and 256. which I subscribe in a foureth 9 . 12 . 16

ranke thus. 27 . 36 . 48 . 64.

Lastly I multiplie euery tearme of 81 . 102 . 144 . 192 . 256

this 4. ranke, first by 3. the Antecedent, and then I iterate the multiplication of the last onely by 4. the Consequent, after the former order, whereby amounteth these products 243. 324. 432. 576. 768. and 1024. which I subscribe vnder all the rest as the whole progression required, and 3 . 4

so the fabrique of the worke sheweth thus. . 9 . 12 . 16 .

. 27. 36. 48. 64 .

And now by your saying, . 81 . 102 . 144 . 192 . 256.

this ranke of tearmes last 243 . 324 . 432 . 576 . 768 . 1024

found, is a progression of 6. the least numbers that can be giuen in this kinde of proportion.

Philo. I suppose you your selfe thinke not otherwise, for you see that the 2. extreames of euery ranke, are still primes together.

Euma. I see they are so, and therefore must needes graunt your saying to be true, but yet I pray you before you proceede any further, set me but onely the like fabrique of a progression, in some kinde of mixt proportion for my learning.

Phi. That I will not sticke to do, behold here a progression of 6. tearmes in proportion Subdupla Sesquiquarta, the primes or

originall

originall tearmes, whereof are 4. and 9. as appeareth.

4 . 9
16 . 36 . 81
64 . 144 . 321 . 729
256 . 576 . 1284 . 2916. 6561
1024 . 3204. 5136 . 11664 . 26244 . 59049.

Euma. Sith the tearmes of this last ranke be 6. the least numbers that can be giuen in this proportion continuall, it appeareth that the numbers would mount exceeding great in a progression of many tearmes.

Phi. There is no doubt of that, but now touching the 4. or last rule, which teacheth the summing of this or any other progression of what proportion soeuer it consist, it is thus.

240. At last the totall sum to know,
and all the termes in one to knit,
Diuide some one terme of the row,
by next thereto but lesse then it.
VVhose quotient then take heedely,
and thereby multiplie the last,
And from that product speedely,
abate the first agayne as fast,
Then take the remnant and diuide it,
by 1. lesse then you multiply it,
So doth the quotient make account,
whereto the totall sum doth mount.

200

So that the last point of progression generally, is to sum it, that is, to finde the toall sum of all the tearmes, as was declared heretofore in the notes of the 220. verse, which thing this rule teacheth. And for example thereof, I will sum this Subtripla progression insuing viz. 4. 12. 36, 108. 324. 972. 2916, 8748, wherefore to performe the same I diuide 12. the second tearme by 4. the first, which is the next tearme to 12. lesse then it, and so the quotient is 3. (for although it skilles not, which of the greater termes I diuide by the next lesser,) yet seeing that small numbers are easilier wrought then great, as the glosse of the 27. verse testifieth, therefore I say, I chuse the 2. least tearmes, diuiding the greater of them by the lesser, as the rule importeth: and so the quotient (as I sayd) is 3. by which quotient I multiplie 8748. the last tearme

tearme of this progreſſion, as the rule alſo willeth, and the product yeeldeth 26244. as here appeareth by the worke.

From which product I ſpeedely abate 4. the firſt tearme of the progreſſion, and ſo remayneth 26240. then finallye I take the ſame remnant, 26240. and diuide it by 1. leſſe then I multiplied it, but I multiplied before by 3. as appeareth, wherefore now I muſt diuide it but by 2. (which is 1. leſſe then 3.) and ſo the Coſient bringeth 13120. for the totall ſumme of this progreſſion as the worke of the diuiſion here ſheweth.

8748
3
26244

26240(13120.
zzzzz
0000

And for further triall thereof to ſatiſfie you the better, I haue alſo collected the totall, by ordinarie Addition of all the tearmes together, as by the fabrique therefore here annexed appeareth, whereby you may ſee the agreement of both workes, and the vndoubted certaintie of this rule.

4
12
36
108
324
972
2916
8748

13120

Euma. Indeede the triall by Addition, agreeing with the other forme of worke, maketh the totall found by this rule to appeare very true, but me thinkes the totall is aſſoone or rather ſooner found by ſuch playne Addition, then by this rule of progreſſion.

Philoma. It is ſo, when the tearmes are few in number, as here they are, but if a Progreſſion ſhould conſiſt of 200. or 300. tearmes, you ſhould finde it otherwiſe, for here are but fewe tearmes put of purpoſe to make the examples the more eaſie.

And agayne, euery Progreſſion, hath not all the tearmes thereof alwayes expreſſe or knowen, by reaſon whereof, their totall cannot be found by Addition, but as in Progreſſion Arithmeticall, the knowledge of the 2. extreames with the number of the tearmes ſufficeth to finde the totall ſumme of all the tearmes, how many ſoeuer they are, ſo in this Progreſſion Geometricall, the knowledge of the 2. extreames, and of the proportion whereby the tearmes increaſe (which is alwayes contrarie to the proportion of the Progreſſion) is ſufficient inough to ſumme the whole progreſſion.

For when this rule biddeth me to diuide ſome one tearme of the Progreſſion, by next thereto but leſſe then it, it doth nothing els in effect, but teach me by that meanes, how to

ſumme

finde the proportion whereby the tearmes increase, that so the last tearme may be multiplied by the Inder thereof, as the rule requireth.

And so in this last example, when 12. the greater tearme, diuided by 4. the next lesser, yeelded 3. in the quotient, that quotient being 3. certifieth me that 12. the greater tearme, increaseth aboue 4. the next lesser in triple proportion, and so consequently, that the proportion whereby the tearmes of that progression encreased, is Tripla, for by the 231. verse they increase all alike, and by the glosse of the 182. verse, the Inder or quotient of triple proportion is 3.

And agayne, forasmuch as by supposition euery lesser tearme of the progression continually, is subtripla to the next greater, it followeth thereby necessarilie, that euery greater terme, shall continually be Tripla to the next lesser, that is to say, quite contrarie to the proportion of the progression, as I sayd euen now.

But I sayd that if both the extreames of a progression, and the proportion, whereby the tearmes increase be knowen, then the same progression may be summed by this rule, without knowledge either of any meane terme, or of the number of the termes, which thing addition cannot performe.

Euma. Indeede you sayd so, and therefore I desire much to see the practise thereof by an example.

Phi. Well, then for that purpose I put the case, there be a progression whose extreames are onely knowne to be 8. and 256. and the proportion whereby the termes increase is double or Dupla, and that I am demanded what the sum of this progression is, and also how many tearmes it containeth.

For answere whereto, seeing I knowe the proportion already, whereby the termes encrease to be Dupla, the quotient or Inder of which proportion is 2. I need not now by diuiding ỳ greater term by the next lesser, to seeke this quotient which I knowe already, but without any such diuision, I presently proceed to multiplie 256. the last terme of this progression, by the same 2. the Inder or quotient of the giuen proportion, and it produceth 512. from which product I forthwith abate 8. the first terme, as the rule bideth me, and so remayneth 504. which I diuide by 1. lesse then I multiplied, but I multiplied by 2. wherefore now I must diuide but by 1. and so doing it continueth 504. still, or els I may omit diuision quite as needelesse here, because an vnit diuideth not any

number

number, and so both wayes I haue 504. constantly, for the totall summe of this progression.

And now to finde the number of the termes, I vse the rule prescribed in the 232. verse, doubling or multiplying 8. the first terme by 2. the Index of that proportion whereby they increase, and it produceth 16. for the second terme, which I also double or multiplie by 2. and it produceth 32. for the third terme, and so I continue, still doubling or multiplieng euery terme last produced by 2. vntill I finde 256. to be the sixth terme, which by supposition is the last extreme, wherefore I affirme that this progression contayneth 6. termes, as heere the worke declareth.

Euma. Indeede this is more then addition could haue done, whereby now I see both the necessitie and commoditie of the rule better then I did.

Phi. Well then will I shew you the practise of this rule once more, in summing the Subsesquialtera progression before described, which was this 32. 48. 72. 108. 162. 243. the proportion of whose increase is Sesquialtera, for as I sayd before, if the first terme be Subsesquialtera to the second, then the second (which is the first beginner of encrease) must needes be sesquialtera to the first. And therefore, when any terme heere, is diuided by the next lesser, the quotient yeeldeth alwayes constantly $1\frac{1}{2}$ or rather in simple termes $\frac{3}{2}$. which is the proper quotient or Index of sesquialtera proportion.

But as in the last example, so heere, I need not to vse anye such diuision, because the proportion whereby the termes increase here, is already knowne to be sesquialtera, wherefore I doe onely multiply 243. the last terme of the progression by $\frac{3}{2}$, the quocient or Index of the same proportion, and the product is $\frac{729}{2}$, as heere appeareth by the Fabrique of the Multiplication.

$$\frac{243}{1} \times \frac{3}{2} = \frac{729}{2}$$

From which I subtract 32. the first terme, (that is $\frac{64}{2}$) and so resteth $\frac{665}{2}$, which remainer I must diuide by $\frac{3}{2}$ (for that is 1. lesse then $\frac{3}{2}$. whereby I multiplyed before) and so by the 17. verse, there issueth in the quotient 665. for the totall summe of that progression.

The triall whereof appeareth heere also by ordinary addition as you see.

32	
48	
72	
108	
162	
243	
665	

Euma. Now by your fauor, will I proue to sum the Subsesquitertia progression, which I my selfe found before, whose termes were these heere following viz. 243. 324. 432. 576. 768. 1024.

And heere because the progression is already knowne to be in proportion Subsesquitertia, Therefore you say I may affirme that the proportion, whereby the termes thereof increase, is quite contrarie, that is to say, sesquitertia, wherefore I take $\frac{3}{4}$ the quotient or Inder of that proportion, multiplying 1024. the last terme by it, whereby I produce $\frac{3072}{4}$, as heere appeareth by the order of the multiplication.

1024 4
1 3
3

From which product I presently abate 243. the first terme of the Progression (which by exchanging reduction I finde to be $\frac{972}{4}$.) and so the remainder is $\frac{3367}{4}$. this remainder, the rule biddeth me diuide by 1. lesse then I multiplyed, but I multiplyed by $\frac{4}{3}$, wherefore now I must diuide but by $\frac{3}{4}$. for that is iust 1. lesse then $\frac{4}{3}$. and so diuiding $\frac{3367}{4}$ by $\frac{3}{4}$. the quotient bringeth 3367. for the totall sum of this progression, as the fabrique of the diuision vsed, after the former of the 173. verse here sheweth.

3367
3367 4
3 3
1

Phi. You have done well, and yet for triall thereof, to assure your selfe, vse the ordinary proofe by addition also.

Euma. That will I quickly do, wherefore after that I haue set ten the numbers in their due places, as here you see them, I reiter the perels of euery ranke by the 23. rule, whereby I finde their totall to be 3367. euen as it was before, and in the agreement of the workes argue the totall found, to be vndoubted true.

243
324
432
576
768
1024
3367

Phi. Very well done. Now let vs apply these rules of progression to some further vse, handling a question or twaine and vnfolding them, whereof let this be the first.

14. A question touching the valure of a mariage.

A certaine man, whose daughter was married on Newyeeres day, gaue her husband, towardes his mariage with her 1. shilling, and on the first daye of the next moneth, he gaue him 2. shillings, and the first day of March 4. shillings, and so foorth the first daye of euery moneth, still doubling the former gift, vntill the yeare was ended. Now I demaund the valure of this maydes mariage?

Euma.

Euma. I pray you answer this question your selfe, that so I may learne the better thereby, how to applie the numbers in the like.

Phi. Content: here you see the 12 moneths, are the 12 termes of this progression, the first terme is 1. shilling, and the proportion whereby the termes increase is double or Dupla, the quotient or Index whereof (as you know) is 2. Now therefore here lacketh but the knowledge of the 12. or last terme, and the matter were soone answered.

Eu. That will I finde if you will giue me leaue, and to do it I set downe halfe a doolen of the first termes thereof, with their vndersums or gradnall numbers, as the 233. verse teacheth, thus.

1. 2. 4. 8. 16. 32. 64.
0 1 2 3 4 5 6

And now multiplieng together 32. and 64. the product is 2048. which by the same rule is the very last terme (because 5 + 6. their vndersums make 11.) for as touching diuision by the first terme, here needeth none, sith the first terme heere is but 1. which doth neyther multiply nor diuide.

Phi. That is true, and it was well remembred and considered of you, wherefore now to proceede to the summing of the progression. I multiply the same 2048. the last terme, by 2. the Index of the proportion, whereby the termes thereof increase, and it produceth 4096. and 4096 —— 1. is 4095. which by this verse should be diuided by 1. lesse then I multiplyed, that is to say, by 1. but sithe 1. neither multiplyeth nor diuideth, as you said euen now, I omit such diuision heere as needelesse, taking the same 4095. for the verye true totall number of shillings giuen to the mariage.

Euma. That is, by reduction to grosser denames (after the 154. verse) 204. pounds, 15. shillings.

Philo. It is iust so, and now what saye you to this question? A certayne man which had 10. sonnes and a number of angels, which I know not, in his death bed bequeathed $\frac{1}{2}$ of the same angels to his eldest sonne, and to his second sonne $\frac{1}{3}$ of the angels remaining, and to his third sonne $\frac{1}{4}$ of the residue, and so to euery sonne $\frac{1}{5}$ of the remaynder continually, and the last halfe he gaue to a stranger his executor, for his paines to see the same legacies truly distributed, which when the Executor had done, he had but 1. angel for his labour. Now the question is, what euery sonnes portion was, and how many angels this testator had?

15. A question of a testament.

Euma. If the executor hauing the last halfe, had but one angell, then had the youngest sonne but 1. angell also, for the executors legacie was iust the one halfe of that which the 9 sonne left.

And heere, forasmuch as this progression commeth as it were backwardly, decreasing from the greatest terme to the least, (for it beginneth first with the eldest sonnes portion which was greatest) therefore by your former doctrine I must turne it directly, beginning at the least tearme, and so proceede increasingly to the greatest, that so the least terme may be the first, and the greatest the last.

Philo. You haue said very true, wherefore do you so.

Euma. Againe, forasmuch as heere is not onely required the totall sum of the tearmes, which do represent the number of angels which the testator had, but also euery meane terme, which representeth euery sonnes portion. Therefore I say, heere is no way be by finding the last terme interruptly, in such sort as the 23. verse teacheth, but without remedy, I must needes double euery terme, continually, according to the 23. verse, till I come to the last or greatest, and must set them forth expressely. And so euery sonnes part will appeare.

Philo. You must needes doe so.

Euma. Therefore beginning at the youngest sonnes portion, which was but 1. angell, I double it, and it yeeldeth 2. for the 9. sonnes portion, which I double also, and it yeeldeth 4. for the 8. sonnes portion: and so I continue still doubling euery terme last produced, till I haue found all the 10. termes, as this figure sheweth, where the greatest or last terme belongeth to the eldest sonne.

The tearmes.	The executor hauing the last halfe, had but one angell, then had the youngest sonne but 1. angell also, for the executors legacie was iust the one halfe of that which the 9 sonne left. Portions
Tenth.	1
Ninth.	2
Eighth.	4
Seuenth.	8
Sixth.	16
Fifth.	32
Fourth.	64
Third.	128
Second.	256
The eldest.	512

Phil. You haue done well, and made a good occularie demonstration of the children's portions, wherefore now proceed to the finding of the totall, as this last rehearsed verse taught you.

Eu. To do that, I multiply 512 the last terme by 2. (the Index of the proportion, whereby the terms increase) and it produceth 1024. and then 1024 — 1. leaueth 1023. for the sum of all the 10 termes, that is to say, of all the children's portions, for as for diuision

diuision by 1. lesse then I multiplyed is needlesse here, sith such diuision is to be made but by 1. which neyther multiplieth nor diuideth, wherfore if to the same 1023. there be added the 1. angell which the executor had, then will the totall be 1024. representing the true number of all the testators angels, as the former figure heere newly described with the totall annexed thereto declareth.

The executor.	1
The sonnes.	Portions
Tenth.	1
Ninth.	2
Eighth.	4
Seuenth.	8
Sixth.	16
Fifte.	32
Fourth.	64
Third.	128
Second.	256
The eldest.	512
Totall.	1024

Phi. You say truth, and heere although many other questions might be propounded answerable by the rules of Geometricall progression, yet forasmuch as all such questions commonly are to be dissolued after one of the former examples. Therefore referring you to them, as to a sufficient president in all like questions. I will heere conclude progression Geometricall, which riseth of proportionalitie continuall, as the 231. verse declared. And now I will go in hand with Geometricall proportionalitie, discontinuall in order following.

Of Geometricall Proportionalitie discontinuall in generall, and of the golden rule proceeding of the propertie thereof. Chap. 8.

241. In questions of proportion discontinuall,
most commonly, 3. termes are first assign'd
To whom is sought a 4. proportionall,
which being found they are by course of kinde,
Two of one firname, and 2. of another,
so matching by couples like brother and brother,
242. Whereof if the third which matcheth with the first.
require by his greatnesse his portion the greater,
Or els by his smalnesse doth ratably thirst,
to haue his portion the lesse of his debter.
Then is such proportion call'd by that effect,
the golden rule, or rule of 3. direct. } 201
243. But if the third terme exceeding his match,
the more it exceedes craues so much the lesse,
Or being the lesser doth greedely watch,
for so much the more at last to possesse,

Then call such proportions as this last rehearst, the backward rule, or rule of 3. conuerst.

The definition of proportion discontinuall, with the diuision therof into 2. kindes viz. direct and reciprocall or conuerted, is so playnly set foorth in the 199. and 200. verses and in their glosses, that it may seeme meere superfluous to make a new repetion therof agayne heere, wherefore I will proceede to the explication of th'effects of this kind of proportionalitie, which as this preſẽt maxim or proposition importeth, by 3. numbers first named or known, seeketh out a fourth number proportionall, and therefore some men regarding the subiect thereof, call it the rule of 3. but others considering the excellent vse therof, wherein it surpasseth all other rules of Arithmeticke, as far as golde excelleth all other mettals, name it (and that worthely) the golden rule.

Now when such a fourth number proportionall is by this rule found out and known (if the numbers be contract to any sirnames, as in all questions commonly they are,) then 2. of the 4. are alwaies by common course of kinde of one sirname or qualitie, and the other 2. of another sirname or qualitie, so that they match together in sirname or qualitie by couples, as it were 2. brothers of one kinde, agaynst 2. other brothers of another kinde (as the first verse of this maxim telleth you.)

Matche-termes or Gender-like terms

Wherefore each couple of them which so agree and match togeather in like sirname or qualitie, are by the 202. verse properly to be called matchtermes or genderlike termes: for in such cases, the one couple are the antecedents, and the other couple are the consequents.

Whereof, if the third terme being greater then the first (with whom it naturally matcheth, as the second part of this Maxim affirmeth) doth therefore in reason require a greater portion, or being lesser then it, doth therefore require a lesser portion: then by the same verse, both such proportionalities are direct and therefore are called the golden rule or rule of 3. direct, as for example, I put this question.

If 4. weekes commons at Cliffords Inne did cost me 12.s. what will 18. weekes commons there cost me?

Here (as the first verse of this Maxim saith) you see there are 3. termes or numbers first named or assigned, to wit. 4. 12. and 18. whose 4. proportionall by the 73. or 77. Maxims is found to be 54. For as 4. the first terme is in subtripla proportion to 12. the

second

ſecond, ſo is 18. the third terme in ſubtripla proportion to this 54. the fourth terme.

And as 4. and 18. being the firſt and third termes, are both ſirnamed weekes, ſo 12. and 54. being the ſecond and fourth termes, are both ſirnamed ſhillings, wherefore I call the firſt and third numbers Matche termes, and ſo likewiſe the ſecond and fourth, according to the 202. verſe.

And further, becauſe that 18 weekes the third terme, being greater then 4. his proper match-terme, doth by ſuch his greatnes, require a greater portion or number of ſhillings then 4. weekes did, therefore by ꝑ laſt part of this maxim, this proportionality is direct.

Eu. I vnderſtand both the verſes and your ſaying in this ſorte, that if 4. weeks commons coſt you, 12. s. then 18. being a greater number of weekes ought in reaſon to coſt you more then 4. weeks did, and agayne if 18. weekes commons coſt you 54. s. then 4. being a leſſer number of weekes ought by like reaſon to coſt you leſſe then 18. weekes did.

And therfore becauſe in the firſt queſtion the greater number of weekes requireth the greater portion of money, and in the ſecond, the leſſer number of weekes, requireth the leſſer portion of mony, ſo that more requireth more, and leſſe requireth leſſe, according to naturall order, therefore (as I take it) your ſaying and meaning is, that both theſe proportionalities are direct, which thing ſeemeth alſo to be approued by the 200. verſe, for ꝑ the cõference of the terms therein, is frõ the firſt to the ſecond, & from the third to the fourth.

Philo. My meaning is euen ſo as you take it, and you haue well remembred the rule which confirmeth the ſame.

But now by the laſt verſe of this maxim, if the third terme exceeding his match, do yet (notwithſtanding ſuch exceſſe) craue ſo much the leſſe for his portion, or being leſſe then the firſt terme, doth therefore contrarily looke to haue ſo much the greater portion (ſo that more requireth leſſe, and leſſe requireth more) then ſuch a proportionalitie, is called the Golden rule, or rule of 3. alſo, yet not direct, or right forth, as was the other: but backward or conuerſed, becauſe that in this rule, as the firſt terme is to the third directly, ſo is the fourth to the ſecond conuerſedly or backwardly, as for example.

If 6. buſhels of oates ſerue for the prouender of 4. geldings 6. dayes, how many dayes will the ſame finde 8. geldings?

Here the 8. geldings being the third terme, is greater then 4.

Geldings his genderlike match-terme, and yet notwithstanding, requireth in reason a lesser portion of time, then doth 4. Geldings, for by reason 8. geldings will consume 6. bushels of Oates in fewer dayes then 4. geldings.

Wherefore if 6. bushels of Otes serue 4. Geldings 6. dayes, then I conclude (by the said last verse) that the same quantitie shall serue 8. geldings but 3. dayes.

And so the conference of 4. the first terme, to 8. the third directlie, is like to the conference of 3. the fourth terme, to 6. the second backwardly, which thing is confirmed by the 201. verse, where the termes of the proportionalitie being set thus, 4. 6. 8. 3. are compared in this manner, as 4. to 8. so is 3. to 6. So that (as they now stand) the first antecedent is compared to the second directlye, and the last consequent is compared to the first backwardly.

Likewise, if the forme of the question were changed, so that it be said thus. If 6. bushels of Oates serue 8. Geldings for prouender 3. dayes, how many dayes will the same quantitie of Oates serue 4. geldings?

Heere now 4. geldings being the third number, and lesser then 8. his match terme, doth notwithstanding craue and require a greater portion of time or more dayes then 8. geldings did.

For by common reason, 6. bushels of oates shall serue 4. geldings a longer time then it will serue 8. geldings, and therefore this proportionalitie concludeth, that if 6. bushels of oates serue 8. geldings 3. dayes, then the same will suffice to serue 4. geldings 6. dayes, for as 8. the first terme is to 4. his match-terme directly, so is 6. the fourth number to 3. his match-terme conuersedly, and the last proportionall in this rule is found, eyther by the 98. or 99. verses of the former booke.

Euma. All this I vnderstand verye well, and remember the verses also aswell, which teacheth the finding of such a proportionall.

Philo. Then will I proceede to shew you the very order, forme, and ordinarie practise of this Golden rule, both direct and conuersed, but first as it is vulgarly single and directly, with no accompt proceeding.

And although I am not ignorant, that the most proper and naturall conferrence of numbers proportinall, requireth that the Antecedent of each proportion be referred and compared to the consequent of like surname and nature, yet in the golden rule

direct,

direct, the tearmes generally are otherwise conferred, as for example, when I said, if 4. weekes commons cost 12. s. then 18. weekes commons shall cost me 54. s.

Here I say the proportions being naturally conferred, should be thus 4. weekes to 18. weekes, and 12. s. to 54. s. or thus. $\frac{4 \text{ weekes}}{18 \text{ weekes}}$ and $\frac{12. s.}{54. s.}$ for as 4. weekes is to 18. weekes, so is 12 s. to 54. s. that is to say, in proportion Subquadrupla Sesquialtera.

And so by this meanes 4. is the first tearme, 18. the second, 12. the third and 54. the fourth, but in the Golden rule direct, the termes are conferred alternately, So that both the Antecedents are alwayes of one nature and sirname, and the 2. consequents are of another nature or sirname.

And the reason or cause thereof is, for that questions cannot be so well and sensibly framed in numbers of like sirnames, as of dis. like sirnames, for it were no reason to propounde a question thus: if 4. weekes cost 18. weekes, what shall 12. shillings cost, nor yet thus, if 4. weekes be 18. weekes, what shall 12. shillings be?

Wherefore aswell for the better understanding of the learner, as also to make the questions more agreeable to reason, the golden rule conferreth the 4. weekes to the 12. shillings. and the 18. weekes to the 54. shillings, framing the question thus.

If 4. weekes cost 12. shillings, what shall 18. weekes cost, and so in this rule 4. is reputed to be the first tearme, 12. the second, 18. the third, and 54. the fourth, for as 4. weekes is in proportion to 12. s. so is 18. weekes, to 54. shillings.

Euma. That is in proportion Subtripla, which I understand well, persuing with all by the 91. verse that such change of conference doth neither alter the situation of the termes, nor hynder their proportionalitie, for both wayes the fourth proportionall sought, shalbe one selfe same number, to wit 54.

Phi. You have sayd very truly, wherefore now behold the whole rule it self, (as it is vulgarly single and direct, without any accompt proceeding) comprehended in the verses following.

244. The golden rule, first single and direct,
by 3. numbers knowne, the 4. seekes to finde,
Pointing such places to worke that effect,
that each number hath his proper place assignde,
At the ends and angles of a Romane Z,
whereto they make claime, and looke to be led,

Doctor Records grounds of arte.

245. And

Cyclopedia Ioach. fortij Ringelbergij pag. 408.

245. And first the terme wherein a question lieth,
claymes the left angle for his proper place,
Next whome the matchterme of that number cryeth,
to beat the head, for that is his grace,
So that the number, which rests cannot chuse,
but go to the angle, which both they refuse,
246. And so the first terme, and third, euer must,
match in one surname by this instruction,
For if they varie, they must be reduced,
to one like firname by th'art of reduction,
Which being once done, as longs to the case,
and each of them setled in stn his own place,
247. Then multiplie both the anguler sums,
one by another the product diuiding,
By that at the head, so forth with there coms,
cleare in the quotient the fourth number sliding,
VVho claymeth to stand at the tayle of the Z,
VVhere being once plac'd, your purpose is sped.

202

Now to explaine the meaning of this rule, by an example, I will vse the first question propounded before, which was this.

16. A question of the charges of commonning by the week.

If 4. weekes commons at Cliffords Inne cost me 12. shillings, what will 18. weekes commons there cost me? wherfore in applying these 3. giuen numbers to the figure which the verses (for the similitude it hath therto) call a Romane Zed. I consider fyrst that the number whereon the question lieth is 18.

Figure of the golden rule.

For the very effect of the question consisteth onely in these woordes, what will 18. weekes commons cost me?

And therefore as the second verse of this rule importeth I settle the same 18. weekes at the left angle of the figure, which is the third place, or place of the third number thus. weekes.

Then right ouer that at the vpper end or head of the figure, which is the first place and belongeth properly to the fyrst terme or number, I set the 4. weekes, which is the matchterme or numer of like firname to 18. thus. weeks

Head and tayle of the figure, what it signifieth.

So must 12. the other number remayning, needes take the right angle which is the second place, and so these 3. numbers, being placed each in his owne place in a readinesse, to make inquisition for the fourth proportionall, stand thus.

	18 Z
	4 Z
weekes	18 Z
	12

And now (as the last verse of this rule willeth

me

me) I multiplie 18. and 12. the 2. angular numbers together, and they produce 216. which I diuide by 4. the first number situate at the head or fyrst place of the figure, and so there issueth in the quotient 54. which is the fourth number proportionall, and by the last verse of this rule claymeth to stand weekes at the taple or latter end of the figure, where I place it accordingly, as here you see it, and so haue I not onely spedde my purpose in finding the number answering to the question, which I sought for, and desired, but also now the 4. numbers being all knowne, it appeareth that 2. of them are of one sirname and 2. of another, as the 241. verse affirmed, for the one couple at the left side of the figure are both sirnamed weekes, and they are commonly called the antecedentes, and the other couple at the right side are both sirnamed shillings, and they are vsually called the consequentes, and therefore I affirme that if 4. weekes commons do cost me 12. s. is to then 18. weekes commons will cost me 54. as 4 12 shillings. For so 18 54 is to

Tayle or latter end of the figure.

Euma. All this as me seemeth, is very plaine and easie inough, both to vnderstand and to do, and hereby I imagin that the crooked indented lines like Romane Zeds which were put betweene the tearmes before in the 203. and 204. verses was to shewe, which couples were to be compared together, for that purpose I take the right lines of this figure serueth, and the bias line thereof serueth as it were to poynt to the 2. numbers, which are to be multiplied together.

Phi. Your imagination therein is not to be vilyayled.

Euma. Also that the 2. tearmes situate at the first or left side of the figure, are named antecedentes, and the other 2. at the other side Consequentes, is very agreeable to reason, and to the tenor of the 202. verse, and that the proportionalitie in this rule is direct, appeareth also by the 200. verse; but why you call it single, I pray you declare, for I yet vnderstand not the reason thereof.

Phi. 248. Single proportion is that which at first,
Answeres a question with one rule alone,
And it is eyther direct or conuerst,
VVhereof the direct doth passe away prone,
Playne as a pike-staffe no other woorke needing.
Or elsit requires an accompt proceeding. } 203

So

So that when the goulden rule bringeth a full & perfect answer to a question demaunded at one onely operation thereof, then is such a proportionalitie called a single proportion, of which kinde there is both direct & conuerst, as this verse playnly sheweth you.

And if such proportion be direct, thē it is either plain proceeding redily and immediatly without need of any other operation as this question last before did, and as all the rest henceforth in this chapter do, or elle it requireth some accompt to be made before you proceed to it, of which kinde you shall haue examples in the next chapter folowing.

Euma. You haue very well satisfied me herein, but yet I maruell at this, that sith you repute the first end or head of the figure to be the first place, and to belong properly to the first Antecedent, and the right angle to be the second place, and to belong of dutie to the first consequent. And the left angle to be the third place perteining onely to the latter antecedent, & finally, the last end or tayle of the figure, to serue onely for the fourth number or last Consequent, which giueth the answere to the question: yet you depose not the numbers accordingly, but place the 3. number or latter Antecedent first, and then next the first Antecedent, and thirdly, the first Consequent.

Phi. The cause thereof is, for the sure placing of the third number or latter antecedent in his proper place.

For he that knoweth already perfectly, the due placing of euery number in the figure, may set downe the first number first, and the second secondly, and so the third, thirdly, but if the questiō be ouerthwartly and disorderly propounded (as the common people for the most part, vse to propound their questions,) you shall be constrained to seeke out the number whereon the substance of the question dependeth, and to place that first, which by the 245. verse, alwayes claymeth the left angle, or third place of the figure: And next, to settle the number of like stname, which is his match-fearme, at the head or first place, and so the other number remayning must needes settle at the right angle or second place, for none may stand at the tayle of the figure, but fourth or last number, which giueth the answere to the question.

And it is not the first, second, or third number, uttered or named, that maketh the same alwayes to be first, second, or third in place, but that fearme or number whereon the question lieth

lieth or depend, ought alwayes undoubtedly to be taken for the later antecedent or third number, & to be placed in the third place, and next that the number matching with it in firname, ought to be taken for the first Antecedent and setled in the first place.

And then the other tearme or number remayning, which is of dislike firname, must consequently be the second, and supplie the right angle or second place, as for example, let the question be thus propounded.

If it cost me 12. shillings for 4. weekes commons, what will 18. weekes commons cost me.

Here you see 12.s. is the number first named, and 4. weekes the the second, and 18. weekes the third, but if they should be placed in the figure according as I utter them 12.s. 4. weekes. which is thus, as here you see

weekes
18

Then I say the first and third numbers being the 2 Antecedentes, should not match in like firname, as by the golden rule they alwayes ought to doe, neyther should the fourth number succeeding by the operation of the rule, giue true answere to the question.

For if the product of these 2. number numbers (which is 72.) be diuided by 12. the first number, the quotient insuing thereby will be but 6. which is knowne false answere.

Euma. So, now I vnderstand very well, that the reason why you placed the numbers in such order as before, is for that the questions demaunded, are not alwayes orderly propounded, so that if the question were put foorth thus: What will 18. weekes commons cost me, when 4. weeks commons costeth me 12. s. yet I perceiue I am not to iudge, nor accept 18. weekes for the first number, because it is first named, nor 4. weekes for the second number, because it is secondly pronounced, nor yet 12. shillings for the third because it is thirdly vttered: but I must esteeme 18 weeks to be the third number, and due to be placed in the third place, because the question lieth or dependeth on it.

For indeede I see that the very question is no more but what 18. weekes commons will cost, and then sith 4. weekes are the match-tearme thereto, I must accompt that to be the first number, which therefore ought to take the head or first place of the figure, whereby consequently 12. s the other number remayning yet vnplaced must of necessitie be the second, and take his place at the right angle, which is the second place by order.

And

And thus I perceiue that by the placing of the tearmes in such order as you haue prescribed, euery tearme shalbe sure to possesse his proper place, how disorderly or preposterously soeuer the question be propounded, and so by that meanes, the forward auerthwart forme of the question, will also be altered withall and brought to the true and orderly forme thereof, which is thus: If 4. weekes commons cost me 12. shillings, what will 18. weekes commons cost me.

Philo. You haue sayd very truly, and here before I propound any mo questions touching this rule, sith it is the most necessarie and profitable rule of the whole art of Arithmeticke, I would not willingly omit to demonstrate the reason thereof vnto you, if you would not thinke it too tedious, for to knowe a thing, without knowing the cause or reason thereof, is no perfect knowledge in any art. Sire enim proprie, est rem per causam cognoscere.

Euma. I cannot thinke any thing tedious, that tendeth to the furtherance of my knowledge, wherefore I pray you in any wise, proceede to the demonstration thereof.

The reason of the golden rule, demonstrated,

Philo. The reason or cause thereof proceedeth originally from the naturall propriete of Geometricall proportion discontinuall, propounded in the 100. verse, of the former booke, which I will not sticke here to repeate, and is thus.

If 4. numbers be truly proportionall.
The product produced of both the extreames,
Shall neyther be more nor lesse, but iust equall,
vnto the product made of the 2. meanes.

But the ordinarie action, vse, and practise thereof, is deriued from the 101. verse, which sayth thus.

If any 3. numbers what euer they be,
Haue the product made of the later twaine,
Diuided foorthwith by first of the three,
The quotient doth shew the 4. number playne.
In right proportion to those 3. propounded,
And on this verse the golden rule is grounded.

Euma. I remember these 2. verses very well,

Phi. Then when soeuer any 3. numbers are giuen or propounded, to the intent that a fourth proportionall to them should be sought, you know that the first is the first extreame, and the other 2. are the 2. meanes.

Eu-

Eu. Yea I know that very well, and therefore the fourth number, which is lacking to make vp the fourth proportionall, is the latter extreame, as appeareth by your former question, where the 3. giuen numbers, seried duly stood thus. weekes.

So that the first extreame & both ỹ meanes 4 — 12. 8.
are apparent, but the last extreame is it weekes.
which is sought for, and required. 18

Phi. Very well sayd, now for asmuch as by the sayd 100. verse, the products of the 2. meanes, is equall to the product of the 2. extreames, therfore after that (by such multiplication as the 101. verse requireth, ỹ the golden rule direct vseth) we haue obtayned the product of 18. and 12 the 2. angular numbers, which are the 2. meanes. It followeth that we haue the selfe same product or number, which would result by the multiplication of the 2. extremes, but by the 94. verse.

If any number which you would diuide,
be the product made of any 2. other.
Then each of those numbers if they be applied,
diuiding that product shall quote out the tother.

Wherfore sith the product of the 2 means, is also the product of the 2. extreames, therefore if one of the extreames diuide the same product the quotient shall bring the other, but when the first number diuideth the product of the 2. angular numbers, then doth the first extreme diuide the product made of both the extreames, wherfore by such diuision, the quotient of necessitie shalbe the latter extreame or 4. proportionall, which is the thing that was to be proued and demonstrated.

Eu. Truly this demonstration is so playne and sensible, that I cannot chuse but vnderstand it, and it is no lesse certaine true and infallible, for when 18 multiplied by 12. produceth 216. and that the same product diuided by 4. yeeldeth 54 in the quotient for the fourth proportionall or latter extreame, it is manifest the same 4. and 54 multiplied together, will also produce 216. which thing is proued both by the 58. 59. 97. and 100. verses.

Phi. The selfe same demonstration serueth also for the inuention of the third proportionall, in continuall proportion Geometricall, according to the rule here of contayned in the 224. and 225. verses, which there I purposely omitted, to the end that here this one demonstration might serue for both.

For by the 223. verse, the product of the 2. extreames in such

such continuall proportion is equall to the product of the middle tearme by it selfe, but by the golden rule of the 17. Partic. 3. tearmes in continuall proportion, are 4. in power, and the meane or middle tearme seuereth for 2. wherefore it followeth, that the product of such a middle tearme by it selfe, is as much as the product of the 2. meanes in proportion discontinuall, and therefore such a product diuided by the first extreame, shall consequently deliuer the latter extreame in the quotient.

Euma. The reason of this proofe is so manifest that it cannot be denied, which being once well printed in my memorie, I am perswaded I shall thereby proceede in all questions and operations of this rule, with much more facilitie and certaintie, then I should haue done otherwise: wherefore now I pray you (for my better exercise in this golden rule) propound some moe questions thereof.

Phi. That will I do with a good will, for the varietie of examples is one of the chiefest lights in learning, wherefore take you this for the first.

17 A question touching reparatiō of a parish church.

When my neighbours land was tared to pay 12.s. towardes the reparations of the parish Church, my land was tared to pay 3.l. but now my neighbours land being tared to pay 20. shillings towardes the like reparations, I demaund what my land ought to be tared at after that rate.

Euma. Here I perceiue that the whole question is, what your land ought to be tared at? now your neighbours is tared at 20. shillings, wherefore 20.s. being the number whereon the question lieth, must be taken for the third number or latter antecedent, and 12.s. being the match tearme thereto, must therefore needes be the first, whereby it followeth that the 3. pounds must be the first consequent, which numbers I therefore apply to 12.s. 3.l. 20.s. the figure of the golden rule in their due places thus.

And now multiplying 20. by 3. the product is 60. which I diuide by 12. and so the quotient bringeth 5. for the fourth number, which therefore I place at the table or answering place of the figure thus.

Which 5. by the nature of this rule is alike sirname to his matchtearme the former consequent, that is to say, pounds, wherefore if your land payd 3.l. when your neighbours payd but 12.s. then now, sith his land is tared at 20.s. your land ought to be tared

at

at 5. pounds.

Philo. You haue done well, and answered truly.

Euma. I suppose I haue so, but if the question had beene put all in shillings, as it might be thus. When your neighbours land payde 12.s, yours paide 60.s. therefore what shall your land pay, when his payeth 20.s? I say in this case, although I can tell how to finde the third number or latter antecedent, yet cannot I see how which of the other 2. numbers I should take for the first antecedent, because heere they be all genderlike, matching all in one like sirname.

Phi. Your saying seemeth to haue some shew of reason, and therfore, for your better instruction in euery such case, when you see all the 3. termes so of one sirname, where by they seeme to breede some ambignitie in choice of the matchtermes or antecedents, you must diligently consider which couple of them match or ioyne together in some other further qualitie, referrence or epitheton, then only in their bare sirnames.

And so shall you finde the one couple of them to haue some such qualitie, relation or epitheton, as the other hath not, which couple therefore are to be taken onely for the antecedents or first matche-termes. As for example: the 12.s. and 20.s. whereat my neighbours land was tared, are referred to him and his land, and not to me nor my land. Wherefore the same 12.s and 20.s. are the antecedents or first couple of match-termes. But the 60. shillings remayning, haue relation onely to mee and my land, and not to him nor his land, and so likewise haue the 100. shillings, which will be the fourth terme proportionall: wherefore the same 60. shillings, and 100. shillings, are the Consequents or second couple of matche termes.

Likewise, let this be another question, if I gaine 10. pound in 100. pounds, what do I gaine in 40. pounds?

18. A question of gaines in the 100. pounds.

Heere as you see all the 3. termes are of one like sirname, but yet if you consider whereto the 100 £. and 40. £. are referred, you shall finde them to haue relation both to one thing, namely to a principall stocke or banke of money employed for gaines.

And by like consideration, you shall perceiue the second terme and the fourth (yet vnknowne) to haue relation to another thing, namely to the cleare gaines gotten by the employment of such a principall bancke or stocke, eyther in Merchandise, or at interest, or some other such like trade, and because the question dependeth

vppon the 40. pounds, therefore by the 245. verse the same is the third number or latter Antecedent.

Eumathes. That I perceiue well, and confesse to bee verye true, and therefore I thinke I may for more suerties sake, (till I be better practised in this rule) propound the question thus.

If 100. pounds stocke, bring 10. l. gaynes, what gaynes doeth 40. pounds stocke bring, and then the answer wilbe 4. pound gaynes, as heere the figure sheweth.

stock	gaines
100.l.	10.l.
40.l.	4.l.

How to finde the match-termes.

And so the words which reueale and discouer the matchtermes are stock and gaynes, and not pounds onely.

Phi. You haue said well, and it shall not be amysse for you, to vse such addition of words, to the surnames of the numbers, at all tymes needefull, for it bringeth light to the worke, aydeth the memorie, and is not any wise hurtfull to the order of the question.

For euen as diuers persons of like name and surname, cannot aptlye be made knowne the one from the other, but by some further relation or addition, of age, degree, or place: No more cannot the Antecedents or first matche termes in such cases, bee distinguished or knowne from the second Matche-termes or Consequentes, but by such further consideration or note of distinction.

And therefore as we vse to saye, Hugh Spencer the Father, and Hugh Spencer the Sonne, Thomas Smith Esquier, and Thomas Smith Marchant, William Fielde of Yorke, and William Field of Chichester.

So may you likewise say, Pounds of stocke, and poundes of gaynes, pounds of weight, and pounds of money, pounds of Pepper, and pounds of Ginger, pounds of Troye, and pounds of Hauerdupoise.

And likewise yardes of Cloath, and yardes of Lace, yardes of length, yardes of breadth, and yardes of thicknesse: and so of all other things.

Which matters I straine my selfe purposely, to make as plaine to you as I can, for that I knowe it is farre more difficult (especially to learners) to frame their questions in good and due order to the figure, then it is to performe the rule, after that the

question

queſtion is duely framed, and the numbers orderly placed: and therefore if you vnderſtand your owne former wordes perfectlye which you ſpake a little before, then anſwere mee to this queſtion.

If 12. yardes of Buckerham ſuffice to line 16. yardes of Uelluet, how much Buckerham will ſuffice to lyne 24. yardes of Uelluet? 19. A queſtion of Buckerham and Veluet.

Eum. Although all the 3. termes aſſigned are ſirnamed yardes, yet I perceiue by this your laſt doctrine, that 16. and 24. are onelye the antecedentes or firſt couple of matche-termes, becauſe they onely matche or couple together in relation to Uelluet:

And that 24. is the third terme or latter antecedent, becauſe the queſtion dependeth onely therevpon, and therefore 12. being referred onely to Buckerham, muſt needes be the firſt conſequent, and ſtand at the right angle of the figure, ſo that the termes are to bee adopted to the figure thus.

Uelluet.	Buckerham.
yardes 16.	12. yardes.
24	

Wherein, by the opperation of the Rule, I finde the fourth number to be 18. Which therefore I place at the taple of the figure, as the true anſwere vnto your former queſtion, thus.

Uelluet.	Buckerham.
yardes 16.	12. yardes.
24	18.

Whereby I conclude, that if 12. yardes of Buckerham ſuffice to lyne 16. yardes of Uelluet, then 18. yardes of Buckerham ſhall be ſufficient to lyne 24. yardes of Uelluet, and ſo it ſeemeth now, that not yeardes, but rather Uelluet and Buckerham, are the right ſirnames which make the matche-termes.

Phi. Yea, or els yardes of veluet make the antecedents or firſt couple of matcheearmes, and yardes of buckerham make the conſequents or latter couple of match tearmes, but now what ſay you to this queſtion.

If a ſtaffe pitched in playne ground directly vpright 6. foote high aboue the ground, caſt a ſhaddowe 10. foote long, of what height is that ſteeple, whoſe ſhadowe at that inſtant is 250. foote long. 20. A queſtion of ſhadowe.

Euma. First it is manifest, that the number whereon the question hangeth or dependeth, is the 250. foote of shadow made by the steeple, wherefore I assure my selfe that it is the third number or latter antecedent, and the 10. foote referred to the shadowe of the staffe, must be the matchterme or first antecedent, because it is also referred to shadowe, then must the 6. foote referred to highth needes be the second number, wherfore I frame my question thus, saying.

If 10. foote shadow come of 6. foote height, of what height cometh 250. foote shadowe, and so applyeng the numbers to the figure and working by the rule, I finde that 250. shadow. height. shadowe commeth of 150. foote height, as heere appeareth.

10 — 6
250 — 150

Philo. It is euen so as you say, and thus you see how by this most excellent rule, you may by the shadow of your walking staffe, discreetly placed, know the heighth of any Tower, Steeple, Pole, Tree, or other body, whose shadowe you may measure at that instant.

Euma. Truly I must needs confesse, that this rule is both profitable and pleasant, and therefore well worthy both to be greatly esteemed, and also to be called A Golden rule.

Philo. You would say so, if you know how diuerslie it may be applyed, and to how great purposes. But in the meane season answer me to this question.

If 1. pound of maces cost 6. s. 8. d. what will 4. ounces cost after that rate?

Eu. First the number whereon the question lyeth is the 4 ounces, wherefore I take that for the latter antecedent or third number, but now I see no other number of like firname, wherfore I thinke the 1. pound must be reduced to ounces, for so it seemeth the 246. verse of this Golden rule requireth.

21. A question of Maces.

Philo You say truth, you must eyther reduce the 1. pound to ounces, or els the 4 ounces to $\frac{1}{4}$ of a pound, but if you reduce the 1. pound to ounces. Then by the 153. verse, because the pound of Maces is Hauerdupoise weight, you must multiply 1. r. by 16. and so haue you 16. ounces instead of 1. pound.

Eum. Very well, so haue I 16. ounces for the first antecedent, and 4. ounces for the latter, but yet I wot not what to doe with the other number remayning, because it containeth 2. diuers firnames, that is to say, shillings and pence.

Philo.

Phi. What would you do if the same 6 s. 8. d. had beene expressed in particles, I meane in fractions of fractions, thus, ½ ⅓ of 1. pound.

Eu. Marry then I would first reduce both the particles to one simple fraction by the 156 verse of perticuler reduction, and so instead of ½, ⅓ r. I should haue ⅙ r. equall to them both.

Philo You haue answered well, and you ought to doe so as you haue said, but now see you not that this ⅙ r. being the simple fraction resorting by such reduction, beareth the least sirname or denomination of the 2. former.

Euma Yes, I perceiue that very well, for by the 22. verse I know that thirds is a lesser denomination or sirname then is halfs.

Phi. Euen so by the same reason, when any of the 3. geuen termes of this rule, is of sundry sirnames, the best way is to reduce them all, how many soeuer they be, to the subtilest or least sirname (by the 153. verse) which in the question present is pence, and so insteade of 6. s. 8. d. which are 2. sirnames, you shall haue 80. d. in one sirname equall to both the former.

And then the question will be thus, if 16. ounces of Maces cost 80 d. what will 4. ounces cost, and so following the rule you shall find that 4. ounces will cost 20. d. as the fabrique here sheweth.

	Ounces.	pence.
	16	80
	4	20

For euery coine of lesser or subtill sirname, is a fraction of the grosser, as pence, which are fractions of a groate, and groats of testernes, and euery of them are fractions of a shilling, and so forth of others. Subtils are parts or fractions of the grosser.

And therefore (if you will,) because grotes is a meane sirname betweene shillings and pence, whereunto 6. s. 8. d. or 80. d. may be fitly reduced, you may reduce the same to 20. groats for your second terme, in steade of 6. s. 8. d. or of 80. d. and so the question will be thus, if 16. ounces of Mace cost 20. grotes, what will 4. ounces cost, and then by the opperation of the rule you shall produce 5. grots as here appeareth.

	16	20 grotes
	4	5

And so both wayes come to one effect for 5. grotes and 20. pence are equall, but if you will reduce the 4. ounces to the partes of a pound, then shall the question be framed in this order.

If one pound of Mace cost 20. grotes, what shall ¼ r. cost and so applying the numbers to the figure they will stand thus.

	1 r	20 grotes
	¼ r	

And now to accomplishe the rule I multiplie $\frac{20}{1}$. by $\frac{1}{4}$, and the product is $\frac{20}{4}$. which diuided by 1. continueth $\frac{20}{4}$. but by distinct reduction it commeth to 5. integers, that is to 5. grotes, which I set at the figures tayle as heere you see.

1. ℓ. ∠ 20. grotes
∠ 5. grotes

Eu. Indeede it is manyfest that euery of $\frac{1}{4}$. ℓ. these kinde of wayes returne but one answer in effect, and that after the termes of the proportionalitie are once reduced, aswell the antecedents on the one side of a figure, as the consequent on the other side, agree and match still in like sirnames, but you sayd not long sithence, that all the questions in this chapiter, were of single proportion proceeding playnely without neede of any other opperation, and yet here it seemeth that this question could not proceede without reduction.

Philo. I sayd so indeede, and so I say still, for you know by the 152. verse and glosse thereon, that I esteemed not reduction to be any seuerall opperation arithmeticall, for reduction doth neither add any thing to the termes, nor subtract any thing from the, neither multiplieth them, nor diuideth them, but onely changeth their numbers for others proportionally, whereof euery one still keepeth and expresseth his first value signified, wherefore I conclude that this question needeth no speciall operation, but is applyable to the figure foorthwith after reduction. And now to prooue how you can doe the like, I propound vnto you this question.

12. A question of Ware.

If 100 ℓ. weight of ware (accompting 112 ℓ. to the 100. as Hauerdupoise weight requireth) cost me 5. ℓ. 4. ὃ. what dooth 1. pound thereof cost me, or stand me in?

Euma. Heere I soone perceiue that the 1. pound of Ware is the third number, and that the 112. ℓ. is the matche terme thereto, or first Antecedent, because their pounds are both referred to ware. But because the first consequent or number remayning hath 2. sirnames, viz. shillings and pence, therefore before I applie them to the figure, I reduce them both (by the 153. verse) wholy to pence, which are the subtils of least sirname, named in the question, and so instead of 5. ℓ. 4. ὃ. I haue 1204. ὃ. for the second terme. wherfore now the numbers applied to the figure, stand thus.

112. ℓ. ∠ 1204. ὃ
1 ℓ. ∠

The product of whose anguler termes is 1204. ὃ. which diuided by 112. bringeth in the quotient $10\frac{84}{112}$. that is by abridging reduction, $10\frac{3}{4}$. wherefore I place the same at the tayle of the figure,

figure, as the latter consequent or aunswer to the question thus, as heere you see.

112. l.	Z	1204. d.
1. l.		10 ½ d.

Concluding that 1. l. of this ware standeth you in 10. pence halfe-peny farthing, or 10 ½. pence.

Philo. Well done, then what say you to this: A marchant buying a peece of Satin conteining 22 ½ yards, for 8. l. 16. s. in grose, determineth not to sell it againe vnder 10. l. Now a gentle man is desirous to buye 8. yardes thereof onely: wherefore I demaund for how much the marchant shall sell the same 8. yardes, to make 10. pounds of the whole peece?

23. A question of Satin.

Euma. Herein I see that 8. yardes is the third terme or latter Antecedent, because it conteyneth the question, and therefore 22 ½. yardes being the quantitie of the whole peece, and number of like sirname, shall be his match-terme, and so consequently 10. l. being the price of the same 22 ½. yardes, shall be the second terme or first consequent, where-by the scituation of the terms in the figure appeareth thus.

yardes.		pounds.
22 ½	Z	10.
8		

And now to proceede with the rule, I finde the product of the 2. anguler numbers to be 80. which I should diuide by 22 ½. But I remember not that I haue learned any rule yet, how to do it.

Philo. No haue? why then I perceiue you haue forgotten the 176. verse, which telleth you, that Mixt numbers with mixt nor vnmixt will offer, to yeelde to diuision or multiplication: Till both be reduced to fractions improper, or to whole numbers to their contentation, &c.

Euma. In truth I had forgot that verse, or at the least, the vse thereof at this present, in so much that I was amazed at mine owne worke.

Philo. So it seemeth, for forasmuch as 22 ½. is a mixt number, you should haue reduced it to an improper fraction, eyther by the 164. or 65. verses, before you had appplyed it to the figure, & so, insteed of 22 ½, you should haue had 45/2 yardes for your fyrst terme, and so your figure would haue stood thus.

45/2	Z	10. l.
8		

Eu. Very well, I hope my present ouer sight, shall turne to my good hereafter, in making me take better heed, by this reprehension, & therfore now to proceed, taking 45/2. yards for my first terme or antecedent, I diuide 80. by it, which by the sayd 176 verse, must

also be made an improper fraction thus, $\frac{4}{4}$. And so the division being performed, I finde the quotient to be $\frac{3}{4}$, which by distinct reduction is $\frac{3}{4}$ r.

Wherefore I conclude, that for so much the marchant must sell his 8. yardes of Satin, to make 10. r. of the whole peece.

Phi. He must do so, but now know you what $\frac{3}{4}$ r. is in money?

Euma. No indeed, not presently.

Phi. But you know that 1 shilling, is $\frac{1}{20}$. of a pound.

Euma. Yea that I know very perfectly.

Philo. Then by exchanging Reduction, reduce the same $\frac{3}{4}$ r. into $\frac{3}{20}$, and so will they be shillings.

Euma. To do that, I multiply (as the verse of Exchanging reduction biddeth) $\frac{3}{4}$. by 20. and the product is 100, which I divide by 9. and the quotient bringeth $11\frac{1}{9}$. that is 11. shillings, and $\frac{1}{9}$. of a shilling.

Phi. You say very true, now therefore turne that $\frac{1}{9}$ of a shilling also into $\frac{1}{12}$, that is to say, into pence by the same rule.

Euma. To do that, I multiply 1. by 12. and the product is 12. which divided by 9. bringeth 1. peny and $\frac{3}{9}$, and so the whole money that the marchant must receive for his 8. yardes of Satin is 3. r. 11. s. 1. d. $\frac{3}{9}$. which therefore I set at the figures rule in this manner.

yardes.	pounds.
8	3. r. 11. s. 1. d. $\frac{3}{9}$.

Philo. Yea mary sir, now have you answered the question truly and plainly, and therefore I will trye you with an other somewhat harder, which is this.

24. A question of cambrick.

If $\frac{3}{4}$ of an ell of Cambrick, cost $\frac{1}{2}$ of an Angell, what will $2\frac{1}{2}$. ells cost?

Euma. First I see plainely, that $2\frac{1}{2}$. ells, being the number whereon the question dependeth, is the third number or latter antecedent, and that $\frac{3}{4}$ of an ell being the mate the terme thereof is the first because they are both referred to parts of an ell, and therefore $\frac{1}{2}$ must needes be the second, because it is of like name, for it is referred to parts of an angell.

But before I place them in the figure I thinke I will not any more forget to reduce the mixt numbers to an improper fraction, which being done, I applie them to the figure thus.

$\frac{3}{4}$	$\frac{1}{2}$
$2\frac{1}{2}$	

And now folding together the 2. latter termes, I finde their product by the 169. verse, to be $\frac{5}{2}$. which divided by $\frac{3}{4}$, bringeth in

the

the quotient $\frac{3}{4}$ that is by diſtinct reduction $2\frac{3}{4}$ angels, as here appeareth by the fabrique, which if I ſhall produce vnto money (as I did the $\frac{1}{2}$ r. in the other queſtion) wilbe 6.s.8.d. & ſo the whole $2\frac{1}{2}$ ells wil coſt 26.s. 8.d. reckoning the angels at 10. ſhillings a peece.

ells	angels
$\frac{1}{4}$	$\frac{3}{4}$
$\frac{1}{2}$	$2\frac{3}{4}$

Ph. You haue done this queſtion very well and artificially, but now you are acquainted with the practiſe of exchanging Reduction, and ſee to what purpoſe it is to be applyed, you ſhall not henceforth need to vſe it, except it be ſpecially required, but ſhall annexe the laſt conſequent or anſwering number of any queſtion, to the figures tayle, in ſuch numbers or fractions as the operation of the rule ſhall yeelde, onely reducing them as neere as you can to leaſt termes, becauſe ſo they are moſt ſignificant.

And now I let you wit, that although the order whereby you wrought this laſt queſtion, is very true & certayne, and as it were the high way moſt commonly vſed, yet is there an eaſier rule in ſuch like queſtions, when all the 3. termes or numbers thereof happeneth to be fractions, and it is thus.

249. If in the rule of three direct,
Three fractions fall, this rule ſelect,
And ſeeme not to forſake it.
Fold the firſt Denominator,
In each others Numerator,
Your diuident to make it.
Then fold the toptearme of the firſt,
In both the baſſes of the reſt,
And for diuiſor take it.

} 204

For example whereof I will vſe the laſt queſtion againe ſetting the tearmes thereof to the figure as before, thus.

ells	angels
$\frac{3}{4}$	$\frac{3}{5}$
$\frac{1}{2}$	

Which being done, I fold 4. the firſt denominator in 4. and 5. the numerators of the other 2. fractions (as this rule biddeth me) ſaying 4 times 4. is 16. and 16. times 5. is 80. which 80. I keepe to make my diuident, then do I fold 3. the toptearme of the firſt fraction, in 5. and 2. both the baſſes of the reſt, ſaying 3. times 5. maketh 15. and 15. times 2. is 30, which 30. I take for my diuiſor, now diuiding this diuident 80. by 30. the diuiſor, the quotient is $2\frac{1}{3}$, euen as it was before by the other forme of worke, wherefore I place it, in the due place thus.

ells	angels
$\frac{1}{4}$	$\frac{3}{5}$
$\frac{1}{2}$	$2\frac{1}{3}$

Luna.

Euma. This rule is good and easie, and the worke but short, and it seemeth to me that it will serue aswell when but part of the termes is fractions, as when all 3. are fractions.

25 A question of Satten.

As if I were asked this question, if $3\frac{1}{2}$ yardes of Satin cost 42. shillings, what shall $\frac{1}{4}$ of a yarde cost after that rate.

In this and such like questions I say, it seemeth to me after the mixt number is reduced to a simple, by improper reduction thus $\frac{7}{2}$. that then the Integers of the second tearme, may also induce the shape of an improper fraction, and so may be applied to the figure in this manner.

yardes	shil.
$\frac{7}{2}$	42
$\frac{1}{4}$	Z

Phi. Your coniecture is very good and true, and I am glad that you conceiue the largenesse of the rule so readily, wherefore worke forth your owne question accordingly.

Euma. Then according to this rule, I fold 2. the first Denominator in 42. Numerator of the second terme, and it produceth 84. which I eftsoones refold in 3. the numerator of the third, and so commeth 252. for the Diuidend.

Secondly I fold 7. the toptearme or numerator of the first in 1. and 4. the basses of the second and third tearmes, and so I produce 28. for my Diuisor, and finally, diuiding 252. the diuidend, by this 28. the diuisor, there issueth in the quotient iust 9. shillings for the price of $\frac{1}{4}$ yarde of Sattin, which therefore I place at the figures taile, as answere to my question thus.

yardes	shilling
$\frac{7}{2}$	42
$\frac{1}{4}$	Z 9. s.

Necessitie of transformation in numbers

Philo. You haue done well, and thus you see that if any one of the tearmes be a fraction, and the rest Integers in this rule, yet will the same serue, by transforming such Integers to the fourme or shape of improper fractions, which is an easie thing to doe, and soone performed by the 175. verse.

Euma. It is so, and hereby I see still more and more, both the ease and necessitie of such transformation of numbers, which thing seemed to me very vayne and needlesse curiositie at the first, in our speach of the 18. and 21. verses.

Phi. Well then, thus much may suffice for your Instruction in the vulgar rule of proportion, or common Golden rule, as it is single, direct, and immediate, without any accompt proceeding, wherefore now I will shew you the same rule againe directly, with an accompt proceeding, wherein no other rules are

to

to be giuen, nor other forme of woorke to be vsed, then hath beene alreadie taught, sauing onely there is a certayne accompt or reconing to be made before the application of the numbers to the figure, whereof the next Chapiter shall minister examples sufficient.

Examples of the vulgar Golden rule as it is single, directly, and with an accompt proceeding.

Chapter. 9.

When all the 3. termes required to the opperation of the Golden rule, are not playnely and distinctly geuen and named in any question, then doth the same rule necessarily require a certayne accompt or reckoning before it, whereby is found out such of the sayd 3. termes as seemeth to want, or not to be sufficiently expressed, forthough they be not distinguished and clearely named in the question, yet are they after a sort confusedly (and as it were obscurely) contayned in the giuen numbers, as for example, take this question.

26. A question of Treit allowed in the 100. weight.

If 100. l. weight suttle of any kinde of merchandize be worth 12. l. what is 987. l. suttle woorth, allowing 4. l. weight vppon euery 100. for Treit.

Here the numbers are not foorthwith to be applied to the figure as they are giuen and named, for the first tearme is not here playnely and perfectly expressed, but for the finding thereof, here is to bee considered, that in this allowance 104. l. is deliuered and accompted but for 100. l. wherefore this 4. l. allowance must be added to the 100. l. to make the first tearme, and then I may applie the numbers to the figure, saying thus, if 104. l. be worth 12. l. what shall 987. l. be worth, & so working by the rule as I did before in the other questions, the quotient answereth 113. l. $\frac{11}{13}$. which fraction reduced to coyne, maketh 17. s. 8. d. $\frac{4}{13}$ d. as here the fabrique sheweth, and so much are the 987. l. woorth.

weight.	money.
104. l	12. l
987	113 l. 17 s. 8 d. $\frac{4}{13}$

So likewise in this question, a Dutch falconer solde a Gosshawke to a Gent. for 15 Crownes, wherein he perceiued wel that he had gayned clearely $\frac{1}{3}$. of his principall or price which the hawke cost him, now the question is, what his principall was?

27. A question of an Hawke.

or

or what this hawke cost the Fawlckoner?

Here before any numbers can be applied to the Golden rule, the ⅓ gayned by the Fawlckoner, must of neceſſitie be added to his principall price (that ſo it may make the firſt terme in that rule.

Euma. How can you adde the gaynes to the principall, ſith you know not his principall, for if you knew that, the queſtion were anſwered.

Phi. Though the certaine price of the hawke, or ſum of this fawlckoners principall be knowen in money, yet you may gather by the 22. verſe, that if the ſame principall (what ſoeuer ſum it be) were diuided into halfes, thirds, quarters, or any other parts, then ½ or ⅓ or ¼ &c. thereof, muſt needes be equall to the whole ſum of the principall, for it is an infalible Axiom in nature, that.

250. As all the partes of any thing, is equall to the whole, } 205

So is the whole to all his partes, if it be taken ſole.

Euma. I muſt needes confeſſe this principle to be ſo certayne and true, that it is not poſſible to be falſified.

Phi. Very good, then thereby I enter into this conſideration that if this principall were ⅓ and that by the ſale of his hawke, he hath gayned ⅓ more as the queſtion importeth, that then hath he receiued for her in all ⅔.

Euma. That cannot be denyed.

Phi. But you know the queſtion ſaith, he ſold her for 15 crow.

Euma. That is very true, wherefore it ſeemeth that ⅔ of his principall is 15. Crownes.

Phi. Do you perceiue that, why then now after this reckoning made, come to the golden rule with your numbers (making that your firſt terme thereof) ſaying thus.

If ⅔ of any ſum be 15. crownes, what is ⅓ thereof, and ſo working by the Golden rule of fractions direct, the anſwere bringeth 9. Crownes, as here the fabrique thereof ſheweth.

15. cr. 2 — 3 — 9. cro.

Wherefore ſo much did the hawke coſt the fawlckoner, and ſo much was his principall.

Or if you will vſe a ſhorter way, you may by the 173. verſe reiect the baſſes of the antecedentes, ſith they be equall, and take but their numerators onely as Integers, ſaying thus.

If 2 ſignifie 15. Crownes, what doth 3. ſignifie, and the rule anſwereth 9. Crownes ſtill euen as before, as here you ſee by the fabrique.

2 — 15. cr. / 3 — 9. cro.

Now

Now when he sold the hawke for 15. crownes which cost him but 9. crownes, he gained 6. crownes cleerly, which is iust $\frac{2}{3}$ of 9. crowns, his principall as the question importeth.

Eu. So I see both wayes come to one effect, but the latter is the more easie and briefer.

Philo. Agayne, two wayfaring men, departed from Chichester together, the one Eastwardes towardes Rie, the other westward toward Excester, but not with equall paces: For he that goeth towards Rie, iourneth euery day but 3. miles, but he that trauaileth towardes Excester iourneth euery day 5. myles. Now the question is how many myles these 2. trauaylers are distant, the 9. day after their departure. 23. A question of Iourneying.

In the former questions, the first terme was it which seemed to want, because the same was not distinctly expressed. But here the second terme is that which must be found by accompting the miles which both these traueylers are distant by one dayes iourney before any numbers can be applied to the figure. Wherefore adding 3. miles, the iourney of the one, to 5. miles the iourney of the other, the totall is 8. miles, for 1. dayes iourney of them both.

And now this reckoning being made, I may applie the numbers to the rule framing the question thus.

If 1. dayes iourney make 8. miles distance, how many miles distance shall 9. dayes make? and so working by the rule as before in the other questions, the quotient answereth day. miles. 72. miles as here the figure sheweth.

1	8
9	72

Eu. This I vnderstand very well, & perceiue that such an accompt must necessarily precede the rule by reason.

Phi. Then another question is thus, a marchant of Chichester bought at Lisborne in Portingale 800. l. weight of spices, which cost him there at the first peny 400. ducats, the freight home cost 36. ducats, the costum at Lisborne and Chichester cost 24. ducats, and his costs and expences in the voiage cost 28. ducats, now the question is, how this marchant may sell a pound of these spices, so that he may gayne 160. ducats cleare by the voiage. 29. A question of Spices.

Heere also as you see, is an accompt or reckoning to be made what the totall charges of the spices, together with the gaines required will amount vnto, thereby to make the second terme in the golden rule before the giuen numbers can be applied to the same rule, for the 800. l. weight of spices cost not onely 400. ducats

cats, but it cost 400. + 36. + 24. + 28. ducats, all which added together, with 160. ducats for the appointed gayns, amounteth to 648. Ducats, as here appeareth.

400
36
24
28
160
648

And so now by making this accompt, the totall charge of the 800. l. of spice (which must be the second tearme in the Golden rule) being knowne to be 648. Ducats, now I say I may aptly applie the numbers to the figure saying thus, if 800. l. weight must yeelde 648 Ducats, what must 1. pound yeeld, and so working by the rule, the quotient deliuereth $\frac{648}{800}$ Duc. as here the figure sheweth.

800. l. ⟌ 648. duc.
1. l. ⟌ $\frac{648}{800}$ duc.

And therefore for so much must he sell euery pound of his spices to gayne 160. duc. by the voiage, now what that is in english money, you may easily tell, by exchanging reduction, esteeming euery Ducat at 5. shillings english.

Euma. Then seeing euery ducat is valued at 5. s. I will endeauor first to change $\frac{648}{800}$ duc. into shillings, which to performe, I multiplie 648. by 5. (as the 152. verse willeth me) and so the product is 3240. which I diuide by 800. and the quotient bringeth 4. $\frac{4}{10}$ that is to say, 4. shillings, and $\frac{4}{10}$ or $\frac{1}{10}$ of a shilling, and now to change the same $\frac{1}{10}$ of a shilling into pence also, I multiplie 1. by 12. (because there are so many pence in 1. shilling) and the product is 12. which I diuide by 20. & the quotient is $\frac{12}{20}$ that is $\frac{12}{20}$ of a penny, which by abridging reduction is in least tearme $\frac{3}{5}$ d. and so the merchant must sell his spices for 4. s. $\frac{3}{5}$ d. a pound, to gayne after the rate aforesayd.

30 A question of gaynes in the 100. by sale of silkes.

Phi. You say true, now another question is thus, a merchant selling certayne silkes for 350. l. gayned by the sale of them after the rate of 10. pound vppon euery 100. of his principall disbursed, now I demaund what his principall was.

Here is first this reckoning to be made, that he that gaineth 10. l vppon the 100. l. maketh 110. l. of euery 100. l. wherefore adding the 10. l gayned to the 100. l principall, to make the first antecedent or number proportionall I applie the nũbers to the rules saying thus. if 110. l. principall & gaynes come of 100. l. principall of whom commeth 350. l. principall and gaynes, and so working by the rule the quotient deliuereth 318. $\frac{2}{11}$ l. as here the fabrique expresseth.

110 l. ⟌ 100. l.
350 ⟌ 318. $\frac{2}{11}$.

Which if you list now to subtract from the 350. l. which he made of his silkes, the remayner must needes shew his cleere gaynes.

Euma.

Euma. Then by the glosse of the 163. verse, I take 1. Integer from 350. r. and breake it into $\frac{11}{11}$ so the remainder as it appeareth is thus. $349\frac{11}{11}$. from which now I subtract $318.\frac{2}{11}$ and there remayneth $31\frac{9}{11}$ r. for his cleare gaines, as the subtraction it selfe here sheweth.

tot.	$349.r.\frac{11}{11}$
aba.	$318.\frac{2}{11}$
rell.	$31.\frac{9}{11}$ r.

And here I perceiue that the firname of the antecedents in this question, is pounds of principall and gaynes, and the firname of the consequentes is but pounds of principall onely.

Philo. You haue noted it well, it is euen so, and here as you see that the accompts proceeding the rule in all the former questions were made onely by Addition, so now you shall haue questions wherein the accompt proceeding is made by Subtraction, whereof this is the first.

31. A question of gaynes by sale of Kerseyes

A merchant bought 30. Kerseies for 70. r. which he sold againe for 80. r. now if the same 30 kerseis had cost him 80. r I demaund how he should haue sold them agayne, to gayne after the former rate in his stocke or principall.

Here the first and third tearmes are playnely expressed, but the second is giuen confusedly in the third, wherefore to distinguish it, it is necessarie that 70. r (being the principall stocke) be first subtracted from 80. r the principall and gaynes, that so the cleare gaynes may appeare by it selfe, which being done, there remaineth 10. pound for the cleare gaines, as the worke of the subtraction here sheweth.

80. r.
70
10

Wherefore now applying the numbers to the rule, I order the question thus, saying, if 70. r. principall principall gaynes get, 10. r gaynes, what shall 80. r. principall get, & so working by the rule, the quotient answereth 11. $\frac{1}{2}$ r. as here ỹ fabrick sheweth.

70. r. 10. r.
80 11. $\frac{1}{2}$

Wherefore, if the Kerseyes had cost him 80. r. he must sell them agayne, for 80 r + 11 $\frac{1}{2}$ r. to gayne after the former rate that is for 91 $\frac{1}{2}$ r.

Euma. Indeede I confesse that this proceeding accompt is very necessarie, as this question is wrought, but with your fauour me thinks I can answere the same question, by the very plaine rule of 3. without any such accompt proceeding.

Phi. I graunt you so, for the order of this worke is but for examples sake, but yet I couet much to see, what order you would take to worke this question without any such accompt proceeding.

Euma.

cats, but it cost 400. + 36. + 24. + 28. ducats, all which added together, with 160. ducats for the appointed gayns, amounteth to 648. Ducats, as here appeareth.

And so now by making this accompt, the totall charge of the 800. l. of spice (which must be the second tearme in the Golden rule) being knowne to be 648. Ducats, now I say I may aptly applie the numbers to the figure saying thus, if 800. l. weight must yeelde 648 Ducats, what must 1. pound yeeld, and so working by the rule, the quotient deliuereth $\frac{648}{800}$ Duc. as here the figure sheweth.

400	
36	
24	
28	
160	
648	

800. l. ⟌ 648. duc.
1. l. ⟌ $\frac{648}{800}$ duc.

And therefore for so much must he sell euery pound of his spices to gayne 160. duc. by the voiage, now what that is in english money, you may easily tell, by exchanging reduction, esteeming euery Ducat at 5. shillings english.

Euma. Then seeing euery ducat is valued at 5. s. I will endeauor first to change $\frac{648}{800}$ duc. into shillings, which to performe, I multiplie 648. by 5. (as the 152. verse willeth me) and so the product is 3240. which I diuide by 800. and the quotient bringeth 4. $\frac{4}{5}$ that is to say, 4. shillings, and $\frac{4}{5}$ or $\frac{1}{5}$ of a shilling, and now to change the same $\frac{1}{5}$ of a shilling into pence also, I multiplie 1. by 12. (because there are so many pence in 1. shilling) and the product is 12. which I diuide by 20. & the quotient is $\frac{12}{20}$ that is $\frac{3}{5}$ of a penny, which by abridging reduction is in least tearme $\frac{2}{5}$ d. and so the merchant must sell his spices for 4. s. $\frac{2}{5}$ d. a pound, to gayne after the rate aforesayd.

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Here is first this reckoning to be made, that he that gaineth 10. l vppon the 100. l. maketh 110. l. of euery 100. l. wherefore adding the 10. l gayned to the 100. l principall, to make the first antecedent or number proportionall I applie the nũbers to the rules saying thus. if 110. l. principall & gaynes come of 100. l. principall of whom commeth 350. l. principall and gaynes, and so working by the rule the quotient deliuereth 318. $\frac{2}{11}$ l. as here the fabrique expresseth.

110 l. ⟌ 100. l.
350 ⟌ 318. $\frac{2}{11}$.

Which if you list now to subtract from the 350. l. which he made of his silkes, the remayner must needes shew his cleere gaynes.

Euma.

Euma. Then by the glosse of the 163. verse, I take 1. Integer from 350. r. and breake it into $\frac{11}{11}$ so the remainder as it appeareth is thus. 349 $\frac{11}{11}$. from which now I subtract 318. $\frac{2}{11}$ and there remayneth 31 $\frac{9}{11}$ r. for his cleare gaines, as the subtraction it selfe here sheweth.

tot.	349. r. $\frac{11}{11}$.
aba.	318. $\frac{2}{11}$
rest.	31. $\frac{9}{11}$ r.

And here I perceiue that the sirname of the antecedents in this question, is pounds of principall and gaynes, and the sirname of the consequentes is but pounds of principall onely.

Philo. You haue noted it well, it is euen so, and here as you see that the accompts proceeding the rule in all the former questions were made onely by Addition, so now you shall haue questions wherein the accompt proceeding is made by Subtraction, whereof this is the first.

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Here the first and third tearmes are playnely expressed, but the second is giuen confusedly in the third, wherefore to distinguish it, it is necessarie that 70. r. (being the principall stocke) be first subtracted from 80. r the principall and gaynes, that so the cleare gaynes may appeare by it selfe, which being done, there remaineth 10. pound for the cleare gaines, as the worke of the subtraction here sheweth.

80. r.
70
10

Wherefore now applying the numbers to the rule, I order the question thus, saying, if 70. r. principall principall gaynes get, 10. r gaynes, what shall 80. r. principall get, & so working by the rule, the quotient answereth 11. $\frac{4}{7}$ r. as here ỳ fabrick sheweth.

70. r. / 10. r.
80 / 11. $\frac{4}{7}$

Wherefore, if the Kerseyes had cost him 80. r. he must sell them agayne, for 80 r + 11 $\frac{4}{7}$ r. to gayne after the former rate that is for 91 $\frac{4}{7}$ r.

Euma. Indeede I confesse that this proceeding accompt is very necessarie, as this question is wrought, but with your fauour me thinks I can answere the same question, by the very plaine rule of 3. without any such accompt proceeding.

Phi. I graunt you so, for the order of this worke is but for examples sake, but yet I couet much to see, what order you would take to worke this question without any such accompt proceeding.

Euma.

Euma. That shall you see by and by, For I would say no more but thus, if 70.l. stocke, growe to 80.l. stocke and gaynes, whereto will 80 l. stocke growe? and so working by the rule, the answere wilbe 91. $\frac{3}{7}$ l. agreeable with your former conclusion, as here appeareth.

stock	stock & gaynes
70. l.	80. l.
80	91. $\frac{3}{7}$ l.

Phi. You sayd well, and I cannot but greatly commend your promptnes and dexteritie herein, but you cannot do so in all like questions, as for example take this question following.

32. A question of gaynes on the 100. by sale of Cloues.

I haue 160. l. of Cloues, which cost me 6. s. 8. d. a pound, whereof I dayly retayle parcell at 7. s 6. d the pound, now I demaund, what I gayne vppon the 100 l. of my stocke, after this rate?

Here if you will frame the question in such order as you did the next before saying, if $\frac{1}{3}$ l. stocke (for so much is 6. s. 8. d.) growe to $\frac{3}{8}$ l. in stock and gaines (that is to 7. s. 6. d.) whereto shall $\frac{100}{1}$ l. stocke growe? & so will proceede to worke by the rule immediately, you shal finde the answere to be 112. $\frac{1}{2}$ l. as here the fabricke thereof sheweth.

stock	stock & gaine
$\frac{1}{3}$ l.	$\frac{3}{8}$ l.
$\frac{100}{1}$	112 $\frac{1}{2}$ l.

Which answere doth not satisfie the question demaunded vntill by Subtraction, the 100. l. stocke be seperated from the 112 $\frac{1}{2}$. wherein both the principall stocke and gaynes lye confusedly as it were shuffled together, wherefore when the same 100. l. principall is subtracted from 112. $\frac{1}{2}$ l. which containeth both the stocke and gaynes thereof. Then there remayneth 12. $\frac{1}{2}$ l. for cleare gaynes vppon the 100.

Note that often times, there is an after reckoning to be made either by addition or Subtraction, succeeding the golden rule, aswell as proceeding

And thus you see that if the Subtraction go not before the rule then it must succeede it, for as it cannot be auoided, so it must not be omitted, which after reckoning happeneth so often times euen of necessitie, but not here, for this question might well be answered, by the rule with an accompt proceeding, by subtracting the bare stocke, from the stock and gaynes at the first, saying thus. $\frac{3}{8}$ l —— $\frac{1}{3}$ leaueth $\frac{1}{24}$ l. for cleare gaynes, wherefore I frame the question thus, saying, if $\frac{1}{3}$ l. stock gayne $\frac{1}{24}$ what shall $\frac{100}{1}$ stocke gaine, and so working by the golden rule of fractions, the answere byngeth 12. $\frac{1}{2}$ l. as the fabrique here sheweth, which is the direct answere to the question demaunded.

stocke	gaynes
$\frac{1}{3}$ l.	$\frac{1}{24}$ l.
$\frac{100}{1}$	12. $\frac{1}{2}$

And because the reason and forme of worke is like in losse, as it

is

is in gaynes, I pray you answer me to this question.

A marchant bought Nutmegs for 5.s.4.d. the pound, which he was forced to sell agayne for 4.s.9.d wherfore I demaund what he loseth in euery pound of his principall.

Euma. Here I perceiue that the first and third termes are both playnely expressed, but the second is geuen more obscurely, wherefore to follow your order for finding the same cleerely, fyrst I subtract 4. s. 9. d. from 5. s. 4. d. and so resteth 7. d. as the cleere losse of 5 s 4. d. wherefore now I come to the Golden rule, saying thus, if 5 $\frac{1}{3}$. s principall, incur 7. d. losse, what shall 20. s. loose? And so reducing the mixt number, and working by the same rule, the operation thereof answereth 26 $\frac{1}{4}$ d. that is 2. s. 2. d. $\frac{1}{4}$. And therefore so much doth he loose, in euery pound of his principall, as the fabrique heere annexed declareth.

stock.	losse.
$\frac{16}{3}$ s	7. d.
20	$26\frac{1}{4}$

Which by further computation, I finde to be 10. l. 18. s. 9. d. losse in the 100. pound.

Philo. Because you haue done so well, you shall haue another question, wherein your selfe (I doubt not) will confesse, that an accompt must necessarilie proceede, and it is thus.

There is a water Conduct with a Cock, flowing into a Boll or Cesterne of 250. Gallons, in which cesterne is also another cocke to auoide the water, by the vppermost cock there floweth euery houre into the boll or cesterne 24. Gallons, and by the lower cock, there issueth out of the cesterne euery houre 16. gallons: now the cesterne being emptie, and both cocks set open together, I demaund of you in what time the same shall be filled. 33. A question of a water conduct.

Euma. Indeede here it is manyfest, that before any numbers can be applied to the rule, There is an accompt to be made, how much the excesse of the filling cocke surmounteth the emptieng cocke, that so the remayner may serue for the first antecedent, wherefore sith the one filleth 24. gallons in 1. howr, whereof the other in the meane season emptieth 16. gallons, therefore by subtracting the 16. gallons which runne out, from the 24. gallons which fall in to the Cesterne, it appeareth that there remayneth cleare euery houre in the Cesterne 8. gallons, aboue all that runneth out.

Which being so, I frame the question thus, saying if 8. gallons come in one houre, in how many houers will 250. gallons come? And so working by the rule, the operation thereof bringeth

31. ¼. houres, wherefore in ſo many houres the Ceſterne ſhall be filled, as heere the fabricke manifeſtly ſheweth.

gallons. houres
8 — 1
250 — 31. ¼

Philo. This was well done of you alſo, wherefore you ſhall anſwer me one queſtion more of like nature, which is this.

34. A queſtion of gaines in the 100. by ſale of Sugar.

A marchant ſold as many cheſts of Sugar as yeelded him 600. l. ready money, wherein he gayned cleare aboue his principall 60. l. Now the queſtion is, what he hath gained vpon euery 100 l. of his principall?

Fuma. The whole ſumme receiued by him, was 600. l. whereof 60. l. you ſay, was cleare gaines: therefore to know what his principall was, I muſt needs ſubtract the 60. l. gained from the 600 l. & ſo will his principall appeare diſtinctly, which being done there remayneth 540. l. for his principall, which being knowne, ſerueth for the firſt terme of the Golden rule, whereunto now applying the numbers I frame the queſtion thus.

If 540 l. principall, get 60 l. gaines, what doth 100 l. principall get? and ſo by vſing the rule, I finde the anſwere 11 ⅑ l. as heere the figure expreſſeth.

principall. gaines.
540 l. — 60 l.
100 l. — 11 ⅑. l.

Wherefore I ſay, he gained ſo much vpon euery 100. pound.

Philo. Your anſwer is true, and now I will propound a queſtion or 2. which I will alſo anſwer my ſelfe, for your better inſtruction, wherein you ſhall ſee the accompt or reckoning proceeding the Golden rule, to be made both by addition and ſubtraction, and it is thus.

35. A queſtion of a Tower.

There is a Tower whereof ⅛ is vnder the earth, ⅕ vnder water, and the reſt appearing in ſight is 54. foot, now the queſtion is, how many foote is vnder earth, and how many vnder water.

An vnit repreſenteth any thing entirely, and when the Numerator and denominator are equall: then doth ſuch fraction returne to an vnit, by the 21. verſe.

For anſwer whereto, firſt I vſe addition, ſaying (by the 131. and 133. maximes) ⅛ + ⅕. makes ¹³/₄₀. Secondly I ſubtract the ſame ¹³/₄₀. (being the totall of all that is vnder the earth and water) from ⁴⁰/₄₀. which repreſenth the entire Tower, and ſo remayneth ²⁷/₄₀. as the very reſidue of the Tower appearing in ſight, which by the propoſt of this queſtion is 54. foote, wherfore this reckoning being thus made, and the firſt terme proportionall thereby found, now I come to the golden rule with my termes, framing the queſtion thereto in this manner ſaying.

If ²⁷/₄₀. be 54. foote, what is ⅛? and what is ⅕? and ſo by twice working with the rule, according to the 249. verſe, I finde,

that

that $\frac{1}{3}$. is 10. foote, and that $\frac{1}{5}$. is 16. foote, wherefore I affirme, that 10. foot is vnder the earth, & 16. foot vnder the water, as the fabriques shew.

$\frac{27}{40}$ ⟷ $\frac{54}{1}$ feete.
$\frac{1}{3}$ ⟷ 10 vnder earth.
$\frac{1}{5}$ ⟷ 16. vnder water.

Eu. So then by addition of 10. 16. and 54. feete together, it appeareth that from the foundation of the Tower to the top. is 80. foote, and it is very easie to finde, that $\frac{1}{3}$. thereof is 10. foote, and that $\frac{1}{5}$. thereof is 16. foote, euen iust as the question importeth.

Phi. Then my second question is thus, there is a certayne hall $\frac{2}{3}$. of the length of whose roofe is couered with leade, and $\frac{5}{12}$ thereof is couered with blew Slates, and the rest being 15. foote long, is couered with tyles, now I demaund how many foote long of this roofe is couered with leade, and how many with Slates? 36. A question of a Roofe.

For answer whereto, first I must make my accompt by addition as before in the last question, saying (by the 135. maxim) $\frac{2}{3} + \frac{5}{12}$ of any integer maketh $\frac{21}{12}$, secondly I subtract this $\frac{21}{12}$ from $\frac{24}{12}$, which representeth the whole integer or length of the hall, saying thus. $\frac{24}{12}$ — $\frac{21}{12}$, leaueth $\frac{3}{12}$, my first terme proportionall or portion couered with tyle, which by the contents of this question is affirmed to be 15. foote, thus sithe that by this reckoning $\frac{3}{12}$, is found to be 15. foote, now I come to the Golden rule with my numbers, framing my question thus.

$\frac{3}{12}$ ⟷ $\frac{15}{1}$. foote
$\frac{2}{3}$ ⟷ 80. foote

If $\frac{3}{12}$. of any thing be 15. foote, then what is $\frac{2}{3}$? and what is $\frac{5}{12}$? and so by twice vsing the rule as before I finde, that $\frac{2}{3}$. of the hall is 80. foote long, and that $\frac{5}{12}$ thereof is 25. foote, as heere the worke declareth. Wherefore I affirme, that 80. foote of the roofe is couered with leade, and 25. foote with Slates.

$\frac{2}{3}$ ⟷ $\frac{15}{1}$ foote.
$\frac{5}{12}$ ⟷ 25. foote

Eu. I vnderstand the order of proceeding in these questions verie well, by these your playne examples, but me thinkes that these 2. last questions, cannot aptly be said to be single proportions, because the 248. verse affirmeth, that single proportion is that which at first answers a question with one rule alone, but heere you are constreyned to vse the rule twice.

Phi. It is true that I vse the rule twice, but not to answer one question, for if you examyne the matter diligently, you shall finde that there are 2. seuerall questions conteyned in each of these latter questions, whereof as each of them require a seuerall answer, so are they seuerally answered with one onely rule, according to the prescript of the said 248. verse.

Euma. Indeede I thinke it be so as you say, for in the former question of the Tower, was demanded first what is $\frac{1}{7}$? and secondly, what is $\frac{1}{7}$? and in this present question is demaunded, first what is $\frac{1}{7}$? and next, what is $\frac{1}{17}$? wherefore I thanke you much for disoluing my scruple.

Phi. Now shall you haue a question, wherein both the antecedents are found, and distinctly expresst by the accompt proceeding, which accompt is made by subtraction and multiplication, and it is thus.

37. A question of theft.

A certaine Gentlemans seruant hauing the custodie of his maisters plate, conueyed away diuers percels thereof, and one of his maisters Geldings out of his pasture, wherewith he fled away and was gone 3. dayes, before knowledge thereof came to his maister, flyeng continually euery day 40. miles. The 4. day in the morning, the Gentleman hearing thereof, and learning by inquirie which way his man cooked, pursued after him, riding euery day 50. miles till he ouertooke him: wherefore the question now is, first in how many dayes, and secondly after how many miles riding the gentleman ouertooke his man.

For answer whereof, first I reckon thus with my selfe, the gentleman rideth in one day 50. miles, and his man rideth but 40. and then 50 —— 40, is 10. myles, the cleare gaines of one dayes iourney: but then the theefe was gone 3. dayes before his maister began to pursue him. Wherefore to know how many miles he was fled in that space, I multiply the 3. dayes of his former flight, by the 40. miles of his dayly iourney, whereby I produce 120. which argueth, that he was fled so many miles before the gentleman began to pursue him. So now by this accompt, hauing gotten my termes ready for the golden rule, I come thereto, saying thus.

If 10. miles be gained in 1. day, in how many dayes will 120. miles be gained. And so working by the rule, I finde the answere 12. dayes, wherefore I say that at 12. dayes end, the gentleman shall ouer take him, as heere appeareth by the fabrique.

miles.	dayes
10	1
120.	12

And now to know how many miles he rideth in the meane season, I vse the rule againe, saying thus, If 1. day passe 50. miles, what dooth 12. dayes passe? and the rule answereth 600. miles, as heere also appeareth.

dayes.	miles
1	50
12	600

Whereby I say, that the gentleman rode 600. miles before he ouertooke his man, and thus haue you had sufficient

questions

queſtions and examples of the Golden rule, ſingle, directly, as well without an accompt, as with an accompt proceeding: wherefore now I will ſhewe you the practiſe of the ſame rule, backward or conuerſedly.

Of Proportionalitie conuerſed, otherwiſe called The Golden rule, backward, conuerſed, or reciprocall. Chap. 10.

251. The Golden rule backward or conuerſt,
Placeth the termes as doth the rule direct:
But then it foldes the firſt two termes rehearſt. } 206
Diuiding the product got by that effect.
Not by the firſt, but onely by the third,
So is the product the fourth at a word.

Euma. Heere I vnderſtand by this verſe, that in this rule, the numbers muſt keepe the ſelfe ſame places ſtill, that they do in the Golden rule direct, ſo that the number whereon the queſtion dependeth, muſt ſtill be the third, keeping the left angle of the figure, and the Matcheterme thereof muſt be at the head, and ſo conſequently, the diſlike terme muſt take the right angle.

Philo. You ſay very true, onely this is the difference, that whereas in the direct rule you multiplyed the 2. anguler numbers together, diuiding their product by the firſt antecedent. Heere you muſt folde together the 2. vppermoſt termes, for they are the firſt 2. viz. the firſt antecedent and his conſequent, and then muſt diuide the product thereof by the third number or latter antecedent, and ſo take the quotient for anſwering number. to ſet at the taile of the figure, as for example, I put this queſtion.

38. A queſtion of lone or Intereſt.

I lent my friend 40. l. freely 9. moneths, vpon his promiſe that he would doe aſmuch for me at my requeſt: but when I required the performance of his promiſe, he could lend me but 12 l. wherefore I demaund how long he ought to forbeare his 12 l. to make me iuſt recompence for my former curteſie?

Now heere the order of the queſtion is eaſilie framed thus. If 40. l. were forborne 9. moneths, how long ſhall 12. pound be forborne? but now 12. l. being the third number, and leſſer then 40 l. his proper match terme, doth yet looke for a greater portion of time, or number of moneths, then 40 l. doth, for reaſon willeth that I muſt haue my friends 12 l. longer then he had my 40. l.

And therefore by the 201. and 243. verſes, this proportionalitie

is conuerſed, ſo that looke in what proportion my 40 l. exceedeth his 12 l. in like proportion ought my time or number of moneths to exceede his: and therefore, as 40 l. to 12 l. is Triplaſeſquitertia, ſo ought the number of my moneths to be Triplaſeſquitertia to his 9. moneths, which thing the practiſe of this rule perfectly performeth, for the termes being knit to the figure in their due places, as heere you ſee them.

40. l. — 9. months
12 —

I multiply (as this rule imporeth) 40. and 9. the 2. firſt or vppermoſt together, whereby I produce 360. which I diuide by 12. the third terme, and ſo iſſueth 30. in the quotient, which I place at the figures taile, as aunſwer to this queſtion thus.

40 l. — 9. months.
12 — 30.

auouching that my friend ought to forbeare his 12. pounds, 30. moneths, for as 40 is to 12. directly, antecedent to antecedent, ſo is 30. to 9. conuerſedly, conſequent to conſequent, that is to ſay, in proportion Triplaſeſquitertia.

Euma. I ſee very well that the termes are in ſuch proportion together as you ſay, whereby the anſwer cannot be vntrue.

Philo. Then let me ſee how you can anſwer me to this queſtion.

39. A queſtion of the weight of bread.

If when Wheate is ſolde for 12. ſ. a quarter, the halfepeny white loafe ought by ſtatute to weigh 13 ½. ounces, what ought the white loafe of like price to weigh by the equitie of the ſame ſtatute, when Wheate is ſolde for 20. ſ. the quarter?

Euma. Surely I take 20. ſ. heere to be the latter antecedent, becauſe the queſtion conſiſteth therein, and then muſt 12. ſ. being his matche-terme, be the firſt: and ſo of force the 13 ½ ounces, muſt be the ſecond number, and ſettle at the right angle of the figure, but yet I cannot expreſſe this queſtion in apte woordes to my minde.

Philo. You may ſay thus. If 12. ſ. affoorde 13 ½ ounces for ob. what will 20. ſ. affoorde for ob. ? applyeng your numbers ſo to the figure: and heere naturall reaſon teacheth you, that 20. ſ being the greater antecedent, will not affoorde ſo much bread for ob. as 12. ſ the leſſer antecedent doth, for the greater the price of wheate is, the leſſer ſhall be the weight of the bread. For which cauſe this and ſuch like queſtions, are anſwerable onely by the Golden rule conuerſed.

Euma. Indeed I perceiue the reaſon very well, and the better for the 243. verſe, and your large expoſition thereof, wherefore now to proceede with the practiſe thereof, I firſt reduce the mixt

numbers

number into ſimple, and next I breake the ſhillings. ounces
Integers by ſubſcribing 1. and ſo applye
the termes to the figure thus.

And now, as this preſent verſe importeth, I multiply the 2. firſt vppermoſt numbers together, to wit, $\frac{12}{1}$ by $\frac{61}{5}$, and the product is $\frac{732}{5}$, which I diuide by $\frac{20}{1}$ (the third number) and ſo iſſueth in the quotient $8\frac{12}{25}$. which I ſet at the figures taile ſhillings, ounces
as the anſwer to the queſtion thus. 12 $\frac{61}{5}$
Whereby I conclude, that if 12 s. affoorde 20 $8\frac{12}{25}$
$13\frac{1}{5}$. ounces of bread for a halfepeny. Then 20. ſhillings will affoorde but $8\frac{12}{25}$. ounces for an halfepeny.

Philo. Your concluſion is good and true.

Euma. Then I pray you let me ſee the practiſe of the queſtion which you put before, which was this. If 6. buſhels of Otes ſerue 4. geldings 6. dayes, how many dayes will the ſame ſerue 8. geldings? 40. A queſtion of the expences of prouender.

Philo. Go too. geldgins. dayes

Eu. Then after that I haue placed the numbers in the figure in this manner. 4 6 / 8

I enfold 4. and 6. the two firſt numbers together, whereby I produce 24. which diuided by 8. quoteth 3. wherefore annexing the ſame 3. to the taile of the figure, as anſwer to the queſtion, I conclude, that if 6. buſhels of Oates ſerue 4. geldings 6. dayes, then the ſame quantitie of Oates will ſerue 8. geldings. dayes
geldings but 3. dayes, euen as you there 4 6
affirmed, and the fabrique here ſheweth. 8 3

Philo. Well ſayd, but now before I go any further, I will ſhew you ſuch a likeeaſie forme of operation in this rule, whē any of the termes are true fractions, or improper fractions, as I ſhewed you before in the 249. verſe, touching the Goulden rule direct, and it is thus.

252. If in the Golden rule conuerſt,
The termes be fraction like expreſt,
Of labour to excuſe it,
Folde the laſt denominator,
In each others numerator,
Your diuidend inſues it.
Then folde you thoſe three termes beſide,
Which reſt as yet vnmultiplied,
And as diuiſor vſe it. 207

Euma: So that whether the termes be fractions properly or improperly, yet such a diuidend and diuisor is found by this rule, as the quotient of their diuision is alwayes the true answering number sought for.

41. A question of a penny weight.

Phi. It is so, wherefore I aske you this question, if when 1. ounce of sterling siluer was worth $\frac{1}{12}$. of a pound in money, 1. peny starling weighed 32. graines. what ought 1. d. sterling to weigh when the ounce sterling is worth $\frac{1}{2}$ of a pound in money.

Eu. For answer, first I breake the integers making them fractionlike with the rest, and so apply the numbers to the figure thus, saying if $\frac{1}{12}$ r. in money, allow $\frac{32}{1}$. graines to a peny, what will $\frac{1}{2}$ r. allow? and now working after the prescript of this present rule first I fold 4. the last denominator in 1. and 32 the numerator of the other 2. termes, and so I produce 128. for my diuidend.

siluer. graines
$\frac{1}{12}$ r $\frac{32}{1}$
$\frac{1}{2}$ r

Secondly, I fold or multiply together 12. 1. and 1. the other 3. termes remayning vntouched before, and they produce 12. for my diuisor, then thirdly diuiding this diuidend 128. by this diuisor 12. I finde the quotient to be 10. $\frac{2}{3}$, wherefore I conclude that when an ounce of sterling siluer is worth $\frac{1}{4}$ r. or 5. s. in money (as it is at this day) then a sterling peny ought to weigh 10 $\frac{2}{3}$. graynes as the worke heere sheweth.

money. graynes
$\frac{1}{12}$ r $\frac{32}{1}$
$\frac{1}{4}$ —— 10 $\frac{2}{3}$.

42. A question of the weight of a bottle of Haye.

Philo. Againe, if when a loade of Haye is worth 13 $\frac{1}{3}$ s. the halfepeny bottle weyeth 4 $\frac{1}{2}$ r. I demaund what the like bottle shall weygh when the loade of Haye is worth 20. s.

Euma. To answer this, after the reduction of the mixt numbers to simple, I apply them to the figure thus, which done, I multiply 40. 9. & 1. together and they produce 360. my diuidend.

s. r.
40 9 1

Next that, I multiply 3. 2. and 20. together, and they produce 120. my diuisor, so the diuision perfourmed, there issueth 3 r. in the quotient for the aunswering number, wherefore I conclude that when the loade of Haye is worth 20. s. the halfpeny bottle ought to weigh 3 r. after the former rate, as heere the fabrique expresseth.

$\frac{1}{2}$ s. $\frac{2}{3}$ r.
3 r.

43. A question of lining Veluet with Taffata.

Philo. You haue done well, but yet once I must oppose you with this question. How much Spanish Taffata of $\frac{3}{4}$ yarde broade, will serue to line 4. yardes of Veluet of $\frac{1}{2}$ of a yarde broade?

Eu. This is as much to say, as how much $\frac{1}{2}$ broade will line 4.

yardes

yardes of ¼ brode, wherein as I may perceiue that theſe woordes (quarters brode) notifie the matchtearmes, ſo may I alſo perceiue that ¾ is the third tearme or latter antecedent (becauſe it includeth the queſtion) which being leſſer then ¾ the firſt antecenent, doth therefore require the greater number of yardes, whereby it followeth by the 243. verſe that this queſtion muſt be aunſwered by the Golden rule conuerſed, wherefore firſt breaking the Integers, and ſetting euery tearme in his proper place of the figure as here you ſee.

yar. broad	yar. long
¼	¾
¾	

I do as this verſe preſent biddeth me

¼	¾
¾	

that is, I multiplie 4. the laſt denominator by 3. and 4. the numerators of the other fractions, wherby I produce 48. for my diuidend which done, I multiplie together 4. 1. & 2. which are the other 3. termes remayning yet vnmultiplied, whereby amounteth 8. for my diuiſor. finally diuiſion being executed there iſſueth 6. yardes in the quotient for the anſwering nũber as here appeareth.

yar. broad	yar. long
¼	¾
¾	6

or ſis becauſe the denominators of both the antecedents are equal, I may reiect them both vterly, and woorke with their numerators onely as with Integers by the 173 verſe, and ſo the quotient is 6. yardes ſtill as it was before, as here appeareth alſo by that kinde of woorke.

quarters.	yardes.
3	4
2	6

Whereupon I conclude that 6. yards of Spaniſh Taffatie of ½ yarde broad, will ſerue to face or line 4. yards of veluet of ¾ of a yard.

Philo. Thus as you ſee there is no difficultie in the woorking of this rule conuerſed, So that you be very circumſpect that you miſtake not in vſing it, when you ſhould vſe the rule directly, nor contrarily, for ſo may you greatly deceiue your ſelfe, and incurre manifold errors.

Demonſtration Arithmeticall.

And to the entent you ſhall perceiue ſome part of the reaſon of this conuerſed or backward woorking in this rule, you ſhall vnderſtand, that as in the Golden rule direct, the 3. firſt proportionals are alwayes giuen, and the 4. onely is that which wanteth, & is ſought for by that rule, ſo in this rule conuerſed the 3. laſt proportionals, are alwayes the very numbers giuen, and that which wanteth, and is ſought and found by the operation thereof, is truely and naturally the firſt in proportion direct, although as the order of the rule requireth, becauſe it is laſt found and is the anſwering nũber, it is placed laſt, and ſet at the tayle of the figure.

As

As for example, take the 3. giuen numbers of the laſt queſtion, as you wrought it with Integers, which are 3. 4. and 2. and were ſetled in the 3. firſt places of the figure there thus.

3 Z 4
2

As though they were the 3. firſt proportionalls, and that the 4. were lacking, but take them I ſay, and place them in the 3. latter places of the figure thus.

Z 3
4 2

And now ſhall you ſee that it is the firſt terme or number proportionall that wanteth in proportion direct, wherefore by the 94. and 100. verſes, and by my demonſtration of the golden rule directly fol. 118. The product of the 2. anguler numbers, which are the 2. meanes, being diuided by any one of the extreames ſhall quote out the other.

And therfore 12. (being the product of 3. and 4.) deuided by 2. the latter extreame, quoteth 6. the firſt extreame, which is the vnknowne number ſought for, who beeing ſetled in his proper place, now maketh the proportion to be direct as here you ſee. For as 6. is to 3. ſo is 4. to 2. that is in proportion Dupla.

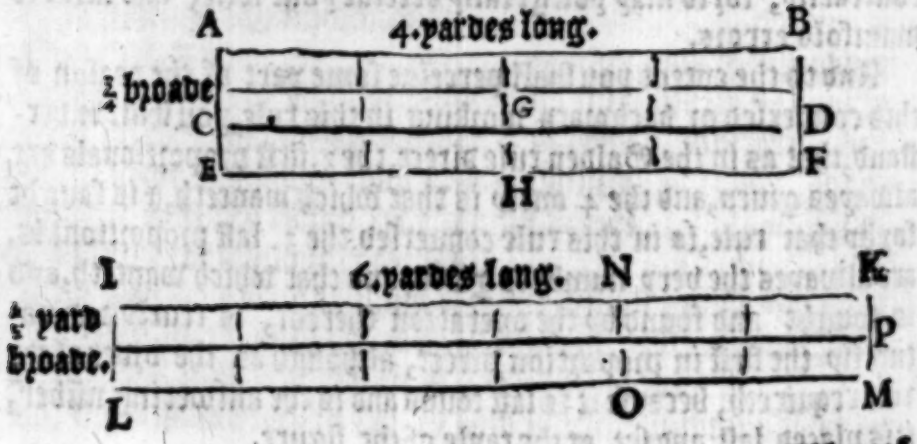
6 Z 3
4 2

Euma. Truly this reaſon is very apparant now, and hereby I perceiue that the 2. vpper numbers in this backward rule, are the 2. meanes, which in the direct rule are the 2. angular numbers, and therefore when they are multiplied together, and their products diuided by the other number remayning, the quotient muſt needes be the number wanting.

Demonſtration Gometrical.

Philo. Becauſe you conceiue it ſo readily, I will alſo linearily demonſtrate the certaynte of this rule vnto you, which I do thus.

Let A. F. and I. M. be 2. paralelograms or long ſquares.



wherevnto

Whereof let A. F. which is ſuppoſed to be ¼ of a yarde broade and 4. yards long, repreſent the yards of veluet, & let I. M. which is ſuppoſed to be ¼ yard broad, & 6. yards long, repreſent the 6. yardes of Taffitie, now the Areas or playnes of A. F. and I. M. are equall as you ſee, for let ¼ and 4 yardes the ſides of A. F. be multiplied together and the product ſhalbe 3. likewiſe, let ¼ and 6. yards the ſides of I. M. be multiplyed together, and the product ſhall alſo be three.

Euclid 1 common ſentence.

Wherefore by Euclid his firſt common ſentence, ſith each of theſe paralelograms are equall to 3. therefore they are equall together, and yet to make the matter more playne, let I. M. be cut of in the line N. O. and layd vpon A. B. C. D. and you ſhall ſee the one couer the other iuſtly, then take N. P. and O. P. and lay the one vppon C. H. and the other vppon G. F. and you ſhall ſee each of them couer the other as iuſtly alſo, wherefore by Euclids 8. common ſentence, their quantities are equall.

If diuers things be ſeuerally, each equal to ſome other. Then are each of thoſe diuers things one equall to another.

Ergo 6. yardes of Taffatie of ¼ yard broad, ſhall iuſtly line or couer 4. yards of Veluet of ¼ of a yarde broad, for looke in what proportion the Veluet exceedeth the Taffatie in breadth, in ſuch proportion doth the Taffatie exceed the veluet in length, that is in proportion Seſquialtera.

Euclid 8. common ſentence. If quantitie or quãtity in al parts ſo agree. That no exceſſe nor want appeare, thẽ equall ſure they be.

En. This demonſtration is ſo manifeſt both to ſence & reaſon, that nothing can be playner, for euery child may vnderſtand it.

Philo. Many other queſtions, might be propounded aunſwerable by the backward golden rule, which I omit here, not onely becauſe theſe may ſuffice for examples and preſidentes in all like caſes, but alſo becauſe I ſhall haue new occaſion to entreate more of this rule hereafter, when I ſhall teach you the golden rule plurall and artificially compounded.

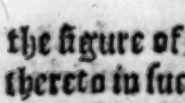
But neuertheleſſe this I muſt not omit to tell you, that in this rule ſometimes both the antecedents, are comprehended and included in one, as thus, I am demaunded what is ⅔ of 15. now here the 2. termes of this fraction ⅔. are the 2. antecedents in conuerſt proportion, ſo that in applyeng theſe numbers to the golden rule, the numerator ſhalbe taken for the firſt antecedent, & the denominator for the latter, and then you know that 15. the other number remayning, muſt needes be the firſt conſequent, which termes being multiplied and diuided as the rule requireth, quoteth the number which bringeth the true anſwer.

Ramus lib 2 chap. 4. Satig. lib. 2. chap. 6. verſe 4.

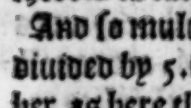
44. A queſtion of fractions.

Euma. Then to declare how I conceiue your ſpeach, I deſcribe the

the figure of the Golden rule, setting the tearmes thereto in such sorte as you sayd, which is thus.



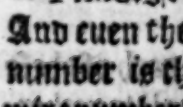
And so multiplying together 4. and 15. the product is 60. which diuided by 5. bringeth 12: in the quotient for the answering number, as here the fabrique expresseth, wherfore by your saying, 12. is $\frac{4}{5}$. of 15.



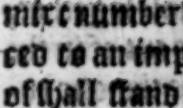
45. Another question touching fraction.

Philo. It is euen so vndoubtedly, you vnderstand me very well. And euen the like order is to be obserued if I should aske you, what number is that which conteyneth 15. 3 tymes and $\frac{3}{4}$. for here the mixt number which is referred to tymes, to wit $3\frac{3}{4}$. being once reduced to an improper fraction thus $\frac{15}{4}$, forthwith the numerator thereof shall stand for the first antecedent, and the denominator for the latter, whereby 15. being the other number remaining, must needs be the first consequent, so that now the question may be vttered thus, what number is that which containeth $15\frac{15}{4}$ of a time, and so vsing the rule, the quotient sheweth the number desired.

Fuma. For triall thereof, I will once more describe the figure with the numbers in such sorte as you say, which is thus.



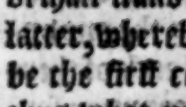
And now folding 19. in 15. I finde the product 285. which I diuide by 5. and the quotient is 57. wherefore 57. is the number which containes 15. $3\frac{3}{4}$ times as here appeareth.



Phi. It is certainely so, for 3. times 15. you know is 45. and the very last question made proofe that $\frac{3}{4}$ of 15. is 12. so then 45 + 12. is iust 57. agreeable with the former operation.

46. Another question touching fractions.

And so likewise if you be asked this question, how many twelue parts are there in $\frac{3}{4}$. you must take the 2. tearmes of the fraction for the 2. antecedentes, and the other number for the first consequent, and so working by the rule conuersed, you shall finde that in $\frac{3}{4}$ there are $\frac{9}{12}$, as heere the fabrik of the work euidently expresseth.



And so heere I will cease to intreate any more of this Golden rule conuersed or reciprocall, and now will go in hand with the Golden rule plurally, and the examples thereof.

Of Plurall Proportionalitie, otherwise called The Golden rule plurally. Chap.11.

253 Plurall proprotion, is that which requires.
the Golden rule wrought, more often then once,

Be-

Before it can answere, to all your desires,
Such questions are put sometime for the nonce,
VVhich rules are sometime all wholy direct,
And sometimes in part conuersedly pact. } 208

This verse hath no obscuritie in it, for it sheweth you playnly that plurall proportion (or the Golden rule plurall) is when by the difficultie of the question propounded, the vse of the Golden rule, is required more then once, before a full solution to the question demaunded can be obtayned.

And further, that when it is so often times to be vsed, then by the entent of this verse, it is sometime wrought altogether directly, and sometime one proportion is wrought directly and another conuersedly, but you shall neuer finde them altogether conuersed.

Euma. Verie good, I suppose I vnderstand the meaning of this verse sufficiently, but I pray you shewe me some examples therof.

Phi. So will I anon, but first I must tell you, that whereas some other men, commonly call this proportion multiplex, I haue thought good to name it plurall, for a difference from that multiplex proportion which is defined by the 189. verse, and that not without reason.

For when the principall terme aunswering fully to any question propounded, cannot be found or explicated by one single proportionalitie, that is to say, by vsing the Golden rule once onely, but requireth the vse of the same rule sometimes twice, and sometimes more often, sometimes altogether directly, and sometimes one rule directly, and another conuersedly, before the last terme of the principall proportionality, which concludeth the question, and yeeldeth the aunswer can be found, then doe I for that cause name such proportionality plurall.

Eu. If the vse of the sayd rule, were neuer required aboue twice in any one question, me thinks it might well be named double proportion, or the double Golden rule.

Phi. Indeede as you say some men do name it the double rule, but because the vse of the rule is many times required, not onely twice, but thrice, and paraduenture more often, aboue the solution of one question, therefore I haue chosen the word plurall, as that which comprehending all the rest, is more agreeable to the nature of the thing, and beside that, there are few questions answerable by double proportion onely, which may not be much easilyer wrought

and

and dissolued by the Golden rule compound, wherefore I will vse the fewer examples thereof here, and will spend the more tyme about them when we come to the chapter of proportion artificially compounded by multiplication. And first nowe for an example of this rule, I propound this question.

47. A question of the first price of an Elle of Holland.

If in 1. Elle of Holland sold for 2.s.10.d. payable at the end of 3. Monethes, there was gained after the rate of 25. in the 100. l. for 12. Monethes, what did that 1. Elle of Holland cost the seller.

Euma. I pray you aunswere this question your selfe, for it passeth my cunning, neyther know I how to dispose the numbers to the figure, for it seemeth to me that 1. Elle of Holland is the number wherin the question lieth, and therfore should be the latter antecedent, but then I see no other number of sirname like to it, nor possible to be reduced to like sirname.

Phi. Indeede this is a good crabbed question for a learner, but here you must consider, that when he that selleth this 1. Elle of Holland, receiueth 2.s.10.d. for it, he doth not receiue the same all as cleare gaynes, nor yet all as his bare principall.

Euma. That is very true, for the same 2.s.10.d. contayneth both his principall paid for that elle, and also the gaines gotten by the sale therof, wherfore if we could declare certainly how much thereof is principall onely, and how much is cleare gaynes onely, the question were soone answered.

Philo. You haue sayd truth, but to do that you must begin here first with the time, framing your question thus, saying if 100. l. stocke in 12. monethes bring 25. l. gaynes, what gaynes will 100 l. stocke bring in 3. monethes, and so

monethes	gaines
12	25. l.
3	6 ¼ l.

working by the Golden rule direct, you shall finde the answere to be 6 ¼ l. as here appeareth.

Euma. So here I perceiue that although 100 l. stock be twice mentioned in the question, yet you vse it not at all in the figure, as any necessarie number pertayning to the operation.

Philo. No that is true, for sith it is as it were common and indifferent to both the antecedentes, it may be indifferently omitted in both places, and so may any like number that is common as indifferent to both consequentes, and therefore if I will, I may (without the mentioning the 100. l at all) say thus.

If 12. moneths bring 25. l. gaynes, then 3. moneths shall bring 6 ¼.

$6\frac{1}{4}$ gaynes, where notwithstanding the 100.l. stocke is twise vnderstood, for neyther the 12. monethes, nor the 3. monethes can bring any gaynes without the stocke.

Euma. That must needes be granted as agreeable to reason.

Philo. Then as you see, I haue vsed the Golden rule here once alreadie about this question, and yet am I nothing nigh the answere that I seeke, for as yet I haue found no more but that 100.l. stocke in 3. monethes, will bring $6\frac{1}{4}$ l. of gaynes, after the rate of 25.l. in the 100. for 12. monethes, which stock and gaines must now be added both together, & so they both being confusedly contayned and shuffled together in one sum, the totall may be of sirname, qualitie, & nature, like to the price of the 1. Elle of cloth sold (to wit principall and gaynes) which Addition being made accordingly, the totall is $106\frac{1}{4}$ l.

Wherefore now going about agayne the second time, I take this totall for the first tearme in the Golden rule, saying thus, if $106\frac{1}{4}$ l. stocke and gaynes together, came of 100. l. bare stock in 3. monethes, of what bare stocke came 2.s. 10.d. stocke and gaynes together in 3. Monethes.

Or els thus, (not mentioning the 3. Monethes at all, because they are common and indifferent to both the consequents) if 106. $\frac{1}{4}$ l. stocke and gaynes. come of 100. l stocke, of what stocke came 2 s. 10. d. stocke and gaynes, in which latter forme of speech, the time of 3. Monethes is twice to be vnderstood silently.

Euma. This I trust I conceiue well inough now, and therefore if it please you, I will worke out the second rule.

Philo. Go to, let me see what course you will take therein.

Euma. Then before I proceed, as I must of force reduce the mixt number to one simple fraction, so I thinke good to make the integers fractionlike by the 175. verse, and then to conuert the 2.s 10. d. into partes of a pound as here you see I haue done, for so now the tearmes being all fractionlike, I may worke by the golden rule of fractions direct, wherein to proceed, I fold 4. the first denominator in 100. and 34. the numerators of the other 2. tearmes, whereby I produce 13600. for the diuidend, the next I folde 425. the top tearme of the first in 1. and 240. being both the balles of the rest, and so haue I 102000. for my diuisor.

$\frac{425}{4}$ l. $\frac{100}{1}$ l. Z $\frac{34}{34}$

And now because the Diuidend is lesse then the Diuisor, the quotient shall be a Fraction by the 78. verse, wherefore

I worke the diuision (as the glosse of that verse willeth) onely by putting a line betweene them thus. $\frac{1.1500}{1.100}$.

Which being done I abridge the same fraction immediately first by the glosse of the 161. verse in this maner, $\frac{1.15|00}{1.1|00}$, and next by the same verse it selfe, I bring it in least termes to $\frac{3}{17}$ l. which I anner to the figures of the rule for answering number thus.

$\frac{425}{4}$ l. $\frac{100}{1}$ l.
$\frac{14}{140}$ $\frac{3}{17}$

And so now it appeareth, that the cleare stocke which brought the $\frac{14}{140}$ l. or 2.s.10.d. both for stocke and gaynes of the Elle of Holland was $\frac{3}{17}$ pound.

Philo. You haue proceeded very well and orderly in this worke and concluded truly, wherefore now declare what $\frac{3}{17}$ l. is in money or knowne coyne, for so shall you know what the same elle cost the seller.

Euma. That can I doe, eyther by exchanging Reduction, or by the Golden rule conuerst, but because I would haue further practise of the Golden rule, I will rather vse that at this time, wherein I see it is all one thing to say what is $\frac{3}{17}$ of 1. pound, or what is $\frac{3}{17}$. of 20.s. or what is $\frac{3}{17}$ of 40. Testerns, or what is $\frac{3}{17}$ of 60. Grotes, or what is $\frac{3}{17}$ of 240. pence.

But because the answere will returne in such sirname or denomination as the question demaundeth, which I iudge best to be in pence as the subtils of least denomination, therefore I say I will demaund the question in pence onely, saying thus, what is $\frac{3}{17}$. of 240.d. and so working by the rule in such forme as you taught me but in the last Chap. I finde it to be 32.d. as heere the fabrique sheweth.

2 ⟷ 240.d.
15 ⟷ 32.d.

Philo. So then by these 2. seuerall executions of the rule, you see at the last, that the principall stocke or price which this 1. Elle of Holland cost the seller, is found to be 32.d. or 2.s.8.d. which elle when he sold for 2.s.10.d. payable at 3. Monethes, he gayned 2.d. thereby cleare in that time, whereupon I aske you this question.

48. A question of gaynes vpon lone pound in 12. monethes.

If 2.s.8.d. or 32. pence stock, in 3. Monethes gayne 2.d what will 100. l. stocke gayne in 12. Monethes.

Euma. I will assay how I can answere you, and therefore beginning with the time as you did, I frame my question thus.

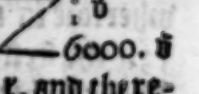
If 3. Monethes gayne 2.d. what will 12. Monethes gayne, and so obseruing the rule, I finde that 12. Monethes will gayne 8.d. as here appeareth.

Mon. gaynes.
3 ⟷ 2.d.
12 ⟷ 8.d.

Both

Both which gaynes is vnderstoode to rise by the 32. ů. stocke, wherefore now seeing I know what gaynes 32. ů. stock will bring in 12. Monethes, I come to the rule the second time saying thus, if 32. ů gayne 8. ů. in 12. monethes, what will 100. ŕ. stock gayne in 12. Monethes, or els (as you sayde before) because the 12. Monethes are common and indifferent to both the consequentes, I vterly omit the mention of them, saying onely thus.

If 32. ů. stock bring 8. ů. gaynes, what gaynes shall 100 ŕ. bring, and so reducing the numbers as before, I applie them to the golden rule, whose operation performed, bringeth for answer 6000 ů. as here the fabricke expresseth, which being reduced to pounds, maketh iust 25. ŕ. and therefore I affirme, that if 32. ů. stock gayne 2. ů. in 3. moneths, then 100. ŕ. stock (after that rate) shall gayne 25. ŕ. in 12. moneths.



Philo. You haue done well, and heereby you may perceiue that the answer of the last question next before is verified and confirmed by the solution of this question.

But heere for ease in worke, you shall vnderstand for a generall rule, that in all questions of gaynes or losse vppon the 100. pound, whether they be without tyme, or with equall tyme, if the numbers be integers, they may be applied to the golden rule, as numbers meerely abstract, vntill you haue wrought the rule, and then may you annexe their propper sirnames to each of them againe, and so may you auoide much labour spent about the reduction of the termes.

Prouided alwayes, that of whatsoeuer sirname each of the antecedents are, of such sirname must you accompt his propper consequent to be, so that in this case aswell the antecedents as the consequents, may be of diuers sirnames, but yet each couple shall continue matchtermes still, either in relation to stock or to gaynes, or els to losse, as the case requireth.

As for example, I will new worke the second operation of the rule of this present question after this fourme, saying onely thus. If 32. gayne 8. what shall 100. gayne? and so following the order of the rule, it answereth me 25. as heere appeareth by the fabrique.

Now if I will imagine 32. the first antecedent heere, to be pounds, then must I take his consequent 8. to be pounds also, but if I take the same 32 to be shillings, gr oates, or pence, then such

as I take it to bee, such must I esteeme 8. his consequent to bee also.

Likewise, if I reckon 100. the latter antecedent to be pounds, thē must I also accept 25. his consequent to be pounds also, but if I take the same 100. to be but shillings, grotes or pence, then so must I esteeme 25. his consequent to be also, but yet the 2. antecedents heere represent stock onely, and the consequents represent onely gaynes, wherefore in that respect they cease not to be matcheermes still as I sayd before, wherefore in the figure here present, sith I take 32. (the first antecedent) to be pence as they are indeede, therefore I take 8. his consequent to be also pence, and so they are indeede, wherefore I annexe the sirname of pence vnto them, and agayne because I take 200. the latter antecedent to be pounds (as they truly are) therefore I accompt. 25. his proper consequent to be pounds also, and so they truly are, wherefore I haue annexed the sirname of pounds to each of them, as you may see by the former fabrique.

stock	gaynes
32 d.	8 d.
100 l.	25 l.

Euma. Indeede this abridgeth a great part of the labor that I tooke, and reason teacheth me that this order of worke is true, for when 32. d. stock, bringeth 8. d. gaynes, then 32. grotes stock, shall bring 8. grotes gaynes, and 32. s. stock shall bring 8. s. gaines, and 32. l. stock 8. l. gaines and so consequently when 32. d. stock bringeth 8 d. gaynes, then 100. l. stock shall bring 35. l. gaynes, for they are all proportionall, and the like I conceyue of losse.

49. A question of interest, and of the principall lent.

Philo. Your conceipt is good, and now partly commeth to my minde another question which is this. A certayne man desired to borrow a sum of money of his friend, promising to giue him interest after 10. in the 100. for a peare, vntill he repayed it, which when he had obtayned, at 10 weekes end, he payd his sayd friend 183 ½ l. in full discharge, aswell of the principall, as of his interest, after the rate aforesaid. Now therefore I demaund, what was the principall sum of money lent?

Euma. This question is much like the first question of this chapiter in my opinion, wherfore I thinke it must haue the like forme of working.

Philo. You gesse well at it, wherefore proone how you can answer it accordingly.

Euma. Then I begin with the time, as you did in that question, saying, If 52. weekes, (for so I reckon 1. yeere) require 10 l. for inte-

interest, what will 30. weekes require: and weekes. interest.
so vsing the rule, it aunswereth $5\frac{10}{13}$ l. as 52 ∠ 10 l.
heere the fabrique sheweth. 30 ∠ $5\frac{10}{13}$.

Both which interests growe of 100 l. stock, viz. the 10 l. in 52. weekes, and the $5\frac{10}{13}$ l. in 30. weekes.

And now to imitate your former worke, I must adde the 100 l. stock, to the $5\frac{10}{13}$ l. (the interest of the 30. weekes,) that so both the stock and interest being shuffled together, the totall thereof may be of like sirname and nature to the totall sum repaid, (which is also said to conteyne both the stock and interest) and also become fit to make the first terme of the next proportionalitie.

Which being done, I frame my second question thus, saying, if $105\frac{10}{13}$ l. stock and gaines, come of 100 l. stock, of what stock commeth $183\frac{1}{3}$ l. stock and gaines? and so stock & gain. stock
reducing the mixt nũbers to simples, I set $\frac{1375}{13}$ ∠ $\frac{100}{1}$ l.
them to the figure, where they shew thus. $\frac{55}{3}$ ∠

And here I would faine ease my selfe of some labour, after your late instruction, but I cannot see any meanes how to do it in this question, because the antecedents are of such sirnames as represent neither pounds, nor shillings, nor pence. Wherefore I proceede by the ordinary Golden rule of Fractions direct, taking the product of 13. 100. 550. which is 715000. for my Diuident, and the product of 1375. 1. and 3. which is 4125. for my diuisor, and so performing the diuision, I finde the aunswering number to be 173 l. $\frac{1125}{4125}$. or in lesser termes $173\frac{1}{3}$ l.

Wherefore it seemeth that so much was the principall sum of money lent, but how may I bee sure thereof? for these questions are so intricate, that I cannot perceiue when I doe well or ill.

Philo. When I haue done with this plurall proportionalitie, I will shew you, how you shall examine and trie all workes, generallie proceeding from the golden rule, and in the meane season assure your selfe, you haue answered the question well inough.

Wherefore you needc not say it seemeth, but may conclude for certeinty, that the principall sum lent, was stoc. & gain. stock.
$173\frac{1}{3}$ l. annexing it boldly vnto the figure $\frac{1375}{13}$ l. ∠ $\frac{100}{1}$ l.
in his due place, as I haue done heere. $\frac{55}{3}$ ∠ $173\frac{1}{3}$.

50. A question touching treit allowed vpon the 100. waite

But now what can you say to this question?

A marchant solde Almonds vnto 2. chapmen, severally to eache of them 572. l. suttle, for 3 l. 6 s. 8 d. the suttle 100. wherein

each of them craning allowance for Treit, he bad them choose whether they would haue 4.r. ouer weight vpon euery 100. or els to be abated the price of 4.r. in euery 100. weight.

Whereupon the first Chapman chose the allowance of 4. r. weight in the 100. But the second desired to be abated 4.r. in the 100. r. Now I demand what each of these Chapmen ought to pay, and which of them made the better choise?

Euma. Why I see no difference betweene their paiments, nor yet diuersitie in their choise seeing they had both like weight, and both at like price, for their allowance seemeth to me all one.

Philo. But you shall finde it otherwise, if you weigh the matter wel, for you must consider, that he which is allowed 4.r. in the 100. taketh 104 r. but for 100. and he that is abated 4.r. in the 100. taketh 100. but for 96.r. and therefore touching the first mans debt, I frame the question thus. If 104 r. be accompted but 100 r. what shall 572. be accompted? and the rule answereth me 550 r. as heere the fabrique sheweth.

weight.	weight.
104 r.	Z 100 r.
572	Z 550

Then secondly I say. If 100. pound weight, cost $\frac{10}{1}$ r. what will 550 r. coste? & the operation of the rule answereth $18\frac{4}{13}$ r. That is 18 r. 6 s. 8 d. as heere appeareth.

weight.	money.
$\frac{100}{1}$ r.	Z $\frac{10}{1}$ r.
$\frac{550}{1}$	Z 18 r. 6 s. 8 d

Now for the second mans debt, I frame the first part of the question thus, saying if 100. r. be taken but for 96 r. for how much shall 572. r be taken, and the rule answereth $549\frac{3}{13}$ r. as heere appeareth.

100 r.	Z 96. r.
572	Z $549\frac{3}{13}$.

This done, I come to the second proportionalitie, saying thus. If 100 r. cost $\frac{10}{1}$, what shall $549\frac{3}{13}$ r. cost? and the answere is, $18\frac{38}{130}$ r. which maketh in money by reduction 18 pounds, 6. shillings, $\frac{24}{130}$ d. as the fabrique heere sheweth.

weight.	money.
$\frac{100}{1}$	Z $\frac{10}{1}$ r.
$\frac{5497\frac{3}{13}}{1}$	Z 18 r. 6 s. $\frac{24}{130}$ d

Whereby as their seuerall payments are made manifest, so it appeareth, that the second Chapman which tooke the abatement of 4 r. in the 100. made the better choise, for that he saued $7\frac{1}{130}$ d. of that which the first paide, for the first paid full 18 r. 6 s. 8 d.

51. A question touching gaines vpō the 100. pound by Salte sold.

Euma. Indeed now heere appeareth a difference, and further I perceiue heereby, that it is better for the seller to giue 4 r. to the 100. for Treit, then to abate 4 r. in 100.

Phil. Well then I proceede to another question, which is thus. A marchant comming to a poyte with a ship of Salte, solde parcell

thereof

thereof for 8. d. a bushell, wherein he gained after the rate of 10. in the 100. but afterwards seeing his market very quick, he raised his price, selling the rest at 10. d. a bushell, wherefore it is demanded, what he gained then in the 100?

Now for answer, heere is first such like consideration to be had, as was vsed in the 47. question touching the ell of Holland, that is to say, heere is to be considered, that when this marchant selleth a bushell of salt for 8. d. he receiueth not that 8. d. as cleare gaines onely, but partly as his principall, and partly as the gaines gotten by his principall after the rate of 10. in the 100.

Wherefore to know how much of this 8. d. is principall, and how much thereof is gaines. First I adde the 100 l. stock to the 10. l. gaines, so is the totall 110. Then say I thus, if 110. stocke and gaines, growe of 100. stocke, of whome groweth 8. stocke and gaines? and working by the golden rule, stock & gain, stock I finde it to growe of $7\frac{1}{11}$ stocke, as heere the figure sheweth.

110 l. — 100 — 8 — $7\frac{1}{11}$.

So that his stock disbursed as the price of a bushell of Salte is $7\frac{1}{11}$ d. for the which when he receiued 8. d. he hath in cleere gaines $\frac{1}{11}$ d. for $7\frac{1}{11}$ d. + $\frac{1}{11}$ d. make iust 8. d.

Thus his cleare stock or principall disbursed for 1. bushell being seperated and knowne from the gaines gotten in the sale thereof. Now thereby I seeke what he gaineth on the 100. when he raiseth his price to 10 d. a bushell, vsing the rule the second time, saying thus. If $7\frac{1}{11}$ stock, encrease to 10. in stock and gaines, whereto shall 100. stock encrease? and the opperation of the rule aunswereth me $137\frac{1}{11}$, as heere the fabrique expresseth.

stock. stock & gain

$\frac{10}{11}$ — $\frac{10}{1}$

$\frac{10}{1}$ — $137\frac{1}{11}$.

Wherefore sith this $137\frac{1}{11}$ conteineth both the stock & gaines of the 100. I subtract the same 100 stock, from the $137\frac{1}{11}$. and so resteth $37\frac{1}{11}$. for cleere gaines, wherefore I say, that this Marchant getteth so much in the 100. when he rayseth his price to 10. pence, as is aforesaid.

Or els if I will, I may worke the subtraction, before I attempt the rule the second time. Saying, 10 d. —— $7\frac{1}{11}$ d. leaueth $2\frac{1}{11}$ d. (for when I am assured by the first vse of the rule, that all his principall disbursed for 1. bushell of salt is but $7\frac{1}{11}$ d. which when he selleth againe for 8. d. his cleare gaines gotten in that principall is $\frac{1}{11}$ d. It followeth that when he selleth a bushell for 10. d. (which is 2 d. aboue the former price) his cleare gaines then gotten, by

disbursing the same 7 1/17. d. is 2 1/17. d.) whereuppon I may now repayre to the rule agayne saying thus, if 7 1/17. stock bring 2 3/11. gaynes, what gayns shall 100. stock bring? and so the mixt numbers being reducted, and the rule executed, the stock. gaynes aunswere is immediately 37. 1/11. as the fabrike heere declareth.

8.d. — 110
10. — 137 1/11

Wherein although I haue expressed the termes partly in improper fractions, to the end you might the easelier conceiue the figure of the rule with his numbers according to the position of the question, yet in the very worke I did reiect the equall basses of the first antecedent and his consequent, according their tops or numerators onely as integers, & so multiplying 30. by 100. I produced 3000. which diuided by 80. forthwith quoted 30. 1/2. that is 1/2.

Or thirdly I may (by a yet shorter way) worke ỳ question, saying thus, if 8 d. stock & gaynes yeeld 110. r. for 100. r. what will 10. d stock and gaynes yeeld for 100. r. and so at one single operation the golden rule aunswereth 137. r. 1/11. as here appeareth, from which if you subtract the 100. r. stock, then will the gaynes shew it selfe to be 37 1/11. on the 100. pound.

Euma. You haue dilated the order of proceeding in this question (sundry wayes) so sensibly and plainly, that I cannot choose but conceiue it, insomuch that I trust I am now able my selfe to answer any like question, and although the second way seemeth not fully so easie at the first, as the former, yet doe I like it better, for that it bringeth a direct answer to the question immediatly. And touching the omission of the equall basses of the improper fractions, I see you had good reason so to doe by the 173. verse, for as 12 is to 10, so is 80. to 30.

52. A question of a ship of come, and of losse by the sale thereof.

Philo. You haue said well, and because you haue so good hope of your owne abilitie, I put you this question. A ship arriued at Lisborne in Portingall with 200. quarters of wheate, whereof the marchant sold the one halfe for 4 1/2 vingtins the alkere.

Euma. What meane you by an alkere and a vingtin.

Philo. An alkere is a certeine Portingall measure, whereof 21. or thereabouts makes 1. quarter or 8. bushels English, and a vingtin is a Portingall coine, worth an English trip, or peece of three pence.

Euma. Very good. Now I pray you proceede as you were about to do.

Philo.

Philo. The marchant perceiued well, that in the sale of his Wheate after this price, he lost cleare 9. in the 100. of his principall, which the corne, with the freight, licence, custome, and other charges cost. Neuertheless the market grew dayly worse & worse. so that he was faine to sell the other halfe for 4. ringgins an alkere. Now I demaund what he lost in the 100. by his latter price?

Euma. For answer heereto, I gather thus much by reason, that like as when the question was of gaines gotten vppon the 100. the totall both of stock and gaines ioyned by addition, made the first terme in the golden rule. So by congruitie of reason now the question is of losse (whereby the stocke is diminished,) the remainer of the stocke after subtraction of the losse, ought to make the first terme, for as he which gaineth 10. in the 100. maketh 110. of his 100. so he that looseth 9. in the 100. maketh but 91. of his 100.

Philo. Your reason is very good. Wherefore proceede accordinglie.

Euma. Then subtracting the 9. lost, from the 100. stock, the remainer is 91. so that now, as the whole 100. is decreased to 91. by the losse of 9. therein, so likewise euery particuler alkere solde, is partaker of this losse. and beareth a parcell thereof in the sale, for finding of which particuler losse, I frame my question thus, saying. If 91. be remainer of 100. of whom is 13. d. the remainer? or thus. If 91. were 100. before the losse, what was 13. d. before the losse? (for so I esteeme the $4\frac{1}{7}$ ringgins by your former valuing of them.)

And by working with the golden rule, I finde it to be $14\frac{2}{7}$ d. as heere appeareth.

91 | 100
13 | $14\frac{2}{7}$

Wherefore I conclude, that euery alkere standeth the marchant in so much, which when he selleth for 13. d. he loseth thereby cleerely $1\frac{2}{7}$ of his principall.

Philo. Well said, hetherto you haue done very well.

Euma. Then to proceede, when he is driuen to abate $\frac{1}{7}$ of a ringgin that is, 1. penny more in euery alkere, then it is manifest he looseth $2\frac{2}{7}$ d. of his principall, which the same alkere cost him, which being knowne, I come now to the golden rule the second time, saying. If $14\frac{2}{7}$ of the stocke loose $2\frac{2}{7}$. what dooth 100. stocke loose? And so reducing the mixt numbers to simples, and working by the rule with them, either as fractions as Integers (as I may, because the denominators of the first antecedents and his

consequent are equall) both wayes the aunswer is 16. Wherefore I affirme that he loseth 16. in the 100. by his latter price, as here the fabrique expresseth.

stock. losse.
100 ⁄ 7 Z 16 ⁄ 7
10 ⁄ 1 Z 16

Or thirdly abating the losse heere (which is 9. r.) from the 100. r. stock, I may say thus. If 4 ⁄ 13 vingtins stocke and losse, yeeld but 91. r. for 100. r. what shall 4. vingtins (stocke and losse) yeeld? and the aunswer is 84. r. as heere you see it, which subtracted from the 100. r. stock sheweth that there is lost 16. r. vpon the 100. also, agreeable with the other workes.

4 ⁄ 13 Z 91. r.
3 ⁄ 12 Z 84. r.

53. A question of losse by the whole ship lading

But put the case I were demaunded what this marchant lost cleere by his whole lading, I pray you how should I answer that? Philo. Very easily now, for sithe the charge of 1. alkere is already knowne to bee 14. ⁄ 7 d. you seeke next by the golden rule, what 1. quarter English, which contayneth 21. alkeres did cost him, saying if one alkere cost 14. ⁄ 7 d. what doth 21. alkers cost? and the rule will aunswere you 300. d. or 25. s. as here appeareth wherefore you may conclude, that euery quarter standeth the marchant in so much, whereby is soone seene that the whole 200. quarters standeth him in 250. r.

alkeres. pence
7 ⁄ 1 Z 100 ⁄ 7
21 ⁄ 1 Z 300

Note.

And here note how for ease in working, I described the vnit (which is the first antecedent) thus 7 ⁄ 7, that so it being made of like denomination to his consequent, I might performe the rule onely, by the operation of the numerators, which kinde of practise, I vse also in finding the price of the 200. quarters, wheate. money wherein I say thus, if 1. quarter cost 4 ⁄ 1 r. what 200 quart. cost, the rule presently aunswereth 250 r. as y fabrike annexed here sheweth.

wheate. money
4 ⁄ 1 Z 5 r.
200 ⁄ 1 Z 250 r.

Now this being done it behoueth to know how many alkers are in 100. quarters, which by reduction of grosse to subtils, is forthwith found to be 2100. alkers, & those sold at 13. d. the peece, yeeld 27300. d. which by reductiō from subtils to grosse are 113. r. 15. s.

Then the other 100. quarters which containe other 2100. alkers sold but at 12. d. the peece yeeld so many shillings, that is by reduction to grosse 105 r. now 113. r. 15 s + 105. r maketh 218. pound 15. shillings, which subtracted from 250. pound, his principall, sheweth the cleare losse of the whole lading to be 31. pound, 5. shillings.

Whereof I inferre this proofe, first sith his whole stock is knowne

knowen to be 250. l. let one halfe thereof that is 125. l. be allowed for the price of one of the 100. quarters, and the other halfe for the other 100. quarters, and ſuch by the firſt ſale, he loſt 9. in the 100. and by the ſecond ſale 16. in the 100. I ſay thus.

If 100. loſe 9. what loſeth 125. and againe for the ſecond price, if 100. loſe 16. what looſeth 125. and ſo ſtocke. loſſe. working for both by the golden rule, I finde 100 ⟋ 9. l. that by his firſt ſale he loſt clearely 11 ½. 125 ⟋ 11 ½

And by the ſecond 20. l. as here the ſeuerall ſtocke. loſſe. fabrickes declare, and now you knowe that 100 ⟋ 16. l. 11 ½ l. × 20. l. is iuſt 31. l. 5. s. agreeable with 125 ⟋ 20 my former concluſion.

Euma. It is ſo, and as this proofe is very certayne, ſo is your note of eaſie working aſwell worthy to be noted and remembred.

Philo. Well hetherto, you haue had examples of plurall proportionalitie, or of the golden rule plurall, wherin the queſtions haue bene concluded & aunſwered by twiſe vſage of the ſame rule onely, but now for your further inſtruction, there ſhall follow other queſtions, wherein you ſhall ſee how the ſame rule is practiſed more often, before the principall propoſition or aunſwere to the queſtion can be concluded. The firſt tearme of which principall propoſition is ſometimes found by Addition, & ſometime by Subtraction, and ſometimes by both, but I will beginne firſt with the finding of it by Addition, becauſe Addition naturally goeth before Subtraction, and this is the firſt queſtion.

54. A queſtion touching iourneying.

Eu. Two footemen, viz. A. and B. ſet forth together, A. from London, to Yorke, who ends his iourney in 5. dayes, and B from Yorke to London, who finiſheth his iourney in 3. dayes, their affayres being done, they both begine to returne backe after their former paces on monday at none, now the queſtion is, when they ſhall meete, and where.

Euma. I pray you aunſwere this queſtion your ſelfe, that I may ſee the maner thereof.

Philo. Content ed, and firſt touching A. I reaſon with the golden rule thus, if 5. dayes paſſe 1. iourney, what dayes iourney doth 1. day paſſe, and it aunſweres me ⅕ as here the figure ſheweth. 5 ⟋ 1 1 ⟋ ⅕

Then for B. I ſay likewiſe, if 3. dayes paſſe 1. dayes. iourney, what doth 1. day paſſe, and it aunſwereth me ⅓, as here alſo appeareth. 3 ⟋ 1 1 ⟋ ⅓

and

And so it is manifest that betweene them both, there is passed $\frac{1}{2}$ of the iourney in 1. day, which added together maketh $\frac{1}{1}$ the first terme of the principall proportion.

Then come I to the rule the 2. time saying thus, if $\frac{1}{2}$ of the iourney require 24. houres (for so I accompt 1. day) what will $\frac{1}{1}$ (that is the entire iourney) require, & it answereth 48. houres as here appeareth.

Wherefore I conclude that they shall meete 24 houres after their seuering fourth, which is on Wednesday at 9. of the clocke in the morning; from Monday noone till Wednesday at 9. of the clocke are iust 45. houres.

$\frac{1}{2}$	24
$\frac{1}{1}$	48

Now to know where they should meete, or at how many miles passage. I question thus with the Golden rule, if A passe $\frac{1}{2}$ of the iourney in 24. houres, what doth he passe in 45. houres. four. houres (which is the time of their meeting) and it answereth me $\frac{1}{2}$ as appeareth.

24	$\frac{1}{2}$
45	$\frac{1}{4}$

This done I lay againe, if B passe $\frac{1}{2}$ of the iourney in 24. houres, what doth he passe in 45. houres (the time of houres iourney meeting) and the Golden rule answereth me $\frac{1}{4}$ as here also appeareth.

24	$\frac{1}{2}$
45	$\frac{1}{4}$

Whereby I conclude that at $\frac{1}{2}$ of A. his iourney, and at $\frac{1}{4}$ of B. his iourney, they shall meete.

For triall whereof let the distance certayne betweene London and Yorke be supposed to be 180. miles, which being so, tell me now, what is $\frac{1}{2}$, and what is $\frac{1}{4}$ of 180. parts A miles

Eu. That can I tell you quickly by the Golden rule, conuerted, wherein the termes being orderly placed, and the rule twice performed, I finde that $\frac{1}{2}$ of 180. miles is 67. $\frac{1}{2}$ miles, and that $\frac{1}{4}$ thereof is 112 $\frac{1}{2}$ miles, as here by fabricks sheweth.

8	180
5	112 $\frac{1}{2}$

parts B miles

Philo. Then may I bolde to affirme that at their meeting, A hath gone 67. $\frac{1}{2}$ miles of the iourney, and B. 112 $\frac{1}{2}$ miles, which added together make iust 180. miles the very sum of the iourney.

Eu. That should argue the worke to be good & the answere true.

55. A question of the euacuation of water.

Phi. It doth so, now another question is thus, Cesterne hauing 3. cockes emptieth it selfe cleerely at the first onely in $\frac{1}{4}$ of an houre, at the second in $\frac{1}{2}$ an houre, and at the third in one houre iust, now if all the cockes should be opened together, I demaund of you in what time this Cesterne would be emptied.

Euma.

En. I will attempt the answer howsoeuer I speed, for performance whereof I begin with the first cock saying thus.

If ½ houre emptie one Cesterne, what will one houres. Cestern houre empty? and the golden rule answereth me 4. Cesternes as here the work declareth

½ houres. 1
½ 4

Then for the second cock I say likewise, if ⅓ houre emptie one cesterne, what will one houres. 1 Cesterne houre emptie? and it answereth, 2. cesternes as here appeareth: and as for the third cock, is affirmed already to emptie one cesterne iustly in 1 houre, so that I may easily gather, that al these 3. cocks running together, would emptie 7. cesternes in 1. houre, or 1. cesterne 7. times in an houre, for 4 + 2 + 1. is 7. which I take for the first terme of my principall proportion, which shal conclude the answere, saying thus, if 7. cesternes emptie themselues in 1 houre, in what time will 1. cesterne emptie it selfe, and it answereth in ⅐ of an houre as appeareth.

⅓ houres. 1
1 7

Wherefore I affirme that all these 3. cocks running together, the cesterne shalbe emptie in ⅐. of an houre.

Philo. This was well done of you and more then I beleeued you could haue done so perfectly and orderly, but sith you are so ripe in these questions, you shall haue another which is this. Foure Carpenters viz. A. B. C. D. being talked withall about the framing of a Curious roofe, & the time wherin it might be finished A. sayd he would vndertake to finish it in 1. yeere, B. required 2. yeeres to do it, C. 3. yeeres, and D. would not vndertake to end it in lesse then 4. yeeres, now if all 4. should be hired to frame this roofe together, in what time would it be finished.

16. A question of Carpenters worke and the time therto requisite.

Fuma. I like my former order of worke so well that I minde to follow it againe in the answering of this question, for me thinks they are not much vnlike.

And therefore touching A. I question with the golden rule, thus, saying, if A. finish 1. roofe in 1. yeere what will be yeers A roofe finish in 4 yeeres, and the rule answereth me 4. roofes as heere appeareth.

yeers A roofe
1 1
4 4

Then for B. I say likewise if 2. yeere finish 1. roofe, what will 4. yeeres finish, and I am answered 2. roofes as here appeareth.

yeers B roofe
2 1
4 2

Againe for C I say likewise if 3. yeere finish

finish 1. roofe, what shall 4. peere finish, and it aunswereth $\frac{1}{4}$ or $1\frac{1}{4}$ of a roofe, as here also appeareth.

peeres B. roofes $\frac{3}{4}$ Z $1\frac{1}{4}$

And as for D. he is alreadie affirmed by the question, to performe the roofe iustly in 4. peere, wherby it is euident that all these 4. Carpenters working together, would finish $8\frac{1}{3}$ roofes in 4. peere, for $4 + 2 + 1\frac{1}{3} + 1$. is iust $8\frac{1}{3}$.

Wherefore I take the same $8\frac{1}{3}$ for the first tearme of my principall proportion which shall conclude the aunswere saying thus, if $\frac{25}{3}$ of a roofe require 4. peeres to finish it, what time will $\frac{1}{3}$ of a roofe require, (for by $\frac{1}{3}$ is represented the entire roofe peeres roofe) and so working by the golden rule, the aunswer is $\frac{4}{25}$ of a peare, as heere appeareth.

$\frac{1}{3}$. I san vnit, by the 21. verse.

$\frac{25}{3}$ Z 4 $\frac{1}{3}$ Z $\frac{4}{25}$

Wherefore I conclude, that if all these 4. Carpenters should worke vpon this roofe together, they would end it in $\frac{4}{25}$ of a peere.

The same otherwise.

Philo. You haue done very well and truly, and yet that you may perceiue the liberalitie of this liberall science, you shall see it yeeld the selfe same answere by another varietie of worke which is thus.

Fyrst I aske by the golden rule, if B. will end the worke in 2. peeres, what will he doe thereof in 1. peere, and the aunswere is $\frac{1}{2}$ as here appeareth.

peers B roofe $\frac{2}{1}$ Z $\frac{1}{1}$

Secondly if C. will doe it in 3. peers, what will he do therof in 1. peere, & the answere is $\frac{1}{3}$ as here the fabricke expresseth.

peers C roofe $\frac{3}{1}$ Z $\frac{1}{1}$

Thirdly I say if D. will finish the roofe in 4. peeres, what will he do therof in 1. peere, and the answere is $\frac{1}{4}$ as here appeareth.

peers D roofe $\frac{4}{1}$ Z $\frac{1}{\frac{1}{4}}$

Now as for A he is sayd alreadie, to doe the whole in 1. peere, wherefore I adde all that they 4. will do in 1. peere, viz. $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ and the totall is $\frac{13}{12}$ which I take for the first tearme of my proportionalitie or last Golden rule which concludeth the question saying thus, if $\frac{25}{12}$ of the roofe or worke require 1. roofes peere what will $\frac{13}{12}$ thereof require, and the aunswer is $\frac{13}{25}$ of a peere, iustly agreeable to your former aunswere as here appeareth.

roofes peeres $\frac{25}{12}$ Z $\frac{1}{\frac{13}{25}}$

That is in 6. monethes, 7. dayes, 7. houres, and $\frac{12}{25}$ of an houre, reckoning 28. dayes to a Moneth, and 365. dayes 6. houres to a peere.

Eumath. This varietie is pleasant, and I perceiue that it groweth

groweth by the alteration of the common time, whereunto euery mans worke is reduced and referred, which (as I see well, must be alwayes one equall, and common to all) so in my worke, it was 4. yeeres, and in your worke but one yeere, for I reduced euery mans worke to 4 yeeres, which was the time that D. required, but you reduced euery mans worke to 1. yeere; which is the time that A. required, both which workes concluding, one aunswere cannot be but true, for that trueth is alwayes agreeable to it selfe, whereas commonly one falshoode impugneth another.

Philo. You haue found out the reason of the matter very well, but now answere me to this question, a certayne miller hath 3. mils vnder one roofe, whereof the first will grinde 15. bushels of corne in 12. houres, In which time the second will grind but 10. bushels, and the third 5. Now there chaunced a baker to send 20. bushels of Wheate to this miller, requiring that it may be ground with all speede, and for that purpose he requested that his corne might be so apportioned to the mils, that they might all beginne and end together, the question is how much corne the miller ought to appoynt to euery mill to satisfie this bakers desire. 57. A question of 3. mils.

Euma. Here the common time is alreadie giuen certayne to euery mill vix. 12. houres, so that here needeth no further worke to finde that, wherefore I adde together the quantities which all the milles will grinde in that time, viz. 15 + 10 + 5. bushels, and they make 30. bushels, whereby it is manyfest that all the 3. milles will grind 30. bushels in 12 houres.

Wherefore I say by the golden rule, if 30 bushels require 12. houres, what time will 20. bushels require? & it answereth me 8. houres, as heere appeareth.

bushels	houres
30	12
20	8.

Thus hauing found that all the mils will grinde this 20. bushels of wheate in 8. houres, now to know what euery mill, will grinde thereof in that tyme, I aske the same rule agayne thus. If the first mill grinde 15. bushelles in 12. houres, what will it grinde in 8. houres, and it answereth mee 10. bushels as heere the fabrike declareth.

houres	bush.
12	15
8	10.

Then for the second mill I question likwise, saying if 12. houres grinde 10. bushels, what will 8. houres grinde? & it answereth $6\frac{2}{3}$, as here apeareth.

houres.	bush.
12	10
8	$6\frac{2}{3}$

Lastly,

Lastly for the third mill, I make this demaund saying, if in 12. houres it grinde 5. bushels, what will it grinde in 8. houres, and it aunswereth me $3\frac{1}{3}$ bushels, as heere the figure sheweth.

houres.	bushels.
12	5
8	$3\frac{1}{3}$

Whereby I conclude that the miller must assigne 10. bushels to the first mill, $6\frac{2}{3}$ bushels to the second, and $3\frac{1}{3}$ to the third, all which parcels make iust 20. bushels.

Philo. You haue done well, and thus as you haue had some examples wherein the first tearme of the principall proportion concluding the question hath beene found or made by Addition, So now shall you haue one question for an example, wherein the first terme of the principall proportionalitie is found or made by Subtraction and it is thus.

58. A question of water, and the time that it will fill a Cesterne.

There is a Conduict, with a cocke flowing into a Cesterne, in which Cesterne is another cocke to auoyde the water thence, if the vppermost cocke onely be let run, it filleth the Cesterne in 4. houres, but if the lower cock onely be set open, it emptieth it in 11. houres, now the Cesterne being emptie if both cockes should be set open at once, in what tyme would the Cesterne be full.

This question I will answer my selfe, that you may thereby perceiue the fourme of working the better in any like, which to do, first for the filling cocke, I question with the golden rule thus, saying, if 4. houres will fill one intire Cesterne what will 1. houre fill? and it answereth me $\frac{1}{4}$. of a Cesterne as heere appeareth.

houres	cest.
4	1
1	$\frac{1}{4}$

Next for the empting cocke, I question by the same rule saying, if 11. houres emptie 1. intire Cesterne, what will one houre emptie, and it answereth me $\frac{1}{11}$. as heare appeareth.

houres	cest.
11	1
1	$\frac{1}{11}$

So now it is euident that in one houre the vpper cocke filleth $\frac{1}{4}$ of the Cesterne, and the lower Cocke emptieth $\frac{1}{11}$ thereof. But $\frac{1}{4} - \frac{1}{11}$ is $\frac{7}{44}$ that is to say the cleare remayner or ouerplus left in the Cesterne euery houre aboue all euacuation, wherefore now taking this remayner for the first tearme of my principall proportion which concludeth the question, I say thus to the Golden rule, if $\frac{7}{44}$ of a cestern be filled in 1. houre, in what time shal $\frac{44}{44}$ thereof be filled, and it answereth me in $6\frac{2}{7}$ houres, as here the fabrique witnesseth.

Cest.	houre.
7	1
44	$6\frac{2}{7}$

Whereby

Whereby I affirme that in that time the Cesterne shalbe filled.

Euma. I do not onely vnderstand all the order of proceeding in this question well, but also I see how because the basses of the antecedentes are both equall, you do not once expresse them at all in the golden rule, but vse their numerators only as pure Integers.

Philo. I did so, and here will I put one question more, wherein the first tearme of the principall proportion, is found partly by Addition and partly by Subtraction, and so we will end this plurall proportion directly onely, the question is thus.

59. A question touching the building of a brick wall.

Three Masons viz. A. B. C. entring into speach of a bargayne for the building of a bricke wall, A. assumed to end it in 30. daies, and B. in 40. dayes, and C. said that he with the helpe of A. B. would finish it in 15. dayes, now if C. onely should do the whole worke, I demaund in what time he would performe it.

For aunswere whereof I question thus with the Golden rule, saying, if A. will make this wall in 30. dayes, what will he doe thereof in 15. dayes, and it aunswereth me $\frac{1}{2}$, as here the fabrique declareth.

dayes.	wall.
30	1
15	$\frac{1}{2}$

Then for B. I make the like question saying if 40. dayes finish 1. wall, what will 15. dayes do, and it answereth me $\frac{1}{3}$, as here appeareth.

dayes.	wall.
40	1
15	$\frac{1}{3}$

Thus it is apparant that A. and B. in 15. dayes would dispatch $\frac{1}{2} + \frac{1}{3}$ (that is $\frac{5}{6}$) of the wall, whereby it followeth necessarily that C. hath no more to doe in that 15. dayes, but that which A. and B. leave vndone, which is but $\frac{1}{6}$ (for $\frac{1}{1} - \frac{5}{6}$ is $\frac{1}{6}$) whereupon I conclude by the last or principall proportionalitie, that if C. working alone finish $\frac{1}{6}$ of this wall in 15. dayes, then he alone will finish the whole wall or $\frac{1}{1}$ therof in 120. dayes as the fabricke very evidently declareth.

wal	dayes
1	
8	15
$\frac{1}{8}$	120

And this may suffice for examples of the golden rule plurall directly, now touching the same, when the rule is partly directly, & partly conversedly, the next Chapiter shall minister examples.

Examples of plurall proportionalitie, or the Golden rule Plurall, partly direct, and partly conversed.

Chap. 12.

60. A question of journeying.

If in 10. dayes of 12. houres long, a footeman may easily journey 300. Miles, in how many dayes of 16. houres long

long, may he iourney 500. miles.

For answere hereunto, first I consult with the golden rule, direct, saying thus if 300. miles require 10. dayes trauell, what will 500. miles require, and it answereth me $\frac{1}{3}$ dayes, as here appeareth by the fabrique.

miles.	dayes.
300	10
500	$\frac{1}{3}$

Secondly I aske the Golden rule agayne thus, if 12. houres require $\frac{1}{3}$ dayes, what will 16. houres require, but here because 16. the greater Antecedent requireth a lesser portion of time then doth 12. his lesser matchfearme, I must beware that I vse not the Golden rule directly now, but conuersedly, which being executed according to the 252. verse answereth, 12 $\frac{1}{2}$ dayes, as the figure here present expresseth.

houres.	dayes.
12	$\frac{1}{3}$
16	12 $\frac{1}{2}$

Whereupon I conclude that in 12 $\frac{1}{2}$ dayes of 16. houres long, this footeman may iourney 500. miles.

61. A question touching wine.

Agayne if 12. penniworth of wine satisfie 8. persons commensalls at a meale, when wine is at 6. pence the quart, how many such commensalls, will 20. peniworth of wine satisfie, when it is but at 4. pence the quarte.

Here (as before) the first proportion or rule is direct and the second conuersed, wherefore first I say directly, if 12. pence serue 8. commensals, how many will 20. d serue, and the rule answereth me $\frac{1}{3}$ commensals as here the figure sheweth.

wine.	commens.
12. d.	8
20. d.	$\frac{1}{3}$

This is vnderstoode by wine at 6. d. the quarte.

Secondly I say conuersedly, if 6. d. a quarte affoord wine sufficient for $\frac{1}{3}$ persons, for how many persons shall 4. d. a quart affoord wine sufficient and the rule answereth for 20. persons as heere the worke it selfe declareth.

wine.	commens.
6. d.	$\frac{1}{3}$
4	20

Wherefore I conclude that when 2. d. in wine at 6. pence the quarte serueth 8 persons at one meale, then 20. d. in wine of 4. d. the quart, shall serue 20. persons at one meale.

42. A question of Ploughing.

Agayne if 2. Ploughes eare vp 18. Acres in 6. dayes, how many Ploughes will eare 45. Acres in 3. dayes.

Luma. I pray you let me proue how I can worke this question.

Philo. I pray you doe.

Luma. Then to beginne I say thus, to the Golden rule direct, if 18. Acres require 2. Ploughes, what will 45. Acres require,

and

and the rule anſwereth me 5. ploughes, as the fabrique heere ſheweth, that is, for 6. dayes.

akers. plowes.
18 Z 2
45 Z 5

But nowe ſecondlye, I aſke the rule this queſtion. If 6. dayes require 5. ploughes, what will 3. dayes require? and here (becauſe it is certayne that 3. dayes the leſſer antecedent requireth mo ploughes then 6. dayes the greater antecedent) I vſe the rule backward or conuerſely, and it anſwereth me 10. as heere you ſee.

dayes. plowes.
6 Z 5
3 Z 10

So that if 2. ploughes eare 18. acres in 6. dayes, then 10. ploughes will eare 45. acres in 3. dayes.

Philo. Becauſe you haue done ſo well, let me ſee how you can anſwer this queſtion. When Wheate is worthe 20. s a quarter, the white loafe of an halfpeny ought by law to weigh 8 $\frac{1}{15}$ ounces troy, wherfore I demaund what that white loafe ought to weigh, which is ſold for 2. pence when Wheate is worth but 12. s. the quarter?

63. A queſtion of the weighe of bread.

Euma. I will do my good will to it, and firſt I will oppoſe the golden rule with ſuch kinde of ſpeach as you taught me before in the 10. chapiter which was thus. If 20. s. affoorde 8 $\frac{1}{15}$. ounces for an halfepeny, what will 12. s affoorde?

But here, whileſt I conſider the matche-terms and order of this queſtion, I finde that it muſt be wrought by the backward rule, wherefore doing accordingly, it anſwereth 13 $\frac{1}{3}$ ounces, as heere appeareth.

ſhilling. ounces.
20 Z 8 $\frac{1}{15}$
12 Z 13 $\frac{1}{3}$

That is for an halfepeny.

Secondly, I demaund of the rule againe thus, if 1. halfpeny buye 13 $\frac{1}{3}$. ounces, what will 4. halfe pence buye. And now I perceiue the rule is direct, ſo that in this queſtion, the conuerſed proportion hapned firſt, and the direct afterwards, which is contrarie to all your former examples, wherefore vſing the rule directly, now as I finde the preſent caſe to miniſter occaſion, it anſwereth me 54 $\frac{2}{3}$. ounces, as heere the worke thereof erpreſſeth.

halfepence. ounces
1 Z 13 $\frac{1}{3}$
4 Z 54 $\frac{2}{3}$

Wherefore I conclude that the 2. peny white loafe ought to weigh 54 $\frac{2}{3}$ ounces, when wheate is worth but 12. s. the quarter.

Philo. You haue done well. Now anſwere me likewiſe to this queſtion. If 500. Pioners in 6. houres, caſt a trenche of 300 rods long, how many pioners will caſt a trenche of 160. rodes long in 2. houres?

64. A queſtion of entrenching.

Eum. To aunſwer you, I flee ſtraight to the golden rule, where I begin with the time firſt, ſaying thus, if 6 houres require 500. pioners, what will 2. houres require? and here aſſoone as I conſider the antecedents or firſt couple of match termes, I perceiue by the 243. verſe, that this firſt proportionalitie is alſo ſubiect to the golden rule conuerſed, for 2. houres being the leſſer number of the match termes doth therfore require mo men then doth 6. houres, wherefore performing the operation of the rule accordingly, the aunſwere is 1500 pioners as here the the figure ſheweth.

houres.	pioners
6	500
2	1500

Which being knowne I vrge the rule with the ſecond queſtion, ſaying thus, if 300. rods require 1500. pioners for 2. houres, what doth 160. rods require? and now it is manifeſt, by the 242. verſe that this proportionalitie, is direct, wherefore by working it accordingly, I finde the anſwering number to be 800. as here appeareth.

rodds	pioners
300	1500
160	800

Whereby I conclude that if 500 pioners caſt a trench of 300. rods long in 6 houres then 800 pioners, ſhall caſt a trench of 160. rods in 2 houres.

Phi. It is euen ſo, and thus as you ſee in theſe queſtions of plurall proportionalitie, partly direct and partly conuerſed, ſometimes the foreparr is direct, and the latter conuerſt, and ſometimes the foreyart is conuerſed and the latter direct, and therefore you muſt haue a diſcretion to foreſee & conſider, by the 242. and 243 verſes, when you are to vſe the rule directly and when conuerſedly, ſo ſhall you neuer miſs, but eaſily produce true anſwers to all queſtions propounded: whereas if you miſtaking vſe the one inſteed of the other, you ſhall fall into an abiſme of error, and chaos of confuſion.

And yet, if you take heede, neither are the 2. laſt queſtions, nor generally any other like, ſo addicted to preferre the conuerſt rule before the direct, but may be ſo handled, that the fore parte ſhall alwayes proceede directly, and the conuerſt part ſhall euermore be vrged to come after.

Euma. I pray you ſhew me how I might doe that in the former queſtions.

Philo. You might haue begun with that part of the queſtion firſt, which you vſed laſt, ſaying thus. If 1. ob. buy $\frac{104}{315}$ ounces of bread, what will 4. ob. buy, and the golden rule directly would haue anſwered you $\frac{416}{315}$ ounces, as heere appeareth by the figure.

halfpence.	ounces.
1	$\frac{104}{315}$
4	$\frac{416}{315}$.

Then

Then might you haue tryed the rule the ſecond time, ſaying thus. If 20. ſ. affoord $\frac{16}{25}$. ounces of bread for 4. halfepence, what will 12. ſ. affoord, & the golden rule conuerſedly would haue anſwered you $54\frac{2}{5}$ ounces, agreeable to your former anſwer, as heere appeareth.

ſh.	ounces
20	$\frac{16}{25}$
12	$54\frac{2}{5}$.

Euma. This I perceiue well euen ſince, and therefore I will turne the other queſtion likewiſe, beginning with that part thereof firſt, which before I tooke laſt, ſaying thus. If the caſting of 300. rodes of trenche, require 500. men, what will require, and the direct rule anſwereth me $\frac{1}{3}$. as heere the fabrique expreſſeth.

rodes.	men.
300	500
160	$\frac{1}{3}$.

Then comming to the latter part, I ſay thus. If 6. houres require $\frac{1}{3}$ men, what will 2. houres require? and the backward rule anſwereth 800. as heere the fabrique declareth.

houres.	men
6	$\frac{1}{3}$.
2	800

And ſo I ſee the aunſwers both wayes agree, but I like this way better, becauſe it beginneth continually with the direct rule firſt, and ſo followeth afterward with the conuerſed vnchangeably.

Philo. There is a third way yet behinde (whereto we ſhall arriue anon) which I beleeue you will like better then any of both theſe. But in the meane ſeaſon, touching tryall of the golden rule, whether it be ſingle or plurall, direct or conuerſed, the truthe of the worke is generally known by this verſe.

254. The golden rule in generall to prooue,
Let the antecedents take each others place:
And ſet the laſt terme at the angle aboue,
which done, worke newly as longs to the caſe,
Direct or conuerſt, ſo truth dooth reſtore,
that terme at laſt, which was ſecond before. 209

The trial of the Golden rule.

Euma. The ſcience of this verſe is manifeſt inough, and to declare how I vnderſtand it, I will prooue the worke of your laſt queſtion propounded, as it was firſt wrought: the figure and termes of whoſe forepart was thus.

6	500
2	1500

Wherefore I chaunge the ſcituation of 6. and 2. the antecedents, ſetting the one in the others place, and 1500. being the laſt terme, I ſet at the vpper angle of the figure, thus.

2	1500
6	

And now I worke the rule newlye, as belongs to this caſe. (that is to ſay, conuerſedly) for ſo it was wrought there) multiplyeng 1500. by 2. and the product is 3000. which I diuide by 6.

and the quotient yeelde:h iuſt 500. the ſelfe ſame terme which was the ſecond before, whereby I may conclude, that the firſt worke is true and well wrought.

Phi. You may ſo, now therefore trie the latter part.

Eu. To do that, I take 300. and 160. the antecedents of that proportionalitie, and ſet them likewiſe to the figure 160 Z 800
each in others place, then do I place 800. the 300
laſt terme at the vpper angle thus.

And here I new worke the rule directly (becauſe the caſe now ſo requireth) multipling 300. by 800. whereby I produce 240000. which I diuide by 160. and ſo iſſueth in the quotient 1500. being the very ſelfe ſame number which was the ſecond terme in the former worke, wherefore I conclude by this verſe preſent, that both parts of this queſtion were truelie wrought, and therefore ſo is the aunſwere.

Phi. It is ſo, and here as you ſee that euery ſeuerall operation of the golden rule requireth a ſeuerall triall, the direct directly, and the conuerſed conuerſedly, ſo may you make further experience thereof by the tryall of any other former queſtions at your owne leyſure, and in the meane ſeaſon I will proceede to explicate the golden rule artifically compounded.

Of the rule of proportion, or Golden rule, artificially compounded, firſt by addition, and next by Subtraction. Chap. 13.

255. Proportion artificiall, is of two kindes or ſortes,
Compounded and continued, as authors old reports.
256 the cōpoūd kind is þ which grows by art & operatiō
of addition and ſubtraction, or els multiplication.
whoſe total, reſt, or product yeelds, a third proportionality
iuſt correſpondēt to the work, in nature, kind & quality.

This verſe purſuing the order of my Table, ſheweth you firſt, that proportionalitie artificiall, is of two kindes, to wit, compounded and continued. Secondly, it defineth proportionalitie artificially compounded, to be that proportionalitie which groweth by arte and operation arithmeticall, as namely by the addition of one proportionalitie to another, or by ſubtraction of one proportionalitie from another. Or els thirdly, by the Multiplication of one proportionalitie by another. In which addition the totall, and in which Subtraction the remainder, and in which Multiplication,

the

the product is alwayes iustly agreeable to the nature of the operation vsed, according to the 30. 37. and 47. verses.

Which as it is no common doctrine of Arithmeticke, so hath it not beene hetherto commonly published by any in our toung, to my knowledge.

And although there may a third proportionalitie issue also by the diuision of one proportionalitie by another, yet forasmuch as in some learned mens opinions, such quotient or proportionalitie bringeth no vse nor commodity therwith, I rather chose to passe it ouer in silence then to trouble you with a matter vnnecessary, intending rather to proceed to the explication, first of the addition of proportionalities, and so to the other operations in order, and therefore touching addition, loe the maxim insuing sheweth both wherefore it serueth, and also the manner of working of it.

257. Addition of proportions serues,
to adde proportions quickly,
Whose consequents are equall iust,
and each to other likely.
For if it chaunce both consequents,
of 2. proportionalities,
To be coequall in one rate,
and both alike in qualities,
Then as the totall made of both,
their first 2. antecedents,
Appeareth in proportion to,
the first of both their consequents,
So shall the totall rising of
their latter antecedents,
Be in like rate vnto the last,
of both their giuen consequents. } 211

As for an example whereby to explaine this Maxim. Let 2. proportionalities bee assigned, whose consequents are equall, whereof let the first be as 24. is to 6. and as 36. is to 9. and let the second be as 2. is to 6. and 3. to 9. and for more plainenesse, let the same proportionalities be described also, in the forme and to the figure of the Golden rule, as is heere represented in this present figure, by A. and B.

A.
24 Z 6
36 Z 9
B
2 Z 6
3 Z 9

Which being done, if now I would quicklie or speedily adde

both these proportionalities together, so as the totall may truly conteine them both, I may very well do it by this present maxim, concluding that as 24 + 2. (the totall of both the first Antecedents) which you see linked together, is in proportion to 6. (the first of both their Consequents) so is 36 + 3. the totall of their latter antecedents) linked also together, to 9. the last of both their consequents.

C

And so the proportionallitie or total resulting by this kinde of Addition, is the proportionalitie C. 26 — 6 / 39 — 9

Euma. If I mistake not the meaning of the 182. verse, and the glosse thereof, I vnderstand both by this Maxim and your present example, that as the proportion of 24. is to 6. and of 36. to 9. is Quadrupla, that is to say 4. (for so is the quotient of 24. diuided by 6. and of 36. diuided by 9.) and as the proportion of 2. to 6. and of 3. to 9. is Subtripla, that is to say, $\frac{1}{3}$. (for so is the quotient of 2. diuided by 6. and of 3. diuided by 9.) whereby the totall of both those proportions, by the same 182. verse is $4\frac{1}{3}$. that is quadruplasesquitertia. So likewise the totall proportionalitie made by the addition of these 2. proportionalities according to this maxim, ought to be $4\frac{1}{3}$, or Quadruplasesquitertia.

And indeede it appeareth euidently so to be, for if I diuide either the antecedent 26. by his consequent 6. or the antecedent 39. by his consequent 9. both wayes the quotient remayneth constantly $4.\frac{1}{3}$, so that this addition needeth no further proofe.

Phi. You haue not mistaken the 182. verse at all, but haue rather applied the meaning therof, very fitly to this maxim, shewing your selfe therby to vnderstand them both & all my sayings well, whereof I am very glad.

And although the certayntie of this addition is such, as by your owne confession needeth no further proofe, yet is the same confirmed also by the 36. verse, wherefore concluding that as 24 + 2. is to 6. so is 36 + 3. to 9. I infer another rule following by this maxim, which is thus.

Euclid. 14 d. 5. Sche. grac. 2. ca. 5 Torstall pag. 200. in al which places this is called a conioynd or composed proportionalitie.

258. If foure termes be proprtionall,
the first and second ioynde,
Shall be vnto the second sole,
proportion like inclinde,
As both the third and fourth conioynd,
are to the fourth alone,
And so a new proportion comes,
although the old be gone. } 212

This

This rule or Maxim teacheth a certaine kinde of abridgement of the former addition. For if the 2. consequents of a proportionalitie giuen, should be compared each one to it selfe, they would make another proportionalitie, whose consequents should be equall to the consequents of the giuen proportionalitie, as for example.

Let 3. to 4. be as 6. to 8. Now may I here take the consequents onely, and say, as 4. is to 4. so is 8. to 8. Whereupon it followeth by the 211. maxim, that as 3 + 4. is to 4. so is 6 + 8. to 8. Do you vnderstand this?

Euma. Yea very well, for it is very intelligible.

Phi. Then hereupon I yet induce another rule touching ỹ additiõ of termes proportionall, how many soeuer they be, which is thus.

259. If numbers set by couples,
keepe proportion discontinuall:
Then as their antecedents are,
conioyned all in generall.
To all the giuen consequents,
vnited by like accident.
So is each antecedent sole,
vnto his proper consequent.

213

Euclid. 12. prop. 5. & 12. prop. 7 Salig. li. 2. ca. 8. ver. 5

For the explication of which verse, take this example. As 3. is to 6. so is 4. to 8. and 5. to 10. and 7. to 14. all which 4. couples of numbers, keepe one kinde of proportion discontinuall as you see. Wherefore adding all their antecedents together by themselues, I compare them to the totall of all their consequents, added or vnited also together by themselues, saying by this verse, that as 3 + 4 + 5 + 7. is to 6 + 8 + 10 + 14, so is 3 to 6. For heere first, as 3. is to 6. so is 4. to 8. and alternately: as 3. is to 4. so is 6. to 8. and by the 258. verse, as 3 + 4. is to 4. so is 6 + 8. to 8. And againe, as 4. is to 8. so by supposition is 5. to 10. and alternatly, as 5. is to 4. so is 10. to 8. wherefore by the 257. verse, as 3 + 4 + 5. is to 4. so is 6 + 8 + 10. to 8. Moreouer as 4. is to 8. so by supposition is 7. to 14. and alternately as 7. is to 4. so is 14 to 8. wherefore by the 257. verse, as 3 + 4 + 5 + 7. is to 4. so is 6 + 8 + 10 + 14. to 8. And therefore alternately, as 3 + 4 + 5 + 7. is to 6 + 8 + 10 + 14, so is 4. to 8. that is by supposition as 3. is to 6.

Demonstration

And here note, that I call not this kinde of addition, the addition of proportions, as I did the other before, but I call it onely the addition of termes proportionall.

For the addition of proportions, immitateth the addition of fractions, wherein as the numerators are onely added, and not the denominators, so in the addition of proportions and proportionalities, the Antecedents onely (which stand as numerators) are added together, and not the consequents of the proportions.

But in this addition present, the antecedents are not onely added by themselues, but also the consequents by themselues.

Eu. All this I vnderstand well, and hope to remember it as well.

Phi. Then will I go in hand with subtraction of proportionalities, whose office & forme of working the verse insuing declareth.

260. Subtraction of proportions doth,
abate proportions thus,
Whose consequents are equall iust,
without an ouerplus,
For when it haps both consequents,
of 2. proportionalities,
To be both equall all a like,
and both alike in qualities,
Then as the difference found betweene,
their 2. first antecedents,
Appeareth in proportion to,
the first of both their consequents.
So shall the difference found betweene,
their second antecedents,
Be in proportion to the last,
of both their giuen consequents. } 214

If you consider well the 257. verse, touching the forme of addition, you cannot choose but easilie conceiue the effect of this present rule of Subtraction. Yet for an example thereof, let vs assigne 2. proportionalities, whose consequents are equall, whereof, let the first or totall be as 26. is to 6. and 39. to 9. and let the second being the abatement, be as 24. is to 6. and as 36. is to 9. and to auoide all obscuritie, let them both be also described, after the fashion of the Golden rule, to wit, the totall by A and the abatement by B. thus.



Which being done, now to subtract the proportionalitie B. from the proportionalitie A. I vse no further circumstance, but say presently by this verse, as 26 — 24. is to 6. so is 39 — 36. to 9. that is, as 2. is to 6. so is 3. to 9.

And

And so haue I done the subtraction, and the proportionalitie or remaynder left after this subtraction is the proportion of 2. to 6. and 3. to 9. that is Subtripla as here appeareth by C.

C
2 Z 6
3 Z 9

Euma. This I conceiue very readily, for by the 182. verse, and glosse thereof, the proportion of 26. to 6. and 39. to 9. is Quadrupla sesquitertia, whose quotient or Index is $4\frac{1}{3}$, but the proportion of 24. to 6. and 36. to 9 is Quadrupla onely, whose quotient or Index is 4. wherefore now if I subtract 4. from $4\frac{1}{3}$, the remaynder must needes be $\frac{1}{3}$ which is the quotient or Index of Subtriple proportion, iustly agreeable, with the former remayner found by this present rule.

Wherefore I may be bold to conclude by the 45. verse, that as 26——24. is to 6. so is 39——36. to 9.

Philo. You haue sayd very well, and there vpon it followeth also that.

261. If 4. termes be proportionall,
then as the first terme of them all,
Diminisht by the second shall,
be to the second there.
So shall the third abated lesse,
but by the last terme in succesſe.
Be to that last in rate expresſe,
and like proportion beare. } 215

Euclid 15. d. 5. Tonstall pag. 200. where this is called a portion disioynd or parted proportion.

This verse teacheth a kinde of abridgemēt of the former Subtraction, euen as the 158. verse taught the like before in the Addition of proportionalities: for example whereof let 7. to 4. and 14. to 8. be 4. tearmes proportionall, now hereupon I may frame a new proportionalitie, of their consequents onely, saying thus, as 4 is to 4. so is 8. to 8. & then by the 260. verse, as 7——4 is to 4. so is 14——8. to 8. that is to say as 3. is to 4. so is 6. to 8. and so the proofe of this Subtraction being manifest, I goe forward to shew the subtraction of tearmes proportionall, how many soeuer they be, the rule whereof is thus.

262. If totall to totall, like proportion beare,
As part doth to part from each totall taken,
Then looke what proportion such totals first were,
Such are their remaynders or remnantes forsaken. } 216

Euclid 19. prop 5. & 11. prop. 7 Records whetstone fol. 89.

As for example, let the totall of these parcels 3 + 4 + 5 + 7. be to the totall of these parcels 6 + 8 + 10 + 14. as 3. a part taken from

from the one totall, is to 6. a part taken away from the other, now I say, that as 3 + 4 + 5 + 7 is to 6 + 8 + 10 + 14. so is 4 + 5 + 7 the remaynder of the one totall to 8 + 10 + 14. the remaynder of the other totall. for here (changing the supposition) As 3 + 4 + 5 + 7, is to 3. so is 6 + 8 + 10 + 14, to 6. therefore by the 261 verse as 4 + 5 + 7 is to 3. so is 8 + 10 + 14. to 6. and alternately as 4 + 5 + 7. is to 8 + 10 + 14. so is 3 to 6. which by supposition is so as 3 + 4 + 5 + 7. is to 6 + 8 + 10 + 14.

And heere you are also to note (as before) that this maner of subtraction, is not the subtraction of proportions, but only of terms proportionall, because subtraction of proportions immitating the like operation in fractions, doth not abate the consequents, which stand as it were in the place of the Denominators, but euen as in subtraction of fractions when their bases are equall, the lesse numerator onely is abated from the greater, without changing the denominators, so likewise in subtraction of proportions whose consequents are equall, the lesser antecedent is onely abated from the greater, without changing the consequents.

But in this kinde of subtraction present, the consequents are aswell subtracted by themselues as the antecedents by themselues.

Eu. In all this I vnderstand you very well and perfectly.

Philo. Then would I now proceede to the multiplication of proportionalities, but that order requireth that I should first here entreate of the rule of societie or partnership, because it is deriued and springeth originally from the 259. verse, and therefore in my iudgement doth not vnaptly challenge place here presently.

Eum. I pray you hartely vse your owne discretion.

Of the rule of societie or partnership, and specially of the first kinde thereof. Chap.14.

Salignac. lib.2.cha.4 ver.1.2.

263. *Phi.* The due addition of termes proportionall,
Did first beget the rule of societie,
VVhich among partners is all in all,
To ballants accompts it hath such propertie,
264. For as their whole stocke is generally,
Vnto the totall thereby gaynd or lost,
So is each mans stock particularly,
To his proper share of gaynes or of cost,
265. The rule of partners hath 2. sundry kindes,

217

VVherof

VVhereof the first doth seuerally expresse,
The last Antecedents, by whose helpe it findes,
The latter Consequents, and so doth surcease.

Now one example shall expound all this matter, and let the same be thus.

65. A question of partition of gaines gotten by tinne.

A. and B. being 2. partners, bought a certayne quantitie of tinne for 20. l. wherof A. layd out 12 l. & B. 8. l. by the sale of which tin, they gayned cleare 5. l. now it is demaunded how much of this 5. l. each of them ought to haue, for it is not reason that B. which layd out but 8. l. should haue so much of the gaynes as A. who layd out 12. l.

Euma. That is very true.

Phi. Now first that the rule which deciderh this and such like questions is deriued and begotten by the addition of the termes proportionall (as the first verse of this maxim sayth) will presently appeare, for by the second verse thereof, as 12. l. + 8. l. their whole stock, is to 5. l. the total gained by the same stocke, so is each mans particuler stock to his propper share of the gaynes.

Wherupon I frame the questions to the golden rule, by adding the termes, saying thus, if 12. l. + 8. l. that is 20. l. stocke, bring 5. l. gaynes what doth 12. l. stocke gayne, and what doth 8. l. stocke gayne, and so working for each partner seuerally, the rule bringeth 3. l. for A. and 2. l. for B. as here the seuerall fabrickes playnely declare.

stocke A.	gaines.
20 l.	5 l.
12	3.

stocke B	gaynes.
20. l.	5. l.
8	2

And here for asmuch as 3. l. + 2. l. the particuler Consequentes, make 5. l. equall to the generall consequent, therefore I affirme this partition to be good, and the answere true.

And finally as the third verse of this Maxim sayth, that the rule of partnership hath 2. kindes, whereof the first doth seuerally expresse the latter antecedents, by whose helpe the latter consequents (or answering numbers) are found, so you see here that the latter antecedents in the golden rules, are 12. l. and 8. l. which were both seuerally giuen and expresse at the first, and serue by the operation of the rule to help to finde out 3 l. and 2. l. the latter consequents, whereby I may conclude this rule to be of the same first kinde.

Eu. All this I vnderstand clearely by this example, and doe perceiue withall, that the whole processe of the worke, is iustly agreeable with the wordes of the maxim.

Philo.

Philo. Then let me see how you can answere me a question of losse wherein is no difference of worke, by the second verse of this Maxim for the reason of both is one, my question is thus.

66. A question of losse by wine cast ouer bourd.

Three English merchants to wit, A. B. and C. freighted a ship at Burdeux with 70. tunnes of wine, whereof A. did lade 30. tun, B. 24. tunne, and C. 16. tunne, now there happened such extremitie of weather in the voyage homeward, that 10. tunne of this wine was cast ouerboord for safegard of their liues and ship; wherefore the question is, how much euery of these merchants ought to beare of this 10. tunnes lost.

Eu. I will giue the attempt to answer according to the proport of this rule, whereby I am taught, that as 30 + 24 + 16. their totall stocke, is in proportion to 10. their totall losse, so is 30. to his propper losse, and so is 24. to his particuler losse, and so is 16. to the losse thereof, wherefore adding the tearmes together, viz. 30 + 24 + 16. I take their totall which is 70. for the first and generall Antecedent in the golden rule to euery operation thereof, and 10. the totall losse, for the first and generall consequent thereto, and euery mans particuler stocke or goods for the latter Antecedents, and so working by the golden rule for euery man seuerally, I finde the latter consequents or aunswering numbers, whereby I cõclude that A. must beare of this losse $4\frac{2}{7}$ tun, B. $3\frac{3}{7}$ tun, and C. $2\frac{3}{7}$ tuns, as here the seuerall figures testifie. And because I finde by the Addition of the latter consequents that they make iust 10. tun, equall to the generall consequent, therefore I thinke I haue done well and truly.

A		B		C	
70	10	70	10	70	10
30	$4\frac{2}{7}$	24	$3\frac{3}{7}$	16	$2\frac{3}{7}$

Philo. You haue so, and indeede as the ordinarie and most vsuall triall of this rule and worke is by such addition, so is the same triall exact and certayne inough, except when any man hath found the consequent due to A. he will for the nonce assigne it to B. or C. and so will attribute one mans proportion to another, which no man, I thinke will doe, that hath his right wits, and therefore I will rehearse no other proofe thereof, but will rather propound mo questions for your more perfect instruction in this rule, wherefore let this be one.

Proofe.

67. A question of building a ship.

Foure partners viz. A. B. C. and D. build a ship at their common charge, which will cost them 600. pounds, whereof because their habilities are not like, they are agreed, that as often as A. payeth

6.r.so often B.shall pay 4 r. C shall pay 8.r.and D.but 3. r.now it is demaunded howmuch euery of them must pay of this 600. poundes.

For answere whereto, first I adde these seuerall termes of their paymentes, saying 6 + 4 + 8 + 3. make 21. the first Antecedent generall to euery operation of the golden rule, then taking 600.r. the totall charge for generall Consequent thereto, and euery mans particuler tearme for the third number or latter Antecedent, I conclude euery mans seuerall charge to be as here is expreſt by the seuerall figures following.

r.stoc. A	r.charg.	r.stoc. B	r.char.	r.sto. C	r.char.	r.sto. Dr.	ch.
21	600	21	600	21	600	21	600
6	171 3/7	4	114 2/7	8	228 4/7	3	85 5/7

For triall whereof, first as 3/7 + 2/7 + 4/7 + 5/7 is to 7. make 1 that is 2. which added with the reſt of the Integers make iuſt 600. 171 poundes, the totall charge as here the fabrique of the Addition declareth. 114 228 85 2 600

Agayne 4. ships being of a conſort togeather in time of warre, tooke a price, which being ſold did procede in money 8190.r. but because these ships are not all of like burden, force, nor furniture, the ſ shares of this ſumme muſt not be equall, wherefore it is agreed by them that A the shippe of the greatest burden and force ſhall haue 5/10 + 1/10 of the valure, B the ſecond ſhall haue 1/4 + 1/10 of the money, C the third ſhall haue 1/8 thereof, and D the fourth, being but a ſmall barque, ſhall haue onely 1/10 of this price, now what ought euery of these ſhips or Companies to haue for their ſhares.

68. A queſtiō of the taking of a priſe at the Seas.

For answere, first here 5/10 + 1/10 maketh 6/10. for the latter antecedent of A. Secondly 1/4 + 4/10 maketh 7/10 for B. thirdly 1/8 ſtretch ſingly as it is for C, and ſo doth 1/10 for D.

Now the totall of 6/10 + 7/10 + 1/8 + 1/10 maketh the firſt and generall Antecedent, by the nature and purport of this rule, but that totall by the 155. verſe cannot be ſummed till these fractions be all reduced to like baſſes by conforming reductiō which being performed (by the 159. verſe) ſtand thus 24/40 + 28/40 + 5/40 + 4/40 or thus, as 24 + 28 + 5 + 3. is to 60. ſo that now reiecting their equall baſſes, I take their numerators only as pure Integers aſſigning 60. the totall of them for the firſt Antecedent generall in the Golden rule, and 8190.r. the totall gaynes for the generall conſequent thereto, and euery particular numerator for the latter antecedent in the golden rule,

and

and so worcking thereby for euery ship seuerally, I finde that A. shall haue 3549 l. of this 8190 l. B. 2866 ½ l. C. 1365 l. and D. 409 ½ l. as the seuerall figures of the Golden rule heere annexed sheweth.

A	B	C	D
60 ℔ 8190	60 ℔ 8190	60 ℔ 8190	60 ℔ 8190
26 ℔ 3549	21 ℔ 2866 ½	10 ℔ 1365	3 ℔ 409 ½

And that this worcke is true, the totall of the particuler consequentes doth euidently declare, which maketh iust 8190. equall to the generall consequent, whereof euery of them are parcels.

Emma. Truly this rule of partnership or societie, proceedeth very orderly, sensibly, and probably in all the operations thereof, and I thinke I do conceiue both the order and reason thereof, so well by these your examples proceeding, that I can answere any like question subiect to this rule.

Philo. I would willingly put a question to trie you, but because it may seeme somwhat harder then any that you haue yet heard, I will answere it my selfe, and it is thus.

69. A question of legacies.

Cicero pleading for Cecinna saith thus, Titius by his Testament bequeathed to Cecinna $\frac{1}{10} + \frac{1}{2}\frac{1}{10}$ of all his substance (which was prised at 2500. Ducats) and to Marcus Fulcinus his late bondman, he willed $\frac{1}{3}\frac{1}{10}$, and to Ebutius $\frac{1}{7}\frac{1}{10}$, now what is the value of euery of their legacies.

Here the particles, or fractions of fractions, must be first reduced to one simple fraction, by particuler reduction, which beeing done, $\frac{1}{2}\frac{1}{10}$ turneth to $\frac{1}{14}$ and then $\frac{1}{10} + \frac{1}{14}$ maketh $\frac{24}{140}$ for the particuler Antecedent of Cecinna. Secondly $\frac{1}{3}\frac{1}{10}$ reduced to one fraction commeth to $\frac{1}{30}$, for the particuler Antecedent of Marcus Fulcinus. Lastly $\frac{1}{7}\frac{1}{10}$ reduced to one fraction, commeth to $\frac{1}{70}$ for the Antecedent of Ebutius.

Now the totall of $\frac{24}{140} + \frac{1}{30} + \frac{1}{70}$ (which must make the first generall Antecedent as before) is found by conforming reduction to be as 69 + 2 + 1, is to 72. but these fractions beeing now transformed to Integers proportionally, by reason of their common base, I reiect the same base (as before) taking 72. the total of their numerators onely, for the first generall Antecedent.

And the totall value of the legacie, for generall consequent thereto, and euery seuerall numerator by it selfe for the last particuler antecedent, and so applying them to the Golden rule, the

seuerall

seuerall operations thereof declare, that the portion of Cecinna is, 2395 1/2 Ducats, of Marcus Fulcinus 69 1/2 Ducats, and of Eburius 34 11/11, as here the figures witnesse.

C	M	E
72 Z 2500	72 Z 2500	72 Z 2500
69 Z 2395 1/2	2 Z 69 1/2	1 Z 34 11/11

For triall whereof, first 1/2 + 1/2 + 11/11 make 2. Integers, which added with the rest of the Integers of the latter consequentes make iust 2500. equall to the generall consequent, as here appeareth.

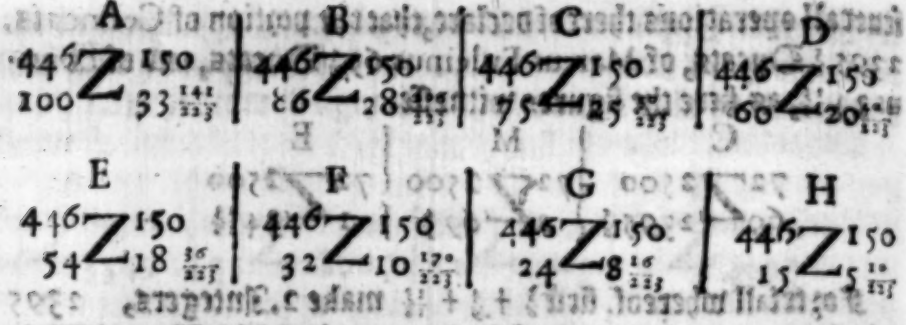
2395
69
34
2
2500

Eumathes. Indeede this question is harder then any of the other aunswerable by this rule, and that is by reason of the fractions of fractions, which perhaps might haue troubled me worse then I looked for, wherefore I am glad that you aunswered it your selfe, but yet hereafter I hope I can do the like.

Philo. Then aunswere me to this question, which is somewhat easier. A certayne merchant died greatly indebted to 8. creditors, to wit, A. B. C. D. E. F. G. and H. for to A. he ought 100. r. to B. 86. r. to C. 75. r. to D. 60. r. to E. 54 r. to F. 32. r. to G. 24. r. and to H. 15. r. but the valure of all his goods amounted onely to 150. r. which as it is too little to satisfie all the creditors, so it may seeme to be neyther equitie nor iustice, that one or 2. of them should be fully payde, and all the rest haue nothing. 70. A question touching a bankrout.

Wherefore as all the creditors are like to be partners of some losse by this debter, so let me see how you can assigne the seuerall portions of his goods which each of them ought to haue by equitie of this rule of partnership.

Euma. To do that, first I say, 100 + 86 + 75 + 60 + 54 + 32 + 24 + 15. make 446. r. wherefore by this rule, as this totall 446. r. is to 150. r. the totall valure of the debters goods, so is 100. to the portion due to A. and so is 86. to the portion due to B. and so is euery other particuler antecedent to his particuler consequent, wherefore by vsing the Golden rule 8. times, I finde the portion of A. to be 33 100/223 r. of B. to be 28 86/223 r. of C. 25 75/223 r. of D. 20. 60/223 r. of E. 18. 54/223 r. of F. 10. 172/223 r. of G. 8 16/223 r. & of H. 5. 30/223 r. as here the figures following of euery seuerall operation declare.



Where all the fractions make 3. integers, which added with the rest of the integers of the latter consequents make iust 150. equall to the generall consequent whereof they are percels.

71. A question of principall stocke disbursed.

Phi. This is well done of you, now what say you to this: three partners, A. B. & C. hauing a cõmon stock betweene them of 500. l. lost 124 l. thereof by creditting of ill debters; for supplie of which losse, they came to a reckoning whereby they found, that A. ought to disburse 20. l and B. 43. towards the said supplie of their stock, wherefore the question is, what euery mans seuerall stock was at the first.

Eu. Why this question is nothing like any of the other that you haue propounded as aunswerable by this rule.

Phi. Yet is it most properly and naturally subiect to the same rule as you shall see by and by: and although here be but 2. of the perticuler antecedents manifestly expiest, yet is the third also giuen after a fashion, which though it seeme somewhat obscure, because it is not named, yet may it easily be brought to light.

For sith the generall antecedent, contayneth all the perticuler antecedents ioynely and iustly, the third may easily be found by subtraction of the other 2. perticuler antecedents from the generall, for so the remeynder sheweth the thing.

Eu. That I consider to be reasonable.

Phi. Then thereuppon, I say that 20. l. + 43. maketh 63. l. and then 124. l. —— 63. l. leaueth 61. l. for the supplie to be made by C. that is to say for his perticuler antecedent.

Wherefore as in the former questions, the seuerall stock of euery partner, was his perticuler Antecedent, whereby his propper gaynes or losse, was found and concluded, so contrary wise here, the losse of euery partner, maketh his perticuler antecedent, whereby his propper stock is found and concluded.

Eu. So then I perceiue now, that this question is neither of losse nor of gaynes, as the 65. and 66. questions of this Chapter were, but

but of the finding of euery partners seuerall stock, wherefore the situation of the termes in the golden rule is changed, so that the antecedents in the former questions are here the consequents, and the consequents there are here become the antecedents, for whereas in those other questions, the gaynes or losse was inquired by each seuerall stock, so now in this question, euery mans seueral stock is enquired by his particuler gaynes or losse.

And so the totall stock (which before was wont to be the first and generall antecedent to the totall gayne or losse) is now become the generall consequent, to the totall gaynes or losse, by reason whereof the particuler antecedents and consequents, become of like nature to their generalls.

Phi. You haue found out all the reason of the diuersitie betweene the questions so perfectly, that I my selfe could not haue told it you more playner.

Eu. Now therfore I doubt not but I can go through with the solution my selfe, which by your fauor I will proue, and first the particuler antecedents being knowne to be 20 + 43 + 61. I take their totall which is 124. for the generall antecedent of losse, and 500 l. the totall stock for generall consequent thereto, & lastly euery mans particuler losse for his particuler antecedent in the golden rule, saying thus.

If 124. l. losse came of 500. l. stock, of whom came 20. l. losse? and of whome came 43 l. losse? and of what stock came 61. l. losse? and the rule answereth distinctly for A. $80\frac{20}{31}$. For B. $173\frac{12}{31}$. and for C. $245\frac{30}{31}$. as heere the seuerall fabriques expresse.

A		B		C	
124	500	124	500	124	500
20	$80\frac{20}{31}$	43	$173\frac{12}{31}$	61	$245\frac{30}{31}$

And sithe the totall of these latter consequents, is equall to the generall consequent (whereof each of them are parcell) therefore I conclude the worke to be good, and the answer true.

Philo. It is so, and heere I might rehearse some such other questions, as some authors propound, and take vpon them to assoyle by this rule and in truth they are properly subiect hereunto, if they were rightly and reasonably propounded, but being proposed so as they commonly are, they are neither dissoluble by this rule, nor any other rule of Arithmetick, but are more worthy to be dissolued and vndone, as Alexander dissolued Gordians knots.

Euma. Why? I pray you what maner of questions are those?

Philo. I am loath to loose so much time as to tell you, but sithe you are so desirous to know them. This is one of them.

72. A question impossible to be truly answered.

A. B. C. and D. agree to diuide 600. crownes betweene them so, that A. shall haue $\frac{1}{2}$ thereof, and 9. crownes ouerplus. B. shall haue $\frac{1}{3}$ thereof and 8. crownes ouerplus. C. $\frac{1}{4}$ and 7. crownes ouer ouerplus: and D. $\frac{1}{5}$ with 6. crownes ouerplus, whereupon they demaund, what euery mans part is. Which question is vtterlie impossible to be answered truly, according to the prescript thereof, for seeing the totall sum to be diuided betweene them 4. is but 600. crownes. If A. haue $\frac{1}{2}$ + 9. crownes thereof, then must he haue for his share 409. crownes, which sum being subtracted from 600. leaueth but 191. crownes remaining, which sufficeth not to make the portion due to B.

So that B. is not onely like to lose part of his share, but also C. and D. are like to haue iust nothing.

Cardanus, li. 10. artes magnæ, fol. 11.

And therefore Cardanus giueth a good note, where he saith, that it is to be holden for a generall rule, that when a question or proposition requireth any thing to be done, which is impossible, and cannot be done, then is that question or proposition also impossible.

Vnitie representeth the totall.

And to make the impossibilitie thereof more manifest, let vs set forth the parts or shares heere appointed to carry of these partners, both in fractions and in crownes, and see how iustly the totall of the one maketh an vnit or integer, and the totall of the other maketh 600. crownes, as they ought to do: and first the fractions added together make $\frac{357}{120}$, as heere the fabrique of their reduction and addition reuiueth.

				357
80	+ 72.	+ 100.	+ 105	
2	3	4	5	7
3	5	6		8
				120

Then the 9. crownes, 8. crownes, 7. crownes, and 6. crownes ouerplus, come to 30. crownes, which maketh $\frac{30}{120}$ more, so that the totall is $\frac{357}{120}$, which amounteth to 3. Integers or entire vnits, and $\frac{1}{4}$ more. All which ought to be but an vnit or 1.

Againe, if I take $\frac{1}{2}$ of 600. crownes, and 9. crownes ouerplus for A. and so for the rest, according to the tenor of the question. Then shall euery man haue so many crownes to his share, as is heere expressed by the letters A.B.C.D.

A.	400 + 9
B.	360 + 8
C.	500 + 7
D.	525 + 6
	1815. crow.

The

The totall of which crowns is not 600. (as it ought to be) but it is 1815. crowns which is alls 3. times 600 crownes & 15. ouerplus.

Now how can any diuision by reason or possibilitie, be made here according to the position of the question when eyther the parts shall exceede the totall, or els the partners shall want of their due shares allowed and assigned vnto them by the Thesis and prescripe order of the question?

Ro. In truth this seemeth to me as absurd, as if you giuing me but a grote, should bid me distribute it to 4. poore men in such sort, that euery of them might haue 3. peece of it. And besides that, it is veterly repugnant to the 250. verse.

Phi. It is so, and therefore cannot be but absurde, and repugnant to common reason, for all mathematicall doctrine shall faile, yea and nature in selfe shalbe destroyed, before that verse or maxim shall fayle in any iote or point.

And therefore Maister Doctor Record (who alwayes esteemed the truth in euery thing more then any authoritie of the author) could in no wise allow of these kinde of questions, as himselfe witnesseth in his Ground of arts, fol. 335.

But as he there excuseth their doings by their meanings, so will I also here, and yet some of them acknowledging the absurdities and impossibilities themselues, do neuerthelesse both propound, and proceede to the answering of this and diuers such other like questions, affirming that although it seeme neuer so much absurde and impossible, yet vnder this rule of fricte bring it to such good passe, and set all things in so good order, that it discusseth the matter without any absurditie at all.

As though the profession of this arte were to make absurdities agreeable to reason, and impossibilities possible.

And yet as I said before, if the question were propounded in such sorte as it may be, then should both the answer be possible, and the meaning of these authors performed but not their words.

And the way to do it is thus. When the fractions representing the parts or shares appointed, are reduced to one firname or common basse, then may the totall (deducting first the odde crownes ouerplus) be diuided amongst the partners in such proportion as their numerators beare each to other.

Note how to make a partition proportionall to the numerators of the fractions being all first reduced to one common denominator.

As for example, it appeareth before, that the fractions here representing euery mans part, being reduced to one common basse are these $\frac{30}{120} + \frac{72}{120} + \frac{112}{120} + \frac{105}{120}$, wherefore I say, that the partition

may be made in such sorte, that as often as A. hath 80. crownes, so often B. shall haue 72. C. 100. and D. 105.

For as 80 + 72 + 100 + 105. is to 600 — 30. so is 80. to the share of A. and euery other particuler Antecedent to his proper consequent yet vnknowne, wherefore the 30. crownes diuided amongst them in ouerplus, must be first deducted as I said before.

Euma. I vnderstand you very well, and therefore now doe I nothing doubt, but I can be auditor to these partners, and answer the question my selfe, which to doe, I begin thus, saying. As 80 + 72 + 100 + 105. is to 600 — 30. that is, as 353. is to 570. so is euery particuler antecedent to his proper consequent vnknowne: wherefore I repaire to the Golden rule, for answer, whereon to applying the numbers orderly, and so working thereby accordingly, I finde that A. ought to haue 127 $\frac{261}{353}$. crownes. B. 114 $\frac{342}{353}$. C. 159 $\frac{247}{353}$. and D. 167 $\frac{211}{353}$. as the figures here shew.

A	B	C	D
353 Z 570	353 Z 570	353 Z 570	353 Z 570
80 — 127 $\frac{261}{353}$	72 — 114 $\frac{342}{353}$	100 — 159 $\frac{247}{353}$	105 — 167 $\frac{211}{353}$

But now because A. must haue 9 crownes ouerplus, therefore his share is 136 $\frac{261}{353}$. and because B. must haue 8. crownes ouerplus, therefore his share is 122 $\frac{342}{353}$. and because C. must haue 7. crownes ouerplus, therefore his share is 166 $\frac{247}{353}$. and sithe D. is to haue 6. crownes ouerplus, therefore his share is 173 $\frac{211}{353}$. crownes.

Philo. You haue wrought well and orderly, wherefore now if you adde all their shares together, you shall finde that they will make iust 600. crownes, as they ought to doe.

Euma. I perceiue that well, for first the fractions make 3. crownes, and then they with the rest of the crownes make iust 600. But how may I be assured that A. hath iust so many times 80. crownes, as B. hath 72. or C. 100. or D. 105?

Philo. That may you proue by Diuision, for euery mans particuler antecedent diuide his proper share or consequent (I meane as it is produced by the Golden rule before the 30. crownes ouerplus be added to it,) then I say, if the quotients of euery such diuision be equall, you may be assured, that their shares are in such proportion, as the tearme of the question importeth.

Demonstration.

For first by supposition, euery particuler antecedent is to his particuler consequent, as the generall antecedent is to the generall conse-

Consequent, and secondly by the fabrike of the former reduction, euery perticuler antecedent, is the least number proportionall to his proper consequent, wherfore by the, 104. verse. euery particuler antecedent shall measure his proper consequent, all by one quotient, that is by like number of times.

Wherby it followeth that so many times as A. hath 80. crowns, so often B. shall haue 72. C. 100. and D. 105. that is to say, once and $\frac{71}{119}$ of a time, for if any of the said particuler consequents be diuided by his proper antecedent, the quotient shall alwayes be constantly $1.\frac{71}{119}$.

Eu. That will I proone hereafter at more leasure.

Phi. Thus haue I shewed you what the meaning of those men is which propound such manner of questions, that is to say, onely to make a certaine distribution or partition proportionally to the numerators of the giuen fractions.

And that such is their very meaning, their owne aunswers euidently proove, wherein they bring the selfe same termes in all things, that you brought now for answer to the question.

Euma. But such an aunswere is farre from the purport of their question. For as $127\frac{361}{517}$. crownes, is nothing nigh $\frac{2}{3}$ of 600. Crownes, no more is anye other of their shares, anything agreeable to the fractions first assigned for their seuerall parts.

Philo. That is the fault I finde, and the matter I mislike, and yet truelye you shall read very few bookes of Arithmeticke, wherein is not some such like questions, with their aunswers concluded very formally. As though it were some prettie point of Arte to propound a question one way, and to conclude it another waye quite contrarilie. Conclusion contrary to the position.

But letting these passe, forasmuch as I suppose you are sufficiently instructed in the first part of this rule of societie, I will now goe in hand with the second part, in forme following.

Of the second rule of Partnership, or second kinde thereof. Chap. 15.

266. The second rule of partnership doth finde,
euery latter priuate antecedent,
Either by folding the numbers assignde,
wherein as the sum of all products present, Saligna. li. 2. ca. 10.

Is antecedent generall to all,
the losse or gaine that luck shall aduance,
So euery priuate product likewise shall,
be antecedent to his priuate chaunce.
267. Or els by adding such products together,
wherein as the totall of all serueth apt,
For antecedent generall euer,
to that totall gaines or losse which is hapt,
So shall each totall of all products found,
by termes belonging to one man alone,
Be antecedent in like sort compound,
to his propper gaines or losse yet vnknowne. 218

The sence of this maxim is so palpable, that it needeth no exposition, for that I cannot well declare it better in prose then it expoundeth it selfe, for by the first part thereof you may gather, that in this rule sometime the latter particuler antecedents, are found by multiplying of the numbers giuen or assigned & then as the totall of all the products together, is generall antecedent to the totall consequent of gaines or losse, so is euery seuerall product, particuler antecedent to his proper gaines or losse in the golden rule.

And by the second verse you may gather that (at other times) the totall of all such products is generall antecedent in the golden rule to the totall consequent of gaynes or losse, and then the totall of the products made by the giuen numbers belonging to one man onely, shall be particuler antecedent to his proper consequent vnknowne.

Euma. Although I giue a good gesse to the meaning of the maxim, yet I pray you cleere it a little more by an example or two.

Philo. With a good will, and therefore first I will put a question or 2 answerable by the forepart of this maxim which is this.

73. A question of parting a bootie gotten by an army.

There is an armie, wherein are 6000. footemen, and 800. horsemen, the stipend of euery footeman is 4. crownes a moneth, and of euery horseman 9. crownes a moneth: all these together are to diuide a bootie of 2000 crownes, which they haue gotten.

But yet it must be so diuided, that their shares may be rateable or proportionall to their wages. Wherefore the question is, how much therof belongeth to the footmen, & how much to the horsmen?

Heere the latter antecedents is found by multiplication of the giuen numbers, according to the forepart of this maxim, but the giuen numbers are 6000. at 4. crownes, and 800. at 9. crownes,

where-

wherefore the product of 6000. by 4. is 24000. and the product of 800. by 9. is 7200. which seuerall products, by the said forepart, are the particuler or latter antecedents, and the totall of them both, viz. 31200. is the generall or first antecedent in the golden rule, whose generall consequent is the totall losse or gaines, which heere is 2000. crownes gaines.

Wherefore I conclude by the same forepart of this maxim, that as 31200. is to 2000. so is 24000. to the vnknowne share of the footemen. And so is 7200. to the vnknowne share of the horsemen.

Wherefore now to finish the question by the golden rule, I say thus. If 31200. gaine 2000. crownes, what shall 24000. gaine? and what shall 7200. gaine? and the rule answereth $1538\frac{6}{13}$. crownes for the footmen, and $461\frac{7}{13}$. crownes for the horsemen, as heere the figures shew: whereof the first hath an F. for footemen, and the last an H. for horsemen.

F
31200 — 2000. cr.
24000 — $1538\frac{6}{13}$.
H
31200 — 2000. cr.
7200 — $461\frac{7}{13}$.

And heere it is apparant, that the 2. latter consequents added, are iustly equall to 2000. the generall consequent whereof they are parcel.

74. A question of hiring a common pasture.

Another example is thus. Three butchers, A. B. and C. hired a pasture in common together to them all, for 30. pound a yeere, wherein A. fed 20. Oxen, 70. dayes. B. grased 46. Oxen 56. dayes: and C. pastured 32. Oxen 60. dayes. Now the question is, how much of the rent each man ought to paye?

First here the numbers giuen or assigned to A. are 20. and 70. whose product is 1400. his priuate latter antecedent.

Secondly the numbers assigned to B. are 46. and 56. whose product is 2576. his particuler latter antecedent.

And thirdly, the numbers assigned to C. are 32. and 60. whose product is 1920. his proper latter antecedent.

Fourthly the sum of all these products is 5896. which by the said forepart of this maxim maketh the first or generall antecedent in the golden rule, and the generall consequent thereto (by the same verse) is the totall gaines, losse, or charge, gotten, lost or paide (which heere is the 30. pound rent.) So that the latter particuler antecedents, being gotten already by such multiplycation as is aforesaid. Now haue I 3. termes proportionall, by whose application to the golden rule (the cõmon touchstone of all such questions) I finde the latter consequents or answering numbers thus, saying.

If 5896. pay 30. l. what shall 1400. pay? and what shall 2576 pay? and what shall 1920. paye? and the operation of the rule answereth, that A. shall paye 7. $\frac{91}{737}$ l. B. 13. $\frac{729}{737}$ l. and C. 9 $\frac{567}{737}$ l. as the figures heere following expresse.

A	B	C
5896 ⟋ 30 l.	5896 ⟋ 30 l.	5896 ⟋ 30 l.
1400 ⟍ 7. $\frac{91}{737}$.	2576 ⟍ 13 $\frac{729}{737}$.	1920 ⟍ 9 $\frac{567}{737}$.

And forasmuch as the totall of these particuler consequents, maketh iust 30. l. equall to the generall consequent, whereof they are parcels. Therefore I affirme the answer to be good and true.

75. A question of diuidents in a Cathedrall church.

Another example yet is thus. There are in a cathedrall church 20 Canons or Prebendaries, and 30. Peticanons or Vicars, which may spend yeerely 2600 l. but euery Canon hath 5. l. diuidend for 1 l. that euery Vicar hath. Now the question is, how much of this reuenew belongeth to the Canons? and how much to the vicars?

For answer whereof, first it is manifest that the numbers giuen or assigned to the Canons is 20. and 5. whose product is 100. and that the numbers assigned to the vicars is 30. and 1. whose product is 30. and so 100. is the particuler antecedent of the Canons, and 30. of the Vicars. Then 100 + 30. maketh 130. the generall antecedent to both, whose consequent must needs be the 2600 l. by the forepart of this maxim. Wherefore now I conclude the question by the golden rule, saying thus. If 130. receiue 2600 l. what shall 100. haue? and what shall 30. haue? and I am answered, that the Canons shall haue 2000 l. and the Vicars 600 l. as heere the seuerall fabriques plainely notifie.

Canons.	
130 ⟋	2600 l.
100 ⟍	2000.
Vicars.	
130 ⟋	2600 l.
30 ⟍	600.

Euma. Truly the finding of all the 3. termes requisite to the golden rule, and their due application thereto, for the answering of questions subiect to this maxim, hath no difficultie in it, but is so plaine and easie, that I perswade my selfe I can finde the answere to any question answerable by this first verse.

76. A question of partnership, with variete of time.

Philo. For triall thereof, I will put one question more, which I will leaue to you to answere, and it is this. Foure partners, to wit A. B. C. and D. made a banke or stocke of money to traffique in common for a yeere, whereof A. disbursed 600. pound by the space of 10. moneths. B. disbursed 400. pound for 8. moneths. C. layde out 800. pound 6. moneths, and D. laide out 500. pound the whole

12. monethes, now at the yeeres end they had gayned cleerely by their occupying 760. l. wherfore I demaund of you, what portion of this gaynes, each of them ought to haue, according to the rate of his stocke, and time of forbearance.

Euma. I will giue the onset to this question, for me thinks the former examples, teach me so plainly that I cannot misse the way, wherefore here the giuen numbers of A. being 600. l. and 10. monethes, I multiplie them together and they produce 6000. the latter Antecedent particuler to A. then take I 400. & 8. the numbers alligned to B. and folding them together, they produce 3200. for the latter antecedent, particuler to the same B. Thirdly I infold 800. in 6. the numbers alligned to C. and the product is 4800. the latter antecedent particuler to him. Lastly I multiplie 500. and 12. together, being the numbers belonging to D. and they produce 600. for the latter antecedent particuler to him.

Then do I adde all these compound termes or products together, and their totall yeelds 20900. for the first antecedent generall to them all, and so taking 760. l. the totall gaynes for generall consequent thereto, and euery mans particuler product before found for his particuler antecedent, I conclude by the golden rule, that A. shall haue 228. l. B. 121 $\frac{2}{5}$ l. C. 182 $\frac{2}{5}$ l. and D. 228. l. as here the seuerall operations of the same rule testifie.

A. and D.	B	C
20000 Z 760 l.	20000 Z 760 l.	20000 Z 760 l.
6000 Z 228	3200 Z 121 $\frac{2}{5}$	4800 Z 182 $\frac{2}{5}$

And to proue that the worke is good I haue added all the latter consequents found by this rule, and their totall is iust 760, equall to the generall consequent, as here appeareth.

228
228
121
182
1
760

Philo. Well, because I perceiue you are sufficiently instructed in the former part of this present rule or Marim, now I will propound a question or 2. also answerable by the latter part thereof, whereof this is the first. Three merchants viz. A. B. and C. entered partnership together on Newyeeres day, to continue for the yeere insuing, whereupon A. disbursed presently the same day 100. l. whereof he receiues back againe the first of April, 20. l. B. deliuered the first of March 60. l. and more the first of August 100. l. C. layd forth 140. l. the first of July, but the first of October he withdrewe 40. l. againe for his priuate vse. Now at the

77. A question of partnership with diuersitie of time.

the yeeres end, they finde their cleare gaynes to be 142. pound wherefore the question is, how much of this gaynes euerie partner ought to haue in respect of his stocke and tyme of forbearance?

Eu. I pray you answer this question your selfe, that I may the better perceiue the order thereof by your working.

Philo. With a good will, and for answere, heere is first to be considered, that when A. disbursed 100. pound presently the first of Ianuarie, and receyued 20. l. thereof agayne the first of Aprill, then was his 100. l. in bank 3 moneths, and but 80 l. the other 9. moneths residue of the yeare, wherefore here the products of 100 by 3, and of 80. by 9. are 300. and 720. which added together make 1020. for the latter antecedent particuler to A.

Secondly sith B disbursed 60. l. the first of March, and 100. l. more the first of August, therfore he had 60 l. in banke 10 months, and 100. l. 5 moneths of the yeare, now the product of 60. by 10. is 600 and of 100. by 5. is 500. and then 600 + 500. make 1100. the latter antecedent particuler to B.

Thirdly forasmuch as C put in banke 140. l. the first of Julie, whereof he withdrew 40. l. agayne the first of October. Therefore he had 140. l. in banke but 3. monethes and 100. onely other 3. Monethes of the yeere, now the products of 140. by 3. and of 100. by 3. are 420. and 300. whose totall is 720. the latter antecedent particuler to C.

Then the totall of (1020 + 1100 + 720.) euery partners seuerall products together, is 2840. which as it by the latter part of this Maxim serueth for the generall antecedent, so doth 142. l. the totall gaynes serue for the generall consequent thereto, and then euery particuler totall of the products made by the numbers belonging to one man alone, and found as before shalbe the latter antecedent in the golden rule, to whose figure addressing these tearmes, and working orderly thereby, as the rule requireth, I finde that A. shall haue 51. l. B. shall haue 55. l. and C. shall haue 36. l. of this gaynes, as here the seuerall operations expresse.

A		B		C	
2840	142 l.	2840	142 l.	2840	142 l.
1020	51	1100	55	720	36

And because 51 + 55 + 36. make iustly 142. l. therefore I affirme the operations and answere to be true.

Euma.

Eu. This queſtion is nothing ſo eaſie as were the queſtions anſwerable by the foꝛepart of this maxim, foꝛ it requireth much diligence to finde the certaynetie of the time, that euery mans poꝛtion remayneth in banke.

Philo. As that is a matter of no great labour to doe, ſo it being once well & truly examined, there is no great difficultie in the reſt, as you ſhall perceiue by another queſtion, which ſeemeth much moꝛe intricate then this, and yet is eaſily anſwered, and it is thus.

78. Another queſtion of Partnerſhip with more diuerſitie of time.

A. B. C. and D. entring partnerſhip together foꝛ 2. yeeres ſpace, make their banke oꝛ common ſtocke ſo, that A. immediately put in banke 30. l. but at 8. monethes end he receiues back 10. l. thereof, foꝛ the which in the beginning of the 20. moneth he reſtoꝛed 12 l.

B. put in 24 l. immediately alſo, but at the end of 6. monethes he reuoked backe 8. l. foꝛ the which in the beginning of the 16. moneth he ſupplied 14 l.

C. put in 20. l. at the beginning, which at 7. monethes end he receiued againe wholy, but then in the beginning of the 18. moneth he reſtoꝛed 16. l.

D. diſburſed nothing till the beginning of the 7. moneth, what time he put in 18. l. but after 4. monethes expired, he receiued backe 9. l. thereof, foꝛ the which in the entrance of the 17. moneth he reſtoꝛed 15. l.

Now at the 2. yeeres end (which is accompted 24. monethes) they had gayned clearely 100. l. whereupon it is demaunded how much thereof euery partner ought to haue.

Here the numbers which muſt be multiplied together foꝛ A. as thoſe that belong to him alone are firſt 30. l. and 8. monethes, whoſe product is 240. but when he receiues 10 pound againe at the end of 8. monethes, then had he but 20. pound remaining in banke the reſidue of the 24. monethes, that is to ſay 16. monethes, wherefoꝛe his ſecond numbers to be multiplied together are 20. and 16. whoſe product is 320. Thirdly when he reſtoꝛed 12. pound in the beginning of the 20. moneth, then was the ſame 12. pound remaine in banke 5. monethes, and ſo the numbers which make his third product is 12. and 5. and the product it ſelfe is 60. now all theſe 3. products 240 + 320 + 60. ioyned by addition make 620. the latter antecedent particuler to A.

Secondly foꝛ aſmuch as B. put in 24 pound in banke 6. moneths, therefoꝛe his firſt numbers to be folded together is 24. & 6. whoſe product is 144. Againe when he receiueth backe 8. l. at the end

of

of 6. monthes, then hath he but 16. l. left in banke the residue of the 2. yeeres, that is 18. monethes, wherefore his second product, made by 16. and 18. is 288. and so when in the beginning of the 16. moneth, he made a supplie of 14. l. then did the same 14. l. rest in bank 9 monethes of the 2. yeeres, wherefore his third product, being produced of 14. and 9. is 126. and the totall of these his 3. products is 558. serving for the latter antecedent perticulerly to the same B.

Thirdly touching C. the product of 20. by 7. is 140. and the product of 16. by 7. is 112. the totall of both which maketh 252. the latter antecedent particuler to him.

Lastly for D. sith first he layd out 18. l. but 4. monethes his first product, being produced of the same numbers is 72. and forasmuch as when he receiued backe 9. l. he left other 9. l. in banke from thence till the end of the 2. yeeres (that is 14. monethes) therefore his second product made of 9. by 14. is 126. but for asmuch, as in the entrance of the 17. month he restored 15. l. all the rest of the 2. yeeres (that is 8. monethes) therfore the product of 15. by 8. which is 120. shalbe his third product; the totall of which 3. seuerall products is 318. making his latter particuler antecedent.

Thus hauing found out euery mans particuler antecedent, as this present latter verse of this maxim intendeth, now do I take the totall of them all which is 1748. for the first tearme or antecedent generall, in the golden rule, and the 100. l. which is the totall gaynes for generall consequent thereto, and then euery of the sayd particuler antecedents by turne in his proper place. And so applying them to the figure, I do thereby boult out euery mans particuler consequent or seuerall share of the gaynes, saying thus, if 1748. haue 100. l. what shall 620. A. haue, and what shall 558. B. haue, and what 252. C. and what 318. D. and the operations bying such aunsweres as here you see annered to the figures tayle.

A	B
1748 Z 100 l.	1748 Z 100 l.
620 Z 35 $\frac{205}{437}$	558 Z 31 $\frac{411}{437}$
C	**D**
1748 Z 100 l.	1748 Z 100 l.
252 Z 14 $\frac{182}{437}$	318 Z 18 $\frac{54}{437}$

And thus as you see the inuention of the termes requisite for the conducting of this and such other like questions to their finall conclusion by the golden rule, is fully and sufficiently shewed & expressed in the sayd latter part of this present maxim, which sayth, that as the totall

of

of all the products serueth euer aptly, for generall antecedent to the totall gaynes or losse happening, so shall the totall of the products made by the termes belonging to one man onely, be the latter antecedent particuler to his proper consequent vnknowne.

Euma. I cannot deny, but that the woordes of this Maxim doe not onely lead the way directly, to the finding and artificiall compounding of both the Antecedences, but also that they do notifie, and specifie the first consequent in the golden rule, whereby the 3. termes due thereunto are orderly prepared, and placed as the 245. verse requireth, but the scanning of the partners times, of puting in, and taking out their stocks, is that whith troubleth me most.

Philo. That is a matter of nothing, to him that taketh any heed first to the time agreed vpon for continuance of their partnership. and next how many monethes euery partner suffereth his propper stocke to remayne in danke. wherfore leauing that to your priuate practise, I would here end the rules of societie or partnership, saue that there commeth one rule more to my minde, which is a by braunch by it selfe, not springing from any of the former verse nor Maxims, and it is such as followeth in the next Chapiter.

Of a third rule of partnership or societie, not depending nor proceeding from the former Maxim: nor concluded by the Golden rule. Chap. 16.

268. Yet doth there one rule of partnership rest,
VVhere neyther the totall to be diuided,
Nor yet any single part is expresst,
But 2. parts or mo in one sum described,
So that the totall is first to be sought,
VVhereby the parcels are found as they ought.

The 16. & 17. probleames of Diophantus 1. booke reduced to a card by Xylander.

269. And in this rule looke how often you finde,
Each partners name put, or letters therefore,
So oft the totall of numbers assignd,
Contaynes the true totall sought euermore,
From which each mans part, abated saue one,
The rest is his part so left out alone.

219

79. A question of vnknown shares among partners.

As for example, three partners A. B. and C. hauing gotten a certayne number of Angels which I know not, agree to diuide them so, that the shares of A. B. together shal be 20. Angels, & the shares of

B.C. together, shall be 30. and the shares of C. A. together 40. wherefore the question is, how many Angells they haue in all, and how many belongeth to euery man seuerally for his single share or portion.

Now, heere as you see, neyther the totall summe of their angells to be diuided, nor yet any one mans single share or seuerall part is expresst, but 2. mens parts together is expresst in one sum, euen as the first part of this rule importeth, and therefore by the same verse the totall must be first sought, whereby the seuerall parcels or shares shall be also found.

Therefore first for finding of the totall the latter verse of this rule willeth me, to looke how often I finde each partners name put, or letters for it, for so often (saith the verse) doth the totall of the numbers assigned, containe the very true totall sought for.

Wherefore I describe the letters for the partners names with the numbers assigned to them, by the fourme of the question, which is thus, whereby I finde their totall to be 90. as appeareth.

A.B.	20
B.C.	30
C.A.	40
Sum.	90

And because that in this description of the letters for the partners names, with their ioynt parts assignd, as the question importeth, I finde euery of the 3. letters A.B. and C. twice put, therefore I conclude by the latter part of this maxim, that the same totall 90. doth twice containe the true totall of their angels which I seeke: wherefore diuiding 90. by 2. the true totall issueth in the quotient, to wit 45. angels.

Which totall being thus found and known, now if from it, I abate the shares of all the partners saue one, that is to say if I subtract the ioynte parts of any 2. partners, (for there are but 3. in all in this question) the rest remayning must needes be the single share of the third, that is, of him so left out alone.

Wherefore sithe the ioynte parts of A. B. are 20. I abate it from 45. and so remaineth 25. for the single share of C.

Againe, if from the same totall 45. I abate 30. which is sayde to be the ioynte shares of B. C. the remainder being 15. sheweth the single parte of A. the other partner now left out.

Finally, if from the same totall 45. I abate 40. which is affirmed to be the ioynte parts of C. A. the remainder which is 5. expresseth the single share of B. the partner now left out.

Ans

And now for proofe that the same 3. numbers, are in such proportion together as the question requireth, the 3. couples of numbers here linked together do fully declare.

20 { A. 15 / B. 5 } 40
39 { C. 25 }

Whereby you may see, that A.B together haue 20. angells, B.C together haue 30. and C.A together haue 40. euen iustly as the question requireth.

Euma. This question seemed not so intricate and difficult at the first, as the fourme of your explication thereof hath nowe made it easie and light to answere.

Philo. Indeed there is no hardnes in it, to him that vnderstandeth the present maxim well, wherefore let me see you answere another like question, which is thus.

Foure legataries, to wit A.B.C. and D. haue a sum of money bequeathed, to be diuided betweene them, in such sorte that the portions of A.B.C. ioyned together may be 20 l. the portions of B.C.D. 22 l the portions of C.D.A. 24 l. and of D.A.B. 27. l. now tell what the totall sum bequeathed is, and what is euery mans seuerall part thereof. 40 A question of vnknown legacies.

Euma. For answere hereto, first I set foorth the letters for the legataries names, with their ioynt portions assigned as you did, so as here appeareth.

A.B.C. 20. l.
B.C.D. 22.
C.D.A. 24.
D.A.B. 27.

Sum 93.

But because I see euery of the said letters A. B.C.D. three times expresst in this description, therefore I conclude by the latter verse of this maxim, that the same totall containeth the true totall thrice, wherefore diuiding it by 3. (that so I may haue $\frac{1}{3}$ thereof) there issueth in the quotient 31. l. for the true totall sum of money bequeathed to them.

From which totall I subtract 20. l. the ioynt portions of A.B C. and so resteth 11. l. for the single portion of D.

Secondly sith B.C.D. together haue 22. l. I say 31. l. — 22. l. leaueth 9. l. for the share of A. the partner now left out.

Thirdly, for asmuch as C.D.A. haue ioyntly 24. l. I say thus, 31. l. — 24. l. leaueth 7. l. for the single share of B. the partner now omitted.

And lastly, seeing D.A.B. ioyntly haue 27. l. I say 31. l. — 27. l. leaueth 4. l. for the portion of C. the partner now left out, and so their seuerall shares appeareth here annexed to their letters.

A.

A B C. 20.r } Sum 93. whereof is { 31 —— 20 leaueth 11 r. D
B C D. 22. } 31. and { 31 —— 22 leaueth 9. A.
C D A. 24 } { 31 —— 24 leaueth 7. B.
D A B. 27. { 31 —— 27 leaueth 9. C.

And that these numbers satisfie, the question is manifest, for 9 + 7 + 4. r the portions of A. B. C. make 20. pounds.

And 7 + 4 + 11. r. the portions of B C D. make 22. pounds.

And 4 + 11 + 9. r. the portions of C D A. make 24 pounds.

And finally 11 + 9 + 7 r. the portions of D A B. make 27. pounds which are the very numbers the question requireth.

Philo. You haue done well, and I perceiue you vnderstand this rule sufficiently hetherto, wherefore I proceede with that which yet remayneth behinde thereof, which is this.

The 18. 19. & 20. problems of Diophantus first booke reduced to this one rule by Xylander

270 But if the question be propounded so,
That euery mans stock or share sauing one,
Conioynd together shall make alwayes mo,
Then that one partners omitted alone,
By some giuen number be it great or small,
Then is there a difference in finding the totall,
271. For if the partners be onely but three,
All 3. excesses for totall then take.
But if they be 4. then shall you playne see
That halfe the excesses the totall doth make, } 270
If 5. then a third part, and so forth still,
You may well proceede, as farre as you will,
272. Which totall known ech single share to finde,
first from that totall abate the excesse,
Of them which in place are foremost assigned,
So halfe the remnant doth his share expresse
which by his fellows, is so surmounted,
After which rate the rest may be counted.

21. A question of vnknown charges of a ship, by partners.

As for example, three partners to wit A. B and C bought a ship together, which cost them I knowe not how much, neither knowe I what euery seuerall partner disbursed toward the payment thereof, but this I am sure of, that A. and B. together payd more then C alone by 50. r and that B. and C. together, paid more then A. alone by 230. r and that C. and A. together layd out more then B. alone by 150. r.

Now the question is, what this ship cost, and what portion thereof euery partner seuerally payd.

Heere you see (like as before in the 219. maxim,) that neyther the totall, nor yet the seuerall parts or shares of the partners are giuen or expresst, but among these 3. partners, the stock of any couple conioyned, is sayd to be alwayes more then the stocke or share of the other partner ommitted by some number giuen or expresst.

And the excesses or numbers whereby the stock of each couple, doth so exceed the other partners stock ommitted, are heere 50.l. 230.l & 150 l. which added together, make 430 l. wherfore by the middle verse of this maxim (sith the partners are but 3.) I take the same 430.l for the true totall or very sum which the ship cost.

The totall, being thus knowne, now to finde out euery mans single share disbursed, first as the last verse of this maxim biddeth me I abate 50.l. from it, which is the excesse of A.B the 2. formost partners, and the remanent is 380.l. whose halfe or moitie is 190. pound, the share or portion disbursed by C. for C. is he which is here sayd to be exceeded or surmounted by his fellowes A. B. and after this rate the same verse concludeth, that the rest of their shares may be compted and found.

Which is asmuch to saye, as that for the next partners share I must abate the next excesse from the totall, taking the moytie of the remainder for his share, which is presently exceeded or surmounted by his fellowes, and so forth for euery one, how many soeuer they be.

Euma. Then (as I take it) 230.l. must be next abated from the totall 430 l. and so the remainer is 200 l. whose halfe being 100 l. is the portion disbursed by A. because it is he now which is presently said to be surmounted by his fellows B. C.

Philo. You conceiue it rightly. And therefore now it followeth, that 150 l. being lastly abated from the same 430.l. there resteth 280.l. whose halfe or moitie is 140 l. the true sum disbursed by B. because now it is he, whom his fellowes A. and C. is presently said to surmount or exceede.

And that these three numbers, to wit, A. 100.l. B. 140.l. and C. 190 l. satisfie, the conditions of the question is manifest. For first their totall by addition is 430. l. iust equall to the totall found before by this maxim. Secondly A B. together make 240. l. which is more then C. 190. by 50 l. Thirdly B C. together make 330. l. which is more then A. 100. by 230. l. And lastly C. A. together make 290. l. which exceedeth B. 140. by 150 l.

And that you may conceiue the forme and order of the woorke the better, beholde the whole fabrique thereof, as it is wrought, heere annexed.

A B = C + 50.r.
B C = A + 230.
C A = B + 150.
Totall. 430.r.

And then { 430—50. leaueth 380. whose half is 190. C
430—230. leaueth 200. whose half is 100. A
430—150. leaueth 280. whose half is 140. B } Er- go. { A 100, B 140, C 190

Euma. I vnderstand the whole maxim very well, now by this example wrought so plainely, I perceiue the order of the fabrique well also in all things sauing one, and that is in the double lines put thus = betweene the letters, whereby I know not what is meant.

Phi. Those two lines generallie serue to signifie an equalitie betweene the valure of the letters or numbers situate at each end of them: and in pronunciation is asmuch to say as (Equall to) and are often vsed for breuities sake, like as the signes + and — are vsed for more and lesse, as you shall better perceiue hereafter.

Eu. I vnderstand it fully already, by that you haue sayd. for now I perceiue that A B = C + 50.r is asmuch to say as that A B is equall to C. and more 50.r. and so forth of the rest, according to the conditions and propose of the question.

82. A question touching the buying of an horse.

Phi. It is nothing els, but now I would see how you can answer this question. Foure partners, to wit A B C. and D. bought an horse to beare their factor ouerland from Antwerpe to Burdeux, whose price I know not, neither what any one of them paid toward it, but this I know, that A B C. together paid more then D. by 20. crowns and that B C D. together laide out more then A. by 30 crownes, and that C D A. ioyntly disbursed more then B. by 40. crownes. and that D A B. together paid more then C. by 50. crowns, now I demaund what this horse cost. and how many crownes euery seuerall partner paid thereof.

Euma. I will do mine endeuor, as shall appeare, and first I set out the whole effect and conditions of the question according as it is propounded, which is thus.

A B C = D + 20.cro.
B C D = A + 30.
C D A = B + 40.
D A B = C + 50.
Totall 140.

Which being done, I adde all the excesses together, and their totall is 140. as appeareth. Whereof because there be 4. partners, therfore by the second verse of this maxim, $\frac{1}{4}$ shall be the totall sought: so that the totall price which this horse cost, is 70. crownes.

And

And heere, although it is affirmed by the same verse, that there is a diuersitie in finding the totall, yet I perceiue by the last verse, that the finding of euery partners seuerall part or share, is alwayes vniforme and after one fashion, wherefore to proceede, I take the same totall, saying thus.

70—20. leaueth 50. whose halfe is 25. D
70—30 leaueth 40. whose halfe is 20. A
70—40. leaueth 30, whose halfe is 15. B
70—50. leaueth 20. whose halfe is 10. C

Ergo { A. paid. 20. cr. / B. paid. 15. / C. paid. 10. / D. paid. 25.

And here as the perticuler payments amount equall to the total, so do they also satisfie the conditions of the question, for it is apparant that A B C. together paid 45. crownes, which is more then D. 25. by 20. crownes and that B C D. together paid 50. crownes which exceedeth A. 20. by 30. crownes, and that C D A. together paid 55. crownes, which surmounteth B. 15. by 40. crownes, and that D A B. together paid 60. crownes, which is more then C. 10. by 50. crownes, as the conditions of the question require.

Phi. You haue done very well, and euen so if there were 5. partners, then by the second verse of this maxim, you must take $\frac{1}{4}$ of the excesses added together for the totall.

But if there were 6. then must you take $\frac{1}{5}$ thereof, and so forth continually. And as for the particular partes or single portions, eyther to be receiued or payde by any partner seuerally, the finding of them remayneth alwayes vniforme and vnuariable, as your selfe euen now confessed, all which I will referre to your priuate practise, and will now proceede to the multiplication of proportionalities.

Of the rule of Proportion, or golden rule, artificially compounded by multiplication, or the multiplication of proportionalities. Chap. 17.

273. When 2. proportions discontinuall,
are both direct and each so to other,
That the last terme of the first is equall,
vnto the second terme of the tother.
Then as the product duely compounded,
by each of their first antecedents alone,
Is in proportion to like product founded,
by each of their last antecedents in one.

So ſhall the conſequent firſt of all placd,
be in proportion or rate to the laſt,
274. But if ſuch firſt proportion be direct
and tother conuerſt, it breeds this euent,
That as the product compounded by th'act,
of folding the firſt and laſt antecedent,
Is to the product of tother couple,
of antecedents not yet toucht at all,
So likewiſe ſhall the ſecond terme buckle,
in rate to the 8. or laſt terme of all. 221

Now heere, firſt to expound the ſeparte of this Maxim, or the firſt verſe thereof by examples and occulatie demonſtration. Let A and B. be 2. proportions diſcontinuall, both being direct, and ſo each to other, that the laſt terme of A. may be equall to the ſecond terme of B. as here you ſee it is, where the laſt terme of A. is 4. and ſo is the ſecond of B.

A
100 — 10
40 — 4
B
12 — 4
6 — 2

Now ſaith the firſt verſe, that as the product made or compounded of each of their firſt antecedents onely is in proportion to the like product founded or made by each of their laſt antecedents, ſo ſhall the firſt conſequent be to the laſt.

But the firſt antecedent of A. is 100. and the firſt antecedent of B. is 12. (which as you ſee I haue coupled with a lyne as followes to be multiplied together, and their product is 1200. and then their laſt antecedents (as you know well) are 40. and 6. (which I haue alſo coupled with another lyne for the like purpoſe) and their product is 240. wherefore I conclude by this firſt verſe preſent, that as 1200. is to 240. antecedent to antecedent, ſo is 10. the firſt conſequent, to the laſt conſequent 2.

And therefore I affirme that I haue multiplied thoſe 2. proportionalities, A. and B. together and that the product of their multiplication, is the proportionality C. here annexed.

C
1200 — 10
240 — 2

But I finde by the 182. verſe that the proportion of A. is decupla, for ſo is each antecedent thereof to his conſequent, and for like cauſe the proportion of B. is tripla, the leaſt termes of which proportions are as 10. is to 1. and as 3. to 1. which now if I ſhould multiplie together like fractions, as the gloſe of the 182. verſe importeth, the products will be as 30. is to 1. that is Trigintuple,

but

But the proportion C. which you haue produced is Centumvigintupla or as 1201 to 1, wherefore, vnlesse I mistake you either there or here, this product cannot be true.

Phi. Truly you mistake my meaning here very much indeede, for seeing that this maxim contayneth a Theoreme, wherein the chiefest thing taught, is to shew what proportion is betweene the first consequent and the last of 2. proportionalities, that is to say, between two matchtermes or genderlike termes, and seeing also that the meanes thereunto, is onely by the opperation of the antecedents of both proportionalities which are also matchtermes, therefore I say, you are greatly deceiued, if you imagine any other relation or conferrence heere, then onely betweene genderlike termes, that is of antecedent to antecedent, and consequent to consequent, which by the 9. verse are as truly proportionall each to other, as the antecedents are to their proper consequents, though the quantitie of their proportions, bee not both wayes like.

And so in A. whether you take the proportion of 100 to 40. antecedent to antecedent, or of 10. to 4. consequent to consequent, both wayes you shall finde it Duplasesquialtera.

Likewise in B. the proportion of 12. to 6. or of 4. to 2. are both wayes Dupla, the least termes of which proportions are as 5. to 2. and as 2. to 1. whose product by the 169. verse, and by the glose of the 182. verse, is the proportion of 5. to 1. that is Quintupla, and so likewise is the proportion of the matchtermes in C.

For whether you conferre 1200. to 240. antecedent to antecedent, or 10. to 2. consequent to consequent, both wayes you shall see the proportion is Quintupla. Wherefore I say as before, that C. is the true product of A. and B. agreeable to the glosse of the sayd 182. verse.

Euma. I perceiue that a little mistaking may breede a great error, but I pray you to what purpose serueth this kinde of multiplication?

Philo. To an excellent purpose, for this first verse is the ground and foundation of that rule, which is commonly called the rule of 3. composed or compounded, directly: and the latter verse is sometime the ground of the same compound rule conuersedly, as shall appeare hereafter, both which rules the French abridger of Tartalea, calleth, La Reigle de cynque choses, and it booth almuch at

one operation, as the common single golden rule doth at twice for an example whereof I demaund this question.

83. A question of Interest.

If 100. l. stocke in 12. moneths, yeelde me 10 l. for interest, what shall 40. l. stocke yeelde for interest in 6. moneths?

This question by the 253. verse of Golden rule plurally, is answered at twice, or with two operations in this manner. First if 100. l stocke yeelde 10. l. interest, what shall 40. l. stocke yeelde? and the answer is 4. l. as heere appeareth by D.

stock. D. interest
100. l. 10 l.
40. 4.

Secondly, if 12. moneths bring 4. l. interest, what shall 6 moneths bring and the answer is 2. l as heere appeareth by E.

month E. interest
12 4 l.
6 2.

Now (if you marke it) these two proportionalities D. and E. consist of the selfe same termes in all points that A. and B. did before.

Euma. By conferring them together, I see very well it is so.

Philo. Very good, then like as heere, by the present verse whereof we entreat, it appeareth that 2. the last consequent of C. the product of proportionalitie compounded of these 2. proportions by multiplication, is the very number that bringeth the answer to the question. So to the end the same multiplication may proceed with the more speede and facilitie, I haue thought it a very apt forme for that compound worke, to compound the figure of the Golden rule also, and to double it in this fashion.

A B
C D
E F

Which figure thus doubled, hath now as you see 6. places, apt to receiue 6. seuerall numbers, agreeable to the nature of these compound questions, which places for a difference, I will name by the names following, that is to say.

Head of the figure.

The first end or vpper part where A standeth, I call the head or first place. B I name the vppercangle or second place, C. I call the second angle or third place. D the third angle or fourth place, E the lower angle or fifte place. And F. the taple or aunswering place.

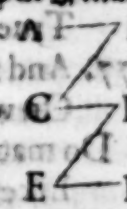
Tayle of the figure.

Meanes or middle termes, by place, and not by quantitie and so likewise are the extremes.

Also when I speake of the termes setled in the said places by couples, I account and name A and B the two vppermost termes. C. and D. the middle termes or meanes. B. and C. and D. and E. the bias termes. B. and E. the extreame angles, and A. B. E. the extreame termes.

Euma, As this distinction of names may serue for necessarie causes,

causes, so it is neither difficult to vnderstand, nor troublesome to remember, especially sithe the very shape of the figure, and sight of the termes, will alwayes easily bring it in memorie. But I marualle you doe not reckon F. among the extreames, sith that both in the 203. verse, & in the 204. verse, where there are figures described, almost like to this, you account the termes of the head and taile, and the 2. extreame angles, the 4 extreames.



Philo. You say well, but I do not name F. here among the extreames, because the number that supplieth that place, is neuer setled there, till all the worke is fully finished, so that it is none of the termes or numbers vsed, or applied to the operation, but it is the onely terme or number sought for, by the operation, and for whose sake, this distinction of names is made, which when we haue found, the rule hath done his duty, and needeth no such distinctions, for the direction thereof.

Euma. You haue shewed me a good reason which I vnderstand very well.

Phi. Now resteth to giue you some rule, whereby to shew you how the termes or giuen numbers of these compound questions are to be placed in this figure, and which of them make the dividend and which the diuisor, by whose diuision there issueth forthwith in the quotient, the true number sought for, as answer to the question, and that not onely at one operation, or by one application of numbers to the figure, but also with much more facilitie then by the golden rule plurall, which some men call the Double rule.

Euma. Such a rule were very commodious.

Philo. Then note well the verses following.

275. As in the golden rule direct,
Three numbers dooth the fourth detect,
Declaring that, with full effect,
which we desire to know.
So in the rule of three compound,
fiue numbers must be laid for ground,
Before the sixt can well be found,
which dooth the answer showe.

276. And of this rule foure kindes there be,
which in their workings disagree,
Wherefore be sure you do foresee,
to vse each kinde in place.

For if you should mistake the kinde,
Wherein the answer is assignd,
You might long seeke but neuer finde,
True answer to the case.
277. And first, when those 2. termes alone.
On which the question stands vppon,
Do match in firname one to one,
Like to the first and second,
The answer by the first rule seeke,
VVho claimes the first and 4. termes like,
And so the extremes angles eke,
All which foreseene and record,
278. Then multiplie the latter three,
Inferior numbers which you see,
Each by other, and so haue ye,
Your diuidend obtaynde,
VVhich done, infold those 2. aboue,
VVhich placed first did neuer mooue,
The product shall diuisor proone,
VVhereby your will is gaynde,

Euma. The first returne ouerse of this maxim is so plaine, that it needeth no exposition, and the second sheweth as plainly, that there are 4. kinds of this compound rule, which haue vnlike formes of working, and therefore it giueth a caueat to the worker, to take heede, that when a question is propounded, answerable by any of the same 4. kindes, that he wisely foresee to worke it by that kinde of rule, whereunto it is properly subiect, for otherwise I perceiue he shall misse in the answere, but the rest I yet vnderstand not.

Philo. Indeed this compound rule is faide to conteyne foure kindes, because it is to be wrought 4. seuerall wayes, whereof the first kinde being it that we haue presently in hand, is called the compound rule direct, for that in it both the proportions are alwayes taken and applied directly.

And aswell the first kinde as the third, proceede originally from the 273. verse, but the second is called the compound rule conuerse, because in it sometimes, one of the proportions is applied directlie, and the other conuerslie, and sometymes it springeth originally from the 274. verse, as I will shew you in due place, and these two rules are primitiues, as for the

other

other two kindes may be called deriuatiues, because they be (after a sorte) deriued from the 2. former rules, but they are not compounded after one sorte, nor yet of like tearmes, and of euery of these 3. I will speake more in their due places.

Now touching the text of this rule or maxim present, which you say you vnderstand not, I say that the third verse or ouerne, sheweth you how you should knowe when a question propounded is answerable by this compound rule direct, as for example, I put the last question which was this, if 100. pounds stocke in 12. moneths bring 10. l. interest, what interest will 40. l. stocke bring in 6. monethes?

Here as you see and as the first verse of this maxim affirmeth, are 3. tearmes or numbers giuen, to wit, 100. 12. 10. 40. and 6. whereof the 3. first are no part of the question, but contayne a condition, or rather a supposition, as when I say thus, if 100. l. stocke in 12. monethes bring 10. l. interest, what then? or suppose that 100. l. stocke in 12. monethes do bring 10. pound interest, what then?

Supposition.

84. A question of interest compounded.

Mary then I say, that thereupon followeth the question, to wit, what interest shall 40. pounds stocke bring in 6. monethes after that supposition.

So that the 3. tearmes which onely make the question, and whereon it standeth and onely dependeth, are the 40. l. stocke, and the 6. monethes, and they ought generally to settle (after the tearmes of the supposition) in the fourth and last places of the figure, as here you see them placed.

Tearmes demaunding or asking the question.

100 l.	12	
interest	stocke	
10. l.	40. l.	
monthes		
6		

Now for asmuch as the same 3. tearmes match in strayne, with the first and second tearmes of the supposition (for the 40. pounds stocke is match tearme to the 100. l. stocke, which is the first tearme, and the 6. monethes is likewise match tearme to the 12. monethes, which is the second tearme) therefore is the answere of this question (sayth the 3. verse) to be sought only by the first kinde of rule, that is by the compound rule direct.

And as by the 245. verse, the single golden rule, requireth the first and third tearmes, to be both of like sirname or qualitie, so here the 277. verse declareth, that this compound rule direct, requireth the first and fourth tearmes to be genderlike or of one sirname, and that the tearmes of the 2. extreame angles be of another

ether surname like together, and then it followeth that the tayle
Tayle terme or answering number. terme or answering number, yet vnknowne, shall match in sir-
name with the other single terme remaining, which as yet hauing
no match, doth as it were expect the tayle terme for his match, and
so when the first number is found, there will appeare 3 couples of
Matchtermes in the figure.

Euma. Very good, all this I vnderstand well, and see by viewe of the former figure, that the first and 4. termes match together in the worde (stocke,) and that the 2. extreame anguler termes match together in the worde (monethes,) and that the 10. r. interest remayneth single without any match as yet, but looketh for the 6. terme to match with him when it is found and knowen as may be partly gathered by the third verse of this Maxim, but is more playnely declared by your speech.

Philo. Then to come to practise the solution of the question by due operation of the rule, first (as the last verse here biddeth me) I multiplie 10. 40. and 6. the 3. last of the 5. numbers, which I see heere in the figure each by other, & they produce 2400. for my diuidend, and then (as it further biddeth me) I multiplie 100. and 12. together, the 2. numbers aboue, which being first placed, were not yet moued or touched, and they produce 1200. for my diuisor, and now finally, diuiding this 2400. the diuidend by 1200. the diuisor, the quotient is 2. which being the number sought for, I ascribe to the figures taile as his due place with his proper surname (of pofids interest) as here you see. concluding thereby, that if 100. r. stock, in 12. moneths gayne 10. pound interest, then 40. r. stocke in 6. moneths shall gayne 2. r. interest and this you see is don at one operation and that very speedy.

100 r.	—	12
stocke		interest
10 r.	—	40 r.
months.		interest
6	—	2 r.

Eu. Indeed this seemeth to be done both easily and quickly wherfore I pray you let me haue the working of some like question, to trie how I can answer in.

Philo. You shall with a good will. and let it be this. If 500. r. weight carriage 200. miles, cost 40. shilling, what will 100. pound weight carriage cost 50. miles?

85. A question of carriage.

Euma. To answer this. I see that here are 5. seuerall termes or numbers giuen as before, viz. 500. 200. 40. 100. & 50. and of these 5. it is euident, that the first 3. containe onely a supposition as you said before, and that the question standeth altogether vpon the

2. last

2. last, which is, what 100. pound weight cariage 50. miles will coſt?

Wherefore making ſuch a figure as you preſcribed, I endeauor to applie the tearmes, thereunto in their due places, beginning firſt with thoſe three which contayne the ſuppoſition, & ſo following with thoſe 2. which contayne the queſtion, whom I place at the 2. latter angles, in ſuch ſorte that it which matcheth with the firſt terme be firſt placed, and th it which matcheth with the ſecond, be laſt placed, and ſo the figure being furniſhed with his termes & ſirnames as here appereth.

weight	miles
500 l.	200
ſhil.	weig.
40	100 l.
miles	
50	

I peruſe them once more how they match together, and finde the firſt and fourth termes like, for they match both in relation to weight, and ſo the extreame angles eke which match together in relation to miles, and thereby knowing that the anſwere is to be ſought by the firſt kinde of this compound rule (as the third verſe of this Warim teacheth) I practiſe the operation thereof, as the laſt verſe biddeth me, multiplying 40. 100. and 50. together which are the 3. laſt numbers of the figure & they produce 200000 which I take for my diuidend.

Then do I multiplie together 500. and 200. the other 2. termes ſtanding vpermoſt, & they produce 100000. for my diuiſor, and ſo performing diuiſion, I finde the quotient 2. which I aſcribe to the tayle of the figure, as true anſwere to the queſtion thus.

weight	miles
500 l.	200
ſhil.	weight.
40	100
miles	ſhilling
50	2.

Concluding now thereby that if 500 l. weight carriage 200. miles coſt 40. ſ. then 100. weight carriage 50. miles ſhal coſt 2. ſ. for by thentent of the queſtion the laſt number muſt match with the third, which is ſhillings, and ſo now there appeareth 3. couples of matchtermes, as you ſayd there would be.

Phi. You haue wrought this queſtion very well, wherefore here before I goe any further, I will deliuer you another rule alſo whereby when the termes of this rule chance to be fractions, or fractionlike, you ſhall notwithſtanding worke it as eaſily as if they were Integers.

Fums. That were very commodious, for if 2. or 3. of the termes happen to be fractions, & the reſt Integers, yet can I ſoone make ſuch Integers fractionlike by ſubſcribing an vnit to them.

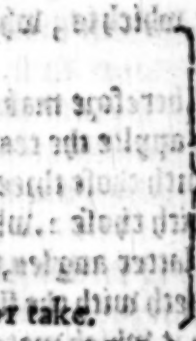
Phi. You may ſo, wherefore behold the rule is thus.

279.

279. If in the compound rule direct,
All termes be fractions in effect,
For ease and playnesse sake.
Fold both the first denominators,
In all the last 3. numerators.
Your dividend to make,
Then fold the 2. first numerators
In latter 3. denominators, which for diuisor take.

86. A question of interest.

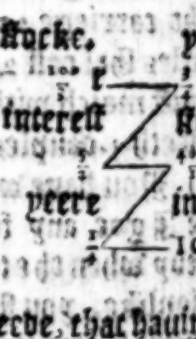
As for example I am asked this question: if 33. l. 6. s. 8. d. stocke in $\frac{3}{4}$ yeere bring 2 $\frac{1}{2}$ l. for interest, what shall 100. l. stocke bring for interest in $\frac{1}{4}$ yeere.



Now herein you see all the terms alloone as they are reduced to simples, are eyther fractions or fractionlike, sauing the 400. l. which is soone made fractionlike, also by subscribing an vnit to it as you sayd.

Eu. I see that very well, and therefore I thinke that the termes must be applied to the figure in such forme as heere appeareth aboue, by which situation the 277. verse informeth me, that the question is subiect to the compound rule direct.

Phi. It is so, and you haue placed the termes well in the figure, wherfore now to proceed to the answer of this questiõ, as this verse present prescribeth. I folde 3. & 4. both the first denominators in 5. 400. and 1. the last 3. numerators, whereby I produce 24000. for my dividend. Then do I fold 100. and 3. the two first numerators in 2. 1. and 4. the latter three denominators, whereby I produce 2400. for my deuisor, finally performing the diuision, the quotient deliuereth 10. l. for the aunswering number which therefore I anner to the figures tayle, as heere you see it.



Concluding that 400. pound stocke, will bring so much interest in $\frac{1}{4}$ of a yeere.

Eumathes. This is a very good rule indeede, that hauing to doe with so many fractions, concludeth the question so playnly and easily.

87. A question of interest.

Phi. Because you commend it so well, answere me now thereby to this question, if 400 l. stocke in $\frac{1}{4}$ yeere, bring 10. l. for interest what interest will $\frac{1}{2}$ l. stocke bring in $\frac{1}{4}$ yeere.

Euma.

Eu. I will do mine endeauor, and for that purpose I addresse the tearmes to the figures thus.

When I see their situation is agreeable to the 277. verse, for the first & fourth termes are matches, and so are the extreame angels also, whereby I am assured that the question is vnder the compound rule direct, wherefore now to come to the operation therof by the tenor of this verse present, first I fold 1. and 4. which are both the first Denominators in 10. 100. and 3. the last three numerators, whereof resulteth 12000. my diuidend.

Stocke yeere
$\frac{400}{1}$ l. — $\frac{1}{4}$
interest stocke
$\frac{10}{1}$ — $\frac{100}{3}$ l.
yeere
$\frac{3}{4}$

Secondly (as this verse also willeth me) I fold 400. and 1. the first 2. numerators, in 1. 3. and 4. the latter three Denominators, whereof amounteth 4800. my diuisor.

Thirdly by executing the diuision there issueth $2\frac{1}{2}$ l. in the quotient for the taple tearme or answering number, wherefore I may conclude, that if 400. l. stocke, in $\frac{1}{4}$ yeere gayne 10. l. for interest, then $\frac{100}{3}$ l. stocke in $\frac{1}{4}$ yeere shall gayne $2\frac{1}{2}$ l. for interest, as the fabrique here sheweth.

Stocke yeere
$\frac{400}{1}$ l. — $\frac{1}{4}$
interest stocke
10 l. — $\frac{100}{3}$ l.
yeere interest
$\frac{1}{4}$ — $2\frac{1}{2}$ l.

Phi. As you haue done this very well, so let the same suffice for examples in the practise of this compound rule direct, aswell in fractions as Integers, vntill I haue likewise explicated the compound rule conuersed, and so the third and fourth compound rules in order.

Euma. May I pray you let me first interrupt you a little with a great doubt that presently troubleth me much, and it is this.

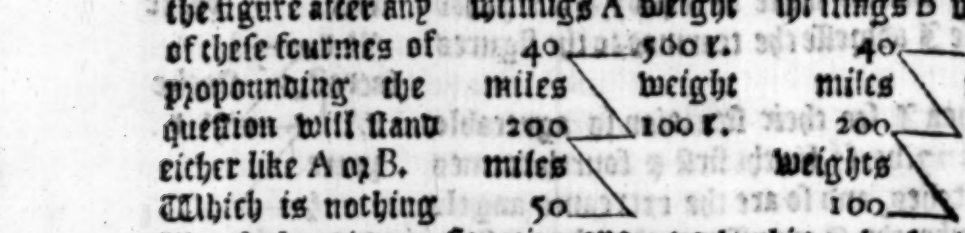
Me thinkes a question may be so propounded, that any man may be easily deceiued, in the right placing of the tearmes thereof, notwithstanding your rule prescribed in the 277. verse.

As for example let the 85. question touching weight and carriage be put forth in this order, when I pay 40. s. for the carriage of 500. pound weight, 200. miles, what will the carriage of 400 l. weight cost 50. miles, or what will 50. miles carriage of 100. l. weight cost.

In both which fourmes the question is the same it was before, and standeth vppon the selfe same tearmes, but neyther of them match with the 40. shillings, which here is now the first tearme of the supposition, insomuch that the tearmes applied to

the

the figure after any of these fourmes of propounding the question will stand either like A or B. Which is nothing like the location or situation appointed by this rule, for in the figure A neyther is the first and fourth termes like of sirname, nor yet the extreame angles.



And in the figure B. although the extreame angles be of like sirname, yet so are not the first and fourth tearmes.

Philo. You haue sayd well, and your doubt is a matter worthie to be moued, but the time aswell to moue it, as to answere it, had bene more fitter after the explication of the compound rule conuersed, because that in aunswering your doubte, I shall haue occasion to speake of that rule which as yet you knewe not.

But neuertheless, sith you are so importunate to ease your hastye desire. I say that howsoeuer the question be propounded, eyther in this first kinde of rule direct, or in the next which I call the conuersed kinde thereof, you shall generally applie that terme of the supposition to the first place, whose sirname is the principall cause of gayne or losse, profit or charge, encrease or decrease, action or passion, and next that put that terme in the second place, whose sirname betokeneth the space of time, distance of place, or other like circumstance, and then the other terme remayning must needs take the thirde place. Note well.

So the supposition being thus placed, now of those 3. termes which containe the question, let that be alwayes first placed, whose sirname is first mentioned in the supposition, wherby it followeth, that the other tearme remayning, whose sirname is mentioned afterward, must needes take the last angle for his place, and so doing, you shall neuer fayle to place your tearmes truly in the figure.

As for example, in the supposition of this question, the principall thing that is cause of the coste or charge, is the 500. pounde weyght, for without it, there were nothing carried, nor nothing to be payd) wherefore that is to be taken for the first tearme, and placed in the first place, next whom the 200. miles

miles cariage betokening the diſtance of place is therefore to be put in the ſecond place, and ſo conſequently the 40. ſhillings remayning, ſhall take the third place.

Now the 2. termes containing the queſtion are 100. l. weight and 50. miles, wherfore noting which of their ſirnames is firſt mentioned in the ſuppoſition, I ſee that weight is mentioned before miles, (for it is the very firſt ſirname there) and therefore I place the 100. pound weight before the 50. miles reſerving the ſame 50. miles to the laſt angle of the figure as here you ſee.

weight.	miles.
50 l.	200
ſhilling	weig
40	100 l
miles	
50	

And ſo the termes are all duly and truly placed, and matcht together, as the rule appoynteth and alloweth.

Fuma. If this be a perfect way to place the termes, I hope I will neuer miſſe, how peruerſely ſo euer the queſtion be propounded.

Phi. It is not onely a very perfect order, but alſo ſuch as doth wayes agree duly both with the 277. verſe, and alſo with the 280. verſe next inſuing, except it happen ſo, that the anſwering tearme or number ſought for, require like ſirname to the firſt, for then it is not poſſible that the firſt and fourth ſhould be ſirnamed like, (without diſagreement, from this order) as ſhall appeare anon. Note this. exception

And in the meane ſeaſon to proceede (as I was about to doe before you interupted me) with the compound rule conuerſt, which as I ſayd before ſpringeth ſometimes from the 274. verſe, and ſometimes from other verſes hereafter mentioned, I put this queſtion for an example thereof.

If 100. pounde ſtocke in 12. moneths bring 10. pound intereſt. in what time or in how many moneths will 40. l. ſtocke bring 2. l intereſt. 88. A queſtion of Intereſt.

Now this queſtion by the 274. verſe conſiſteth of 2. proportionalities, whereof the firſt is direct, and the latter conuerſt, but yet being in ſuch reſpect and behauiour together, that the laſt tearme of A the firſt proportion, and ſecond terme of B the latter proportion, are equall together as here appeareth.

Intereſt. A moneths
10 l. 12
2 2 1/2
ſtocke B moneths
100 l 2 1/2
40 6

Wherefore now by that verſe, as 400. the product of 10. and 40. the firſt

first and last antecedents is to 200. the product of 2. and 100. the other 2. antecedents, so is 12. the second tearme to 6. the last tearme, or so is the first Consequent to the last, that is to say, in proportion Dupla, and therefore by the same verse, if 100. l. stocke in 12. monethes bring 10. pound interest, then 40. pound stock shall bring 2. pound interest in 6. moneths, as the table terme B. before declared.

But now to compound this double woorking so, that one operation may serue for all, and withall, to know when a question propounded is subiect to this compound rule conuersed, I inferre this Maxim following.

280. When those 2. numbers which contayne,
A question put, are match tearmes playne,
To first and third, or to them twaine,
Which second are, and third,
The compound rule conuersed vse,
Who first and 4. termes like doth chuse,
And third to 5. whereby insues,
This bayte to catche the byrde.
281. First take the product of the extreames
for diuident in all such theames.
And then by product of the meanes,
Diuisor vp is sturde.

Hereuppon repeating the question thus, If 100. l stock in 12 moneths, bring 10. l interest, in what tyme will 40. l stock bring 2. pound interest, I consider that the 100 l. stock is the principall cause of the interest gayned, & therfore that it must occupie the first place, & that the 12. moneths being the space of time must be the second tetme, & so consequently the 10 l. interest shalbe the third.

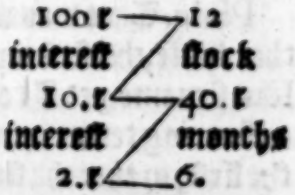
Agayne, because 40. l. stocke and 2. l. interest, (which are the 2. tearmes whereon the question dependeth) are match-tearmes to the first and third tearmes of the supposition, therefore by the 280. verse present, the answere is to be sought onely by the second kinde of rule called the compound rule conuersed. And as I late told you because the sirname stock is mentioned in the suppositiõ before intrest, which stock & intrest are the sirnames of the 2. terms of ỳ question, therfore in annexing the same interrogatiue termes to the figure, I place the 40 l. stocke before the 2 l intrest, as here you see.

stock	moneth
100 l.	12
intrest	stocke
10. l.	40 l.
intrest	
2 l.	

Which

Which termes ſo ſet and placed, the firſt and fourth are like, for they match both in ſtocke as you ſee, and the third and 5. are alſo like, for they both match in intereſt, whereby inſueth the meanes to catch the birde for which we hawke, that is, to ſolue the queſtion, which by the latter verſe of this maxim is to take the product of the 3. extremes for diuidend, and the product of the 2. middle numbers or meanes for diuiſor, the quotient of whoſe diuiſion, is the number for which we ſeeke.

But the product of 100. 12. and 2. which are the extreames, is 2400. and the product of 10. and 40. (the 2. meanes or middle numbers) is 400. wherfore diuiding 2400. by 400. there iſſueth 6. in the quotient which I aſcribe to the figures ſayle, with ſuch ſirname as the ſingle tearme hath which remained yet vnmatched, that is to ſaye, Moneths, as here you ſee.



concluding now that if 100. pound ſtock in 12. months, bring 10. r intereſt, then 40. r. ſtock ſhall bring 2. r. intereſt in 6. moneths.

Euma. So I perceiue that the compound rule conuerſt differeth from the compound rule direct, not onely in ſcituation of the match-termes, but alſo it compoundeth the Diuidend and the diuiſor of other termes, for in the direct rule, the product of the three latter tearmes maketh the diuidend: but in this, the product of the 3. extremes ſerueth to that purpoſe.

Agayne in the direct rule the product of the 2. vppermoſt numbers ſerueth for the diuiſor, which here is made of the product of the 2. midle numbers, but what if the queſtion were propounded thus? if 100. r. ſtock in 12. moneths bring 10. r. intereſt, what ſtock will bring 2. r. intereſt in 6. moneths? 89. Another queſtion of intereſt.

Phi. Then were the ſame alſo to be anſwered by this compound rule conuerſed, for by the 280. verſe preſent, when the 2. terms containing the queſtion are match-termes, either to the firſt and third, or to the ſecond and third termes of the ſuppoſition, then is the conuerſed rule to be vſed.

And in this caſe by my former doctrine the firſt terme of the ſuppoſition, muſt needs be the 100. r. ſtock, becauſe it is the principall cauſe of the intereſt gained, and then the 12. moneths muſt neceſſarily be the ſecond, becauſe it betokeneth the tyme, and ſo conſequently the 10. r. intereſt ſhalbe the third.

Now the ſirname of the 2. interrogatiue termes, are intereſt and

moneths which match with the ſecond and third termes of the ſuppoſition, but becauſe months is mentioned in the ſuppoſition before intereſt, therfore (as I taught you before) the 6. months muſt be placed before the 2. l. intereſt as you ſee it ſet in the figure.

ſtock.	moneths
100. l.	12.
intereſt	moneths
10. l.	6.
intereſt	
2. l.	

Fuma. But now the firſt and fourth termes heere match not in ſirname, as the 280. verſe preſent requireth, for the firſt ſirname is ſtock, and the fourth is moneths: wherefore by the ſame verſe, the termes ſeeme to be diſorderly placed.

Philo. That was well noted of you: but although generally by that verſe, the firſt and fourth termes ought to matche together in like ſirname, yet I tolde you but euen a little before, that when the anſwering terme or number ſought for, requireth like ſirname to the firſt, or that the firſt requireth to match in ſirname with the anſwering terme (which is all one thing) as heere it dooth, then in ſuch caſe it is not poſſible, that any of the 5. giuen numbers ſhould match with the firſt, ſithe the matchterme therof is yet vnknowne, and is the onely number for which we ſeeke.

For there cannot, nor doth not, aboue one couple of terms match together in any one queſtion, wherefore in all ſuch caſes, the two left-handed angles do ordinarily matche together, and ſo do the 2. right-hand angles. For ſeeing that here, the firſt ſirname is ſtock, and the queſtion is what ſtock: that is to ſay, how many pounds ſtocke, will bring 2. pound intereſt in 6. moneths. It is manifeſt that the anſwering number, which as yet we lacke, is the number of pounds ſtock.

And therefore when the 2. anguler termes on the left ſide are matchtermes, and likwiſe the 2. other anguler termes on the right ſide, as here in the former figure you ſee they are, how can the firſt terme by reaſon haue any matcheterme at all in the figure, vntill the laſt terme be found & placed, which remaineth yet vnknowne?

And yet (if you will) you may change places with the firſt and ſecond termes, ſetting each in others place, as heere I haue done. And ſo is al the contents of this preſent maxim accompliſhed, but in ſo dooing, the ſirname of the thing which is cheefeſt cauſe of gaine or loſſe, encreaſe or decreaſe, holdeth

moneths.	ſtocke.
12	100 l.
intreſt	month
10 l.	6
intreſt	
2 l.	

not

not the firſt place, as I taught you before, and as I would wiſh it ſhould be, becauſe it is generally the ſurer way, and yet neuertheleſſe, both theſe ſituations come to one ende, for ſeeing that the product of the extremes onely make the Diuidend, and that whichſoeuer of thoſe two vpper termes be firſt or ſecond, yet alwayes they remaine two of the extreames, the product of them euery way, ſhall be one ſelfe number, and ſo whether you take 100. times 12. twice, or 12. times 100. twice, both wayes, the product or diuidend ſhall be 2400.

Euma. All that I conſider, and further I ſee, that although theſe termes haue thus chaunged their places each with other, yet by the 280. verſe preſent, the queſtion is ſtill ſubiect to the compound rule conuerſed, becauſe now the two demaunding termes of the queſtion are matche termes to the firſt and third termes of the ſuppoſition.

And therefore to worke out this queſtion, as the termes thereof ſtand heere now in the figure.

I diuide 2400. the product of the extremes, by 60. the product of the meanes, and the quotient deliuereth 40. that is 40. l. ſtock, which I place at the figures taile, for the anſwere thus.

moneths.	ſtocke.
12	100 l.
intreſt	moneth
10 l.	6
intreſt	ſtocke.
2 l.	40 l.

Whereby I conclude, that if in 12. moneths 100. l. ſtock bring 10. l. intereſt, then 40. l. is the ſtocke, which ſhall bring 2. pound intereſt in 6. moneths, and ſo it ſeemeth that one of theſe queſtions is the triall of the other.

Phi. You haue ſaid very truly, for there is no better proofe of the direct rule then by the conuerſed, nor of the conuerſed then by the direct, whoſe agreement in returne of termes or numbers, is a certayne argument of the goodneſſe and trueth of the worke. Proofe of the direct rule by the conuerſed, and of the conuerſed by the direct.

Wherefore becauſe you ſee ſo well into theſe matters I will here ſhew you a fourme of wooꝛking in this compound rule conuerſed, like to that I gaue you lately before, touching the compound rule direct, when the tearmes thereof are fractions, or fraction like, whereby you ſhall accompliſh the opperation thereof alſo, as if it were in Integers, and the rule thereof is thus.

282. VVhen in the compound rule conuerſt,
The termes are fraction like diſperſt,
The diuidend to frame.
Infold the tops of all th'extremes,
And both the baſes of the meanes.
Together for the ſame.
Then folde the middle numerators,
And all th'extreme denominators,
Diuiſor ſo to frame. 225

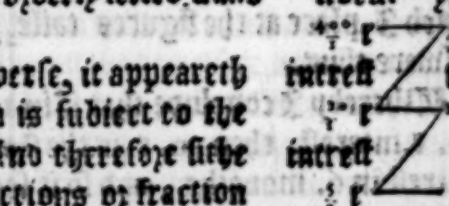
90. A queſtion in what time ſo much intereſt will grow.

As for example, I aſke this queſtion. If 400 l. ſtock in $\frac{1}{4}$ yeare, bring 10. l. intereſt, in what time will 33. l. 6. s. 8. d. ſtocke get me $2\frac{1}{3}$ l. intereſt.

In which queſtion you ſee, ỹ the 400. l. ſtock ought to be the firſt terme of the Suppoſition, becauſe it is the chiefeſt cauſe of the intereſt gained, next whom the $\frac{1}{4}$ yeare being the ſpace of time is to be placed, and ſo the 10. pound intereſt muſt needs be the third.

Then by my former inſtruction the 33. l. 6. s. 8. d. ſtocke, muſt be fourthly placed, becauſe it matcheth with the firſt terme of the ſuppoſition, and ſo conſequently the $2\frac{1}{3}$ l. intereſt, being the fifte terme, ſhall matche with the third.

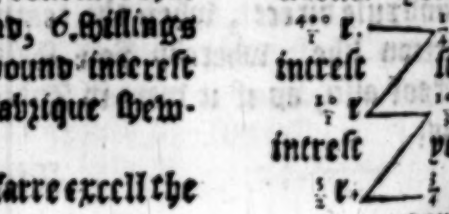
So that the termes being orderly ſetled ſtand as heere you ſee in the figure.



Wherby, and by the 280. verſe, it appeareth forthwith, that this queſtion is ſubiect to the compound rule conuerſed. And therefore ſithe the termes thereof are all fractions or fraction like deſcribed. I ſeeke the taile terme or anſwer thereto, as this verſe preſent biddeth me: that is, I infolde 400. 1. and 5. the tops of all the extreames, and 1. and 3. both the baſes of the meanes together, and their product is 6000. my diuidend.

Secondly I folde together 10. and 100. the middle numerators, and 1. 4. and 2 which are all the extreame denominators, and their product yeeldeth 8000. for my diuiſor.

Finally, the diuiſion ended, there iſſueth $\frac{3}{4}$ yeere, for the anſwering number: wherfore I conclude that $\frac{200}{3}$ pound ſtocke, or 33. pound, 6. ſhillings 8. pence, ſhall bring $2\frac{1}{3}$. pound intereſt in $\frac{3}{4}$ yeere, as heere the Fabrique ſheweth.



Euma. Truly theſe rules farre excell the

double

double working by the golden rule, and I thinke I vnderſtand it very perfectlie.

Phi. Then for triall aſwell of your knowledge therein, as of the certaynte of this laſt aunſwere I put this queſtion, if 400. pound ſtock in ¼ yeere yeelde 10. pound intereſt, what ſtock in ¼ of a yere will yelde ½ r. for intereſt?

91. A queſtion what ſtock in ſuch tyme wil gayne ſo much intereſt.

Euma. Yeere I perceiue the termes of the ſuppoſition are the ſame they were before. But now becauſe the demaunding termes are ¼ yeere, and ½ r. intereſt whereof that muſt be firſt placed which is firſt mentioned in the ſuppoſition. Therefore I put the ¼ yeere in the fourth place, and the ½ pound intereſt in the firſt place thus.

ſtocke.	yeere.
400 r.	¼
intereſt	yeer.
½ r.	¼
intereſt	
½ r.	

Wherein although now the firſt and fourth termes are not gender like, as they cannot be, becauſe the aunſwering terme yet vnknown, is to matche with the firſt: yet I finde by your former doctrine, that the operation of this queſtion belongeth to the compound rule conuerſe.

And therefore to worke it as this verſe preſent requireth, firſt I infolde the tops of all the extremes which are 400. 1. and 5. together, with both the baſſes of the meanes, which are 1. and 4. and they produce 8000. for diuidend.

Secondly I infolde 10. and 3. the middle numerators, and 1. 4. and 2. the extreme denominators togethers, and their product is 240. my diuiſor, and laſtly performing the diuiſion, the quotient deliuereth 33 ⅓ r. which I therefore annex to the figures taple for aunſwering tearme, thus, as heere you ſee.

ſtoc.	yeere
400 r.	¼
intereſt	yeere.
10 r.	¼
intereſt	ſtocke
½ r.	33 ⅓ r

Concluding that if 300. r. ſtock in ¼. yeere gaine 10. r. by intereſt, then that ſtocke which in ¼ yeere ſhall gaine 2 ½ r. for intereſt. muſt be 33 ⅓ r.

Philo. You haue done well: wherefore now to go forwarde to the third kinde of this compound rule (which by particuler name, I call the compound rule diſcending, and as I ſaid before ſpringeth alſo ſecondarilie from the 273. verſe) I propound this queſtion for an Introduction thereunto.

92. A queſtiō of the valure of diuers coynes or moneys.

If two angels counterualue 20 ſ. and 12 ſ. counterualue 2. ▽ that is 2. french crownes (for ſo this character ▽. is ſet to ſignifie)

Note that this Character △. ſignifieth a French Crowne.

how many angels will counterualue 10.▽?

Now this queſtion conſiſteth of two ſuch direct proportionalities as is mentioned in the ſaid verſe, to wit whoſe laſt terme of the firſt, and ſecond of the laſt are equall together, as heere appeareth by theſe 2. proportionalities. A. and B. vnto whoſe tearmes the firnames mentioned in this queſtion are annexed.

shilling. A. angels
20 — 2
12 — 1 1/5
▽. B. angel.
2 — 1 1/5
10 — 6

Wherefore by the ſame verſe, as 40. the product compounded of 20. and 2. their firſt antecedents (heere linked together) is to 120. the product made of 12. and 10. their laſt antecedents (linked alſo together here) ſo ſhall 2. the firſt conſequent, be to 6. the laſt conſequent, that is, in proportion ſubtripla: and therfore by that verſe 6. angels ſhall counterualue 10. French crowns as heere the taile terme of B. declareth.

shilling. A. angles
20 — 2
12 — 1 1/5
▽. B. angel
2 — 1 1/5
10 — 6

But now to compound theſe 2. proportionalities, ſo that one onely worke may ſerue for both, and therewith to know preciſelye, when the aunſwer of a queſtion propounded is to be ſought by this third kinde of rule onelye. The maxim inſuing teacheth, wherefore note it well.

283. Such compound queſtions as permit,
Their firſt 4. termes in place to ſit,
Each with his firname very fit,
And doth but then demaund,
How many of the firſt ſo placde,
Will make ſome number of the laſt,
By the diſcending rule is caſt,
And very truly ſcand,

284. The nature of which rule doth frame,
His bias numbers like of name,
VVhoſe head and tayle do eke the ſame,
By loue they are ſo led,
And thus theſe queſtions all are tried,
Let thoſe three termes be multiplied,
VVhich onely ſtand at lefthand ſide,
Of any compound Zed,

285. VVhoſe product then if you diuide,
By th'offcome of the tother ſide,
The quotient which doth ſo betide,
Doth kill the queſtion dead.

226

Now

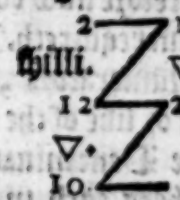
Now if you note it although there are 5. seuerall tearmes or numbers giuen in this present question, as were in the other questions before, yet this question now standeth not vppon 2. terms as the other did, for here the first 4. termes contayne nothing els but the suppositiõ, and the question it self resteth onely on the 5. which is the 10. ▽. for the whole demaund is no more but how many angels will countervalue. 10. △?

And therefore as the first verse of this maxim importeth, this question doth permit, the first 4 termes thereof to be placed each with his sirname to the figure, before it begin to make any demand, and then demaundeth how many of the first sirname, that is to say, how many angells will make some number of the last sirname, which here is 6. French crownes.

Wherby it followeth that such a question is triable onely by this third rule, which for a distinction may aptly be called the compound rule discending, because that in asking the question how many? it beginneth at the very head and first sirname of the figure, and discendeth thence immediately to the verie last sirname of the suppositiõ.

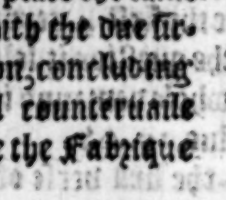
Euma. I vnderstand, your meaning is that in this rule, the questiõ cõmeth from the first sirname to the last, as it were downwards & therfore you thinke good to cal it the compound rule discending.

Phi. You vnderstand me well, now the second verse of this maxim sheweth you, how the nature of this rule is to haue his bias numbers like of sirname, or matchtermes together, which you cannot lightly make otherwise though you would, for they need no other placing commonly, but euen as they are spoaken and metered, and the rest of the maxim doth so playnely shew, how the diuidend and diuisor is compounded and made, that it needeth no interpretation: but for your aide at the first I will place the termes in the figure, as heere you see I haue done, and so leaue the working to your selfe, to performe according as the verses will you.



Euma. Indeed heere it is very manifest that each couple of the bias numbers are matchtermes, wherby it must needs follow, that the head & taile shall also be like: but now to proceede to the operation, as the 284. & 285. verses present will me. I multiply together those 3. terms onely which stand at the lefthand of the figure, viz. 2, 12. & 10. & they produce 240. for my diuidend,

which I diuide by 40. the product of 20. and 2. the termes on the other side, and the quotient is 6. which by the middle deuise must needs be angels, wherefore I place the same 6. at the taile of the figure, with the due sir-name as answer to the question, concluding thereby, that 6. angels will counteruaile 10. French crownes, as heere the Fabrique sheweth.



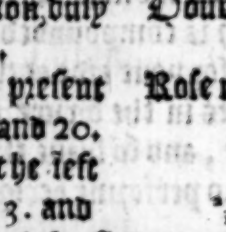
Philo. Thus as you see, there is no difficultie in this rule, wherefore I will put no more examples thereof as yet, but onely I will shew you a very easie forme of operation therein, whereby when there happeneth Fractions in a question. You shall notwithstanding worke it, as if they were all Integers, and it is thus.

286. The compound rule descendingly,
In fractions not offending by,
May thus be wrought the wiser,
Infold the lefthand numerators,
And both the right denominators,
No diuidend preciser.
Then fold the left denominators,
And both the right hand numerators,
And vse it for diuisor.

92. A question of the values of diuers coynes.

Now for an example heereof, take this question. If ¾ of a double Ducat, counterualue ⅔ of a Rose Noble, or olde Royall, and ⅔ of a Rose Noble, counterualue 2. French ▽. How many double Ducats is equivaluable to 20. French crownes.

The termes of which question, duly placed in the figure stand thus. Wherefore now to do as this present Maxim requireth, I take 3. 4. and 20. the Numerators, situate at the left hande or side of the figure, and 3. and 1. the Denominators of the right side, and infolde them all togethers each in other, whereby I produce 720. for the Dividend.



Then doe I also take 4. 5. and 1. the Denominators at the left-hand of the figure, and 2. and 2. the numerators at the right hand therof, & infolde them all togethers, so is their product 80. seruing for my diuisor: finally the diuision being accomplished, there issueth

in

in the quotient 9. double ducats for double duc. Rose Nob.
the aunswere, which therefore I con- 1 2 ▽
uey into their propper place thus, as Rose nob. 1 2 ▽
the figures tayle. Concluding by this ▽ doub. duc.
present maxim, that so many double 1 2 9.
ducats counterualue 20. ▽.

Euma. In truth as you say, this a very easie forme of operation, and it is as easie to know that 9 double ducats are worth 20. ▽. for as 20. ▽. at 6. s. the peece, make 6 l. sterling, so doth 9. double ducats at 13 s. 4 d. the peece make iust 6 l. also.

Philo. Very true, wherefore sith your selfe confesseth the facilitie aswell of the rule, as of the proofe thereof, I will goe to the explication of the fourth and last rule which particulerly is named the compound rule ascending, and is founded originally vppon the first verse of the maxim insuing, and not onely it, but also the compound rule conuersed springeth oftentimes, both from the same first verse, and often tymes from the second.

287. And when two direct proportions are placd
So that in their tearmes such order accrues,
That last of the first, and first of the last,
Are equall toge;her, then thereby insues,
That as the ofcum of second and third,
Is to th'ofcum of first and last saue one,
So shall the sixt terme left standing vnstird,
Be vnto the last compared alone.

288. But if such last proportion be conuerst, 228
And t'other direct, the ofcum then made,
Between those couples of termes last reherst,
Taken directly and not retrograde,
Shalbe each to other in such rate prefixt,
As the last tearme is conuerst to the sixt.

The first verse of this maxim serueth indifferently as foundation both to the conuersed rule already taught.

And also to the fourth or last kinde of the compound rule yet vntaught, for they both spring from it after some sorte, but the conuersed rule, like the triple headed Cerberus proceedeth sometimes from the 274. verse, sometimes from the first verse of this maxim, and sometime from the last thereof, as I will shew you anon. But first I will oculariy demonstrate, how the last kinde of this compound rule proceedeth from the first verse of this present

Maxim

Maxim which is thus.

Let A. and B. be 2. proportionalities direct, placed so that the last terme of A. and the first of B. may be equall together, as here you see they are For the last tearme of A is 1 ½, & so is the first terme of B. wherfore

A
20 s. — 2 angels
12 s. — 1 ½ angels.
angels
1 ½ — 2 ▽.
6 — 10 ▽

I conclude by the first verse of this Maxim, that as 24 the ofcum of 2. and 12. (the second and third termes couple together) is in proportion to 120 the ofcum of 20. and 6. the first terme, and last saue one, so is 2. the first terme to 10 the last, that is in proportion Subquintupla.

Whereupon followeth this conclusion, that if 2. angels counterpayle 20. s and 12. s. counterpayle 2. ▽. then 10. ▽. will counterpayle 6. angels. For by the proportion A. if 20. s. make 2. angels, then 12. s. shall make 1 ½ angell.

And by the proportion B if 1 ½ angell make 2. ▽. then 6. angells shall make 10. ▽. wherefore now to apply this conclusion to our present purpose, in such sorte that it may extend to dissolute questions thereby answerable at one onely operation, as I haue done others before, I will first exhibit the rule thereof, and then shew the practise by an example, the rule is thus.

289. Such compound questions as doe yeelde,
Their first 4. places all well willed,
To be with termes and sirname filled,
And then begins to aske,
How many of the last reherst,
Will counterpayle some of the first,
Th'ascending rule shall quench their thirst,
And ouercome their taske.

290. Which rule ascending so doth craue,
His first and last tearmes like to haue,
And second like the third as braue,
And so the other two.

291. Then make the right side multiplie,
The lowest terme left handedly,
Whose product place downe by and by,
For diuidend to goe,
At last infolding tother twayne,
Which on the left side yet remayne,

229

The

The product is diuisor playne,
Which shall the knot vndoe.

For example of the practise of this rule, I turne the former conclusion into a question, which I moue thus.

If 2. angels counteruayle 20. s. and 12. s. counteruayle 2. ▽. how many French crowney will counteruayle 9. angels. In which question you see 5. termes or numbers giuen, as in others before, & also you may perceiue that the first 4. conteyne but a supposition onely, for the question standeth wholy on the fift number, euen as in the last rule proceeding, wherefore it doth easily yeeld to haue the same 4. first termes placed in the figure with their propper sirnames before it begins to aske any question. 94. A question of the valure of diuers moneys.

And then in asking the question how many? it asketh how many of the sirname last reherst, will counteruayle some certayne number of the first sirname, as here you see it asketh how many ▽ (which is the last sirname of the supposition) will counteruaile 6 angels (which angels is the first sirname of the supposition) wherfore by the 289. verse present such questions are subiect to this latter rule, which as it doth quench them and ouercome them, so for a distinction it may well be named the compound rule ascending, because that in asking the question how many? it ascendeth from the last or lowest sirname of the supposition, to the first sirname and highest terme in the figure.

Eu. I vnderstand both your minde herein, and also the difference betweene this rule and the next former, for in that rule the first 4. termes with their sirnames being placed, the question was, how many of the first sirname would counteruayle a certayne number of the last, but in this rule the selfe same supposition remayning, and the same tearmes contayning in their former places immutably, the question is, how many of the last sirname are equall to some number of the fyrst, and so in that rule the question is from the fyrst to the last, wherefore you reckon it to descende, and call it descending, but in this, it is from the last to the fyrst, and therefore you account it to ascend, and call it ascending.

Philoma. I perceiue by your speech that you vnderstand me in all things very fully, wherefore I cannot thinke but that you do aswell conceiue the meaning of the whole rule present, and therefore I pray you worke the answer of this question your selfe by the teanor thereof.

Euma.

Fuma. To say truly, I finde yet no such obscuritie in the rule, but that I thinke I can execute it, and for a triall thereof I applie the numbers with their surnames to the figure thus.

angels. shillings.
2 20
shilling ▽.
12 2
angels
6

In perusing of whole termes I finde the first and last surnamed like, and likewise the second and third, (euen as the 290. verse requireth) whereby I am assured the other 2. shall also be like anon, when the answering terme is found.

And for the finding thereof (as the same verse biddeth me) I multiplie 20. and 2. the termes of the right side of the figure, by 6. the lowest terme set on the left hand side, and their product is 240. which I keepe for diuidend as the same verse biddeth me, then folding together 2. and 12 the other 2. numbers which remayne on the left side, they produce 24. for the diuisor, which shall vnloose the knot of this question.

And so finally diuiding 240. by 24. the quotient deliuereth 10. for the answering number, which I place at the figures tayle, with his due surname ▽. as here you see.

angels shillings
2 20
shillings ▽.
12 2
angels ▽.
6 10

And so the fourth and last termes now are of like surname, and I thereby conclude that 10. ▽ shall counterualue 6. angels.

Philo. You haue done well, and thus you see how by this rule the same thing is accomplished quickly and easily, with one onelye operation, which by other rule requireth 2. seuerall workings, and moreouer as the compound rules direct and conuerted, were each of them a triall of others truth, euen so are these compound rules discending and ascending, each of them a triall of the other.

Proofe of the rules both descending and ascending

But here I thinke good to shew you also, how this compound rule ascending, may be as easily wrought, when the termes thereof are eyther all or part fractions, as if they were all Integers, which is done by the marim insuing.

95. A question of the valure of English coynes & Forraigne coynes.

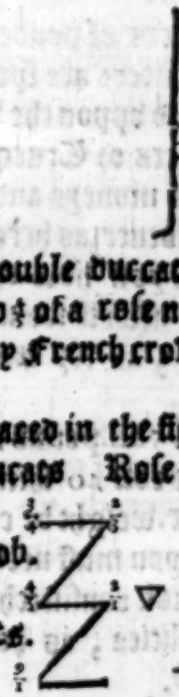
292. When the compound rule ascending,
Is in fractions all depending,
To bring the worke to speedy ending,
Infold together first,
The 2. right handed numerators,
Their opposite denominators,

And

And loweſt lefthand Numerator,
For diuidend moſt iuſt,
Then fold the righthand baſſes light,
In both the tops ſet oppoſite,
And in the loweſt baſſe in ſight,
Which for diuiſor truſt.

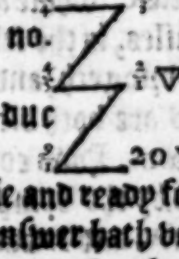
As for example I put this queſtion, if $\frac{3}{4}$ of a double ducat be equiualent to $\frac{2}{1}$ of a roſe noble, or old royall, and $\frac{1}{2}$ of a roſe noble, equall in valure to 2. French crowns, how many French crownes are equauaiuable to 9. double ducats.

Now aſſoone as the tearmes are orderly placed in the figure, their owne ſetling ſheweth by the 290. verſe, that the queſtion is ſubiect to the compound rule aſcending, as here appeareth.



Whereupon by this Maxim preſent, I infolde together firſt 2. and 2. being both the right handed numerators, and 4. and 5. the denominators ſituate directly oppoſite on the other ſide of the figure, and 9. the loweſt lefthand numerator, and their whole product is 720. which I keepe for my diuidend.

Secondly I infold 3. and 1. the righthand baſſes, and 3. and 4. the tops ſet oppoſitely agaynſt them, and 1. the loweſt baſſe (in ſight of all the figure) and their whole product peeldeth 36. for my diuiſor, and finally diuiding 720. the diuident by this diuiſor, the quotient deliucreth 20 for the aunſwering number, as here appeareth by the fabrique.



Whereby I concluue that 20. ▽. are equall or woorth 9. double Ducats.

Eu. In deed I ſee that this is not onely an eaſie and ready fourme of worke, but alſo I perceiue hereby that this anſwer hath verified and confirmed the aunſwere of the queſtion made by the compound rule diſcending, according to your late former ſaying, which was that one of theſe rules is the triall of the other.

Phi. It hath ſo, and the vſe of theſe 4. compound rules is of great moment, for vppon the firſt couple of them hang not onely all queſtions of marchaundiſe, wherein tyme is bought or ſold, but alſo by them, the charge of wars, touching victuals, ſoldiers weges, ex-

erpences of pouder, casting of trenches, and diuers other millitarie matters are speedily and aptly discussed.

And vppon the latter couple dependeth the equation of diuers Barters or Trucques, with the exchange and reduction of all maner of moneys and coynes, weightes and measures of what nation soeuer, as hereafter shall more fully appeare. But first I will shewe you how the compound rule conuerted, springeth sometime aswell from the 287. and 288. verses as from the 274. verse, as for exemple of the 287. verse, this question is asked.

96. A question of carriage.

If 500. pound weight carried 200. miles cost 40. shillings, how farre shall 100. l. weight be caried for 2. shillings: now you must needes graunt that this question consisteth of 2. direct proportionalities, as here you see by A. and B.

weight	A	shillin
500 l.		40.
100		8.
B	miles.	
8 s.		200
2		50

For first if 500. l. weight cost 40. shillings the carriage, then 100 l. weight shall cost 8. s. as appeareth by A. Secondly if 8. s. pay for 200. miles carriage, then 2. s. shall pay for 50. miles carriage as appeareth by B.

But now by the same verse, sith the last terme of A. and the first of B. are equall, therefore as the product of 40. and 100. is to the product of 500. and 2. so shall 200. the first terme, be to 50. the last terme, wherefore the proportionalities A. and B. multiplied together produce the proportionnalitie C. whose aunswering number being 50. miles, is the selfe same number, that the compound rule conuerst bringeth, and yet as you see here, the single proportions A. and B are both direct.

C	
4000	1000
200	50

Eum. How commeth it to passe, that this question is triable onely by the compound rule conuerted, sith that both the proportions A. and B and likewise C. the product rising by their composition, are all direct.

Philo. The reason thereof is because the termes here applied to Multiplication, are not all gende like or match termes. For 40. the second tearme of A. is a consequent, and all the rest are antecedentes, and the cause why the antecedentes here, are all not applied to Multiplication, is because 8. s. which is one of them, is one of the equall termes also.

But

But the equall tearmes are neuer to be vſed, neyther in multiplication, nor yet in relation, but are alwayes vtterly omitted, and ſo the ſame 8. beeing omitted for his equalities ſake, the firſt conſequent is vſed in his ſteade.

Euma. I haue noted it often times before, that the equall termes were alwayes omitted, and thereby I perceiue it commeth to paſſe, that euery compound proportionality occupieth but 6. terms, and hath but 6 places in the figure, whereas 2. ſingle proportionalities, haue 8 termes, and 8. places, but the reaſon thereof I know not.

Philo. The reaſon is, becauſe the worke is concluded as perfectly without the equall termes, as with them, for if you ſhould multiplie any number by 8. and then diuide the product by 8. agayne, would not the quotient be the firſt number.

Euma. Yes that muſt needes be ſo by reaſon.

Philomathes. And euen ſo in this caſe, to multiplie by one number, and to diuide the product by another, equall to the multiplier, would come to like effect, and therefore if any other couple of match tearmes in theſe compound queſtions be equall, they may be alſo omitted, and the operation ſhalbe done aſwell with the reſt of the termes, whereof there is this rule or maxim inſuing.

293. If any queſtion of proportion haue,
One couple of the matchtermes coequall,
Then if ye couet your labor to ſaue,
Let not thoſe equals be touched at all,
But woorke with the other numbers beſide,
VVhich ſerue euen aſwell being well applied.

97. A queſtion of gaines vpon the 100. pound by time. 231

As for example, a queſtion is propounded thus. If 100 l. ſtocke in 12. monethes yeeld 10. l. gaynes, in what time will 400 l. ſtocke yeeld 10. l. gaynes. The termes of which queſtion applied to the figure ſtand thus.

ſtocke.	monethes.
100 l.	12
gaynes	ſtocke
10 l.	400 l.
gaynes	
10 l.	

Now here by the 281. verſe y product of the 3. extremes ſhould make the diuidend, and the product of the 2. meanes ſhould be the diuiſor, but by this verſe, becauſe the third and fift numbers being matchtermes are equall, therefore I omit them both, taking the product of 100. and 12. the other 2. extreames onely for diuidend, and 400. the other meane onely

onely for the diuisor, and so diuiding 1200. by 400. the quotient is 3. monethes, which is the selfe same quotient that would issue if the product of all the 3. extreames were diuided by the product of both the meanes, the like whereof is naturally incident to euery of these 4. compound rules.

Euma. I am glad you haue shewed me this reason, for I begin now to feele, that to worke by rules without knowing the reason is much like the speech of a Parat, who speaketh by customable hearing, but not with vnderstanding.

Or like an Empericke which cureth a man of a feuer by some certayne prescripe medicine, not knowing the cause of the dissease, neyther yet the qualitie of his medicine, but now I pray you let vs returne to the answere of your former question by his proper rule, to trie how it agreeeth with the 224. maxim.

Philo. With a good will, wherefore do you applie the tearmes of that question to the figure, and make the triall your selfe.

Euma. The question was this, if 500. l. weight, carried 200. miles cost 40. s. how farre shall 100. l. weight be carried for 2. s. whose termes being serued in the figure order- ly thus.

weight	miles
500 l.	200
shil.	weig.
40	100 l.
shil.	
2	

It appeareth foorthwith by the first verse, of the 224. maxim that the question is subiect to the compound rule conuerst, and therefore vsing the practise of that rule I take (20000.) the product of the 3. extremes for my dividend, and (4000. the product of the meanes or middle numbers for my diuisor, the quotient of whose diuision is 50. miles, euen iust as you said before that it would be.

98. A question of Ploughing.

Philo. Then finally to shew you by an example how the saide compound rule conuerted proceedeth sometimes from the 288. verse, I put this question? If 2. ploughes eare 18. acres in 6. daies in what time shall 1. plough eare 45. acres.

Here by the Golden rule plurall, partly direct, and partly conuerted, first I say thus, if 18 acres require 2. ploughes, what will 45. acres require, and it answereth me 5. ploughes, as heere appeareth by A.

acres A	ploughs.
18	2
45	5
ploughs B	daies
5	6
1	30

Secondly by the same rule conuerted, I say thus, if 5. ploughes neede 6. dayes, what shall 1. plough neede? and it answereth me 30. dayes, as the figure B. declareth.

Wherefore

Wherefore I conclude that if 2. ploughes eare 18. acres in 6. dayes, then 1. plough shall eare 45. acres in 30. dayes.

But by the 288. verse, as 90. the product of 2. and 45. is to the product of 18. and 1. directly, so is 30. the last terme to 6. the sixth conuersedly, that is in proportion Quintupla.

And so these 2. proportionalities A. and B. being multiplied together in that forme, produce the conuersed proportionalitie C. annexed.

	C.
	90 — 6
From whence proceedeth the compound rule	18 — 30
conuersed, as appeareth by the last former question applyed thereunto, for example whereof, beholde the figure of the same question heere renued.	plowes. dayes. 2 — 6
The product of whose extreames, being diuided by the product of the meanes, quoteth 30. dayes iustly agreeable with the other former worke.	acres plow 18 — 1 acres daies 45 — 30

Euma. It is euen so, and here I note one good commoditie in this compound rule conuersed, which is this, that from which soeuer of the former verses it springeth, and whether the 2. proportionalities whereof it is compounded, be both direct, or but partlye direct and partly conuersed, yet alwayes the operation therof continueth vniforme, & proceedeth after one manner, which is a good assurance, that a man shall neuer erre, by mistaking the vse of the one instead of the other.

Philo. You haue said very truly. And now for your further exercise in all these 4. rules, I will propound plentie of questions, which shall be answerable sometimes by one rule, and sometimes by another, because I would as well see how you can finde out the rule whereunto euery question is subiect, as how you can apply it accordingly, when you haue found it. And this the first.

99. A question of Diaper.

If 15. elles of Diaper of ½ of an ell broade, cost 37. s. 6. d. what shall 40. ells of like stuffe, worke, and fineſſe cost, being ⅔ of an ell broade?

Euma. Heere first I finde, that this question consisteth in two termes, wherefore by the 277. & 280. verses, it must needs be subiect, either to the compound rule direct or conuersed, but because the termes whereon the question dependeth (which I like well to call the demaunding termes) matche in surname with the first and second termes of the supposition. Therefore by the 277. verse, the question is subiect to the compound rule direct: where-

fore the termes being applied to the figure accordingly appeare thus.

Where I finde that the first and fourth termes match as they aught to do, and likwise the 2. extreme anguler termes, wherefore I take $\frac{15}{1}$ the product of the 3. latter termes for my dividend, and $\frac{1}{4}$. the product of the 2. superior termes for my divisor, and so working the division, the quotient or aunswering tearme is 222 $\frac{2}{3}$ s. which I therefore ascribe to the taile of the figure, as to his due place, affirming that the 40. elles of Diaper of $\frac{1}{4}$ of an ell broade, will cost so much, that is, 11. pound 2. shillings $\frac{2}{3}$.

elles. broad
$\frac{15}{1}$ $\frac{1}{4}$
shilling. elles.
$\frac{75}{1}$ $\frac{40}{1}$
broade
$\frac{1}{3}$
elles. broade.
$\frac{15}{1}$ $\frac{1}{4}$
shillin. elles.
$\frac{75}{1}$ $\frac{40}{1}$
broade shilling.
$\frac{1}{3}$ 222 $\frac{2}{3}$.

100. Another question touching the same Diaper.

Philo. Well done, you have both found out the rule, whereto the question is subiect very orderly, and concluded it as truly, wherefore letting the same supposition stand still as it doth now, tell me fyrst, of what breath ought that Diaper to be whereof 40. ells (being of like woorke and fineneſſe) shall cost 222. $\frac{2}{3}$. shillings? and secondly, how many ells of Diaper of $\frac{1}{4}$. of an ell broad, shalbe sold for 222. $\frac{2}{3}$. shillings?

101. A third question touching the same Diaper.

Euma. As this request of yours contayneth 2. questions, so I perceive they require the operation of 2. compound rules for their solutions, wherefore to begin with the first, sith the supposition of the last former question remayneth still as it was before, therefore in applieng the termes to the figure, I place them as they were before, and then after reduction of the mixt numbers. I put the 2. remaunding termes in their proper places also thus.

ells broad
$\frac{15}{1}$ $\frac{1}{4}$
shill. ells.
$\frac{75}{1}$ $\frac{40}{1}$

Which done, the 280. verse assureth me, that the question now is subiect to the compound rule conuerſed, wherefore according as the maxim thereof requireth, I multiply $\frac{15}{1}$ and $\frac{75}{1}$ the 2. extreames together, whereby I produce 2500. for dividend, which I divide by (1500) the product of $\frac{1}{4}$ and $\frac{40}{1}$, the 2. middle numbers or meanes, and the quotient or answering number is $\frac{1}{4}$. of an ell, which I put in his proper place, as the figure here declareth.

shil. broad
$\frac{75}{1}$ $\frac{1}{4}$
shil. elles
$\frac{75}{1}$ $\frac{40}{1}$

Now for the second question, I also place the termes of the supposition in the figure, still as

shil. broad
$\frac{75}{1}$ $\frac{1}{4}$

they

they were before, because the same supposition remaineth vnchangeable, and then reducing the mixt numbers of the demaunding termes, I put them in their due places thus.

elles. broad.
1 1/2 — 3/4
shill. broad.
25/8 — 5/8
shill.
1000/9 — elles.

Which done, the 280. verse, and the glosse thereof assureth me still, that this question is also subiect to the compound rule conuerst. And therefore by that rule, compounding my Diuidend of the 3. extreames, and my diuisor of the 2. meanes as before, the quotient of the diuision bringeth iust 40. elles for the answering terme as heere appeareth.

elles. broad.
15/4 — 1/4
shil. broad
75/8 — 5/8
shil. elles.
2000/9 — 40.

Whereby I conclude, that those 40. elles of Diaper which being of like worke and fineneſſe shall cost 222 2/9 s. ought to be 1/4 of an elle broad, and that 40. elles of Diaper of 1/5 of an ell broade, shall be solde for 222 2/9 s. as their seuerall fabriques declare.

And heere because the answering terme ought to match with the first, therefore I see it was not possible, that the first & fourth terms could be like, as the 277. and 280. verses require, except the first and second termes should change places each with other.

Philo: You haue considered well of it, & hereby you may plainly see, how one of these questions is concluded by the other, no lesse certeinly and cleerly then by a sillogisme. For if by the compound rule direct it be true, that when 15. ells of Diaper of 1/4 of an ell broad cost 37 s. 6. d. then 40. ells of 1/5 of an ell broad, shall cost 222 2/9 s. then by the compound rule conuersed, it must necessarilie follow, that when 40. such ells costeth 222 2/9 s. that then the same 40. elles ought to be 1/5 of an ell broad, & likewise that when 15. ells of Diaper of 1/4 of an ell broad, costeth 37 s. 6. d. then 222 2/9 s. shall buy 40. ells of 1/5 of an ell broade. And therfore Peter Ramus calleth this conclusion of proportionalitie, Analogismus, affirming that as Logicians account a sillogisme to be the strongest and surest kinde of argumẽt, for proofe of any matter in logike, so such Analogisme in Arithmetique, is of like strength and certeintie, for proofe of Arithmeticall conclusions. Analogis-mus.

Eu. It cannot be denied, and here I perceiue further also, that as there is not, nor cannot be but 3. couple of matchtermes in euery compound question, so the same question cannot be asked, but in one of those 3. formes, which you haue demaunded, wherof in the first, the sirname of the answering terme was shillings, but in the second it was breadth, & in the third it was ells, and no sirnames are there

not to be founde in the question, nor in the figure.

Phi. You say truly, and hereby you may learne to vary any like compound question, which I haue put heeretofore, or shall propound hereafter in like manner at your pleasure, but to proceed, I put this question.

102. A question of Fustians.

A marchant hath bought Fustians for 22.s. 6.d. the peece ready mony, which he would sell againe for 24. s. the peece, now I demaund what time he may forbeare his money, and yet gaine after 9. l. vppon the 100. in 12. moneths?

Euma. I pray you answer this question, that I may see how the nombers are applied, for I cannot well tell presently to which rule this question is subiect.

Phi. I am contented, and first seeing this marchant will sell that for 24. s. which cost him but 22. s. 6. d. it is manifest that he will gaine 18. d. in euery 22. s. 6. d. disbursed, or in euery peece of Fustian: wherefore to frame the question to the figure, I say thus. If 4000. testerns (that is 100. pound stock) in 12. moneths, bring 360. testernes (that is 9. ponnd) gaines, in how many moneths will 45. testerns stock (that is 22. shillings 6. pence) bring 3 testerns gaines?

Euma. Now can I answer the question my selfe, for I finde plainely by the 280. verse, that it is subiect to the compound rule conuerted. Wherefore I apply the nombers to the figure thus.

tester. stock. moneths
4000 — 12
tes. gain. — tes. stoc.
360 — 45.
tes. gain.
3

And so first folding together the 3. extremes, I produce 144000. for diuidend, and next folding the 2. means, I produce 16200. for diuisor. Finally the quotient of that diuision is $8\frac{1}{9}$. moneths, which I annexe to the taile of the figure, affirming that so long the marchant may forbeare his money, and yet gaine, after 9. in the 100. as the fabrique heere expresseth.

tester. stock. moneths
4000 — 12
tes. gain. — tes. stoc.
360 — 45
tes. gain. — moneth
3 — $8\frac{1}{9}$.

103. A question of Satin solde for time.

Philo. Then answer me to this question. A marchant bought 20. yardes of Satin for 12 l. 10. s. ready money, now I demaund, for how much more then it cost him may he sell the yarde, giuing 2. monethes day of payment, and yet gaine after 10. pound vppon the 100. in a yeere, or 12. moneths?

But here I must tell you, that because the position is of 20. yards,

and the question is but what he may raise in the sale of 1. yard onely, and not in the whole 20. yards. Therefore you must first learne the price which 1. yarde cost him, by the single golden rule, and so by the price thereof, you may goe forward to the conclusion of the question.

Eu. Then first I say thus, if 20. yardes cost ½ r. what dooth 1. yarde cost? and the operation of the rule answereth ½ r. as here the fabrique sheweth.

yardes.	pounds
20	½
1	?

This done, I come to the compound rule saying thus. If ½ r. stock in ½ moneths, bring ½ r. gaines, what gaines will ½ r. stock bring in ½ moneths?

And by the order of the question, and 277. verse, I finde it to be subiect to the compound rule direct, wherefore by that rule, I multiplie the 3. latter inferior termes together, and they produce ½ for my diuidend, and then next I multiplie the 2. uppermost terms together, and they produce 1200. for my diuisor: Finally diuision being performed, the quotient or answering terme is ½ r. which I ascribe in his due place, as heere appeareth. Concluding that he must sell euery yarde of his Satin, for so much aboue the ½ r. that it cost him, that is, for ½ + ½ pound, which by addition is ½ pound, and in knowne coine 12. shillings, 8. pence halfepenny.

stock.	moneths
½ r.	½
gain.	stock
½ r.	½ r.
mon.	
½	

stocke.	moneths.
½ r.	½
gains.	stocke.
½ r.	½ r
mont.	
½	½ r.

Philo. A gayne a marchant hauing Cinamon which cost him 8. s. a pound ready money, would sell 100. r. weight thereof, and giue 3. moneths day for payment, so that he might gayne after 10. r in the 100. for a yeare by the sale, wherefore it is demaunded for how much more then it cost him, he shall sell the same 100. r. weight? or how much he ought to gaine thereby?

104. A question of Cinamon solde for time.

And here because 8. s the giuen price, is but the price of 1. pound, and the question is of 100 r. therefore the price of 100. r. must first be found by the single golden rule as you did before in the other question.

Eu. Then first I say thus, if 1. pound cost 8. s. what shall 100. r. cost, and the golden rule answereth 800. shillings, as here appeareth.

pound	shilli.
1	8
100	800

This being knowne, now to come to the compound rule, I

frame the question thus, saying. If 100 l. stock in 12. moneths, bring 10. l. gaines, what gaines shall 40. l. stocke (that is 800.s) bring in 3. moneths?

stocke.	moneths.
100.l.	12.
gains	stock.
10.l.	40.l.
mon.	
3.	

And heere, by the 277. verse, I finde this question subiect also to the compound rule direct, wherefore as that rule willeth, I multiply together the 3. latter numbers and they produce 1200. for diuidend. Marke that I multiply the 2. vppermost numbers, and they produce 1200. also for diuisor, wherefore by the 77. verse, the quotient or answering number is 1. l. which I ascribe to the figures taile, as heere you see. Concluding that the marchant must sell 100. l. weight of his Cinamon for 1. l. more then it cost him (that is, for 41. pounds) to gaine after the rate afore required.

stocke.	moneths.
100.l.	12
gains.	stocke
10.l.	40. l.
mont.	gains.
3	1.l.

105. A question of victuals in a towne besieged.

Phi. You haue done well, wherefore now answer me to this.

A Towne is besieged wherein is 3000. soldiers which haue victailes sufficient but for 2. moneths onely, and they looke for no ayde or reliefe to rayse the siege, vntill the end of 6. moneths. I demaund therefore how many soldyers the Captayne may retayne, to make their victailes serue them so long, that is to say, vntill the end of 6. moneths.

Ph. Although I haue not had the like question before, yet I consider that the soldiers are the chiefe and onely cause of the expences of the victails, and therefore I take it that their number must be the first terme of the supposition, and then the 2. moneths which betoken the time must be the second, and so consequently the victail shalbe the third.

Againe because the firname months, is named and placed in the supposition before the victail, therefore I place 6. moneths, the demaunding tearme of like firname in the figure before the victail, which victail therefore must needes settle at the lower angle in such order as heere appeareth.

soldiers.	moneths
3000	2
victall	mone.
1	6
victall	
1	

Whereuppon I frame the question thus, saying If 3000. soldiers be fed 2. moneths with 1. victail, how many soldiers shalbe fed 6. months with 1. victail (for by 1. vnit or intiger, the entire quantity of victails is represented) heere although the first

and

and 4. termes agree not in ſirname, as they cannot, becauſe the anſwering terme muſt needes match with the firſt, yet I finde by the 280. verſe, and your inſtruction thereupon, that this queſtion is ſubiect to the compound rule conuerſed, whoſe vſe is to compound the diuidend of the 3. extreames, and the diuiſor of the 2. meanes. But becauſe one couple of the matchtermes heere are equall, to wit, 1. victuall and 1. victuall, therefore I omit them quite, taking (by the 293. verſe) 6000. the product of the other 2. extreames, onely for diuidend, and 6. the other mid-number for diuiſor, and ſo diuiſion accompliſhed, the quotient or anſwering number is 1000. which therefore I put in his due place, at the figures taple thus,

ſoldiers.	moneths
3000	2
victail.	month
x	6
victail	ſoldi.
x	1000.

Concluding, that the Captaine may retain 1000. ſoldiers onely to make ꝑ victailes ſerue them 6. months.

106. A queſtion of Entrenching.

Philo. Agayne, if 300. pioners in 8. houres, caſt a trench of 200. rodds long, how many will caſt a trench of 3400. rods long in 6. houres?

Euma. The 3. termes of the ſuppoſition, being firſt orderly placed in the figure, I finde by former doctrine, that the 6. houres is to be placed next, and the 3400. rods laſt as heere I haue ſet them.

Pioners.	houres.
300	8
rodds	houre
200	6
rodds	
3400	

And becauſe the anſwering terme muſt needes match with the firſt terme in ſirname, I know that the firſt and fourth termes heere cannot be of like ſirname, for in ſuch caſes you tolde me, that the 2. left hand angles matche together, and likewiſe the 2. right ſide angles, & yet neuertheles by the 280. verſe the queſtion is apparantly ſubiect to the compoſid rule conuerſed.

Wherefore according to that rule, I diuide 8160000. the product of the 3 extreems, by 1200. the product of the 2. meanes, and the quotient or anſwering number is 6800, which I annexe to the taile of the figure thus: concluding that ſo many pioners, in 6. houres may caſt a trenche of 3400. rodds in length.

Pioners.	houres.
300	8
rodds	houres
200	6
rodds	pioners
3400	6800

Stratioticos diggei fol. 31.

Phi. Item if 5. Cannons in 2. dayes battrie, ſpend 60 barrels of Gunpowder, what will 14. Cannons ſpend in 5. dayes battery.

107. A queſtion of the expences of powder at a battry.

Euma. This queſtion appeareth forthwith, to be ſubiect to the com-

compound rule direct, by whose operation duly vsed according to that rule, I finde the aunswering number to be 420. barrels, which I haue placed heere at the figures. taile accordingly.

Canons.	dayes.
5	2
barrel	canon
60	14
dayes	barrel
5	420

108. A question of Horsemes wages.

Philo. If 1. horsman take 3. pounds a moneth for his wages in the warres, how much treasure will pay the wages of 4000. horsemen for 9. moneths?

Euma. This question by the 227. verse, is also subiect to the compound rule direct, wherefore for answer thereto (after that I haue placed the termes in the figure) I take the product of the 3. last inferior termes onely, which is 108000 r. without any further worke, as here you see.

horsmen.	moneth
1	1
posib	horsm.
3	4000
moneth	pounds
9	108000

For seeing the product of the 2. superior termes is but 1. there needeth no diuision by it, because 1. neither diuideth nor multiplieth, wherefore I conclude, that 108000. r. will pay 4000. horsemen for 9. moneths.

Philo. Thus haue you had some taste of the vse of these 2. rules, both in matters of peace and of war, in marchandise and martiall affaires, now let vs see what vse it hath in other trades, and first I ask this question.

109. A question of a turne broche.

If the plummet of a clock or of a turnbroche, whose weight is 56. r. being wound vp to the highest, discend 16. foote in 1. heure, in what time shall a plummet of 84. r. weight, discend therevpon 24. foote?

Euma. This question by the 280. verse, is manifestlie subiect to the compound rule conuerst, wherefore as that rule biddeth after I haue furnished my figure with his termes thus.

pounds.	houres
56	1
foote	pound
16	84
foote	houre.
24	1

I diuide the product of the extreames, which is 1344. by the product of the meanes, which is also 1344. and so the quotient is 1. arguing that 84. pound weight, shall discend 24. foote in 1. houre, and so 84. r. weight shall discend 24. foote. in as little time as 56. r. weight discendeth 16. foote.

Philo. You say truth, and the reason is euident, for as 84. pound weight to 56. r weight, is in proportion Sesquialtera, so is 24. foot also in like proportion to 16. foote. But now answer me to this.

110. A question of expences of wine.

There are 8 commensals, which when wine is at 6. d the quart, spend

ſpend euery meale at their ordinarie 12 d. in wine, wherefore wine being fallen to 4 d. the quart, I demaund what they ſhall now neede to ſpend at a meale, not bating their vſuall quantitie.

Euma. For anſwere to this queſtion, firſt becauſe the 8. commenſals are the chiefe cauſe of the expences of the wine, therefore I put their number for the firſt terme of the ſuppoſition. And

Philo. May ſtay you are deceiued, for although the 8. commenſals be the chiefe cauſe of ſpending the wine, yet ſith their number remayneth alwayes one, and ſith alſo they drinke no more at one time then another, the cauſe of the increaſe or decreaſe of their expences cannot be principally attributed vnto them, but rather to the price of the quart of wine, whoſe alteration is the chiefeſt cauſe of ỳ encreaſe or decreaſe of their charges, wherfore you ſhal not make the 8 commenſals the firſt terme, but the price of the quart of wine, framing your queſtion thus, if 6. d. a quart affoorde wyne to 8. perſons for 12. d. for how much will 4. d. a quart affoord wyne to 8. perſons.

a quart commenſall
6 d. — 8
a quart
12 d. — 4 d.
commen. 8.

The termes of which queſtion, being orderly annexed to the figure, as here you ſee them, argue euidently that the queſtion is ſubiect to the compound rule direct, wherein for aſmuch as one couple of the matcheterms are equall, the diuidend by the 293 verſe, ſhall be made of the other 2. lower termes onely, which in this queſtion is the 2. middle numbers, and the diuiſor ſhalbe 6. the other ſuperior number onely.

So is the diuidend 48. and the diuiſor 6. the quotient or anſwering terme of which diuiſion is 8. d. wherefore you may conclude, that if 8. commenſalls ſpend 12. d. in wine when it is at 6. d. a quart, then they ſhall neede to ſpend but 8. d. in wine when it is at 4. pence a quart, and yet drinke aſmuch as they did before, as the figure here declareth.

a quart commenſal
6 d — 8
a quarte
12 d — 4 d.
comme. 8 — 8 d.

The like conſideration is to be had in all other like queſtions, as for example, if 4. penniworth of breade ſuffice 12. perſons at a meale, when Wheate is at 20 ſhillings the quarter, how many penniworth ſhall ſuffice them when it is at 15. ſhillings the quarter.

111. A queſtion of the weight of bread.

Here the price of the quarter of wheate is it, that chiefly breedeth the

the encrease of costs by the dearenesse, or decrease of costs by the cheapenesse (and not the men, being alwayes like many) wherfore I put that for the first terme of the supposition, framing the question in this order.

If 20 s a quarter, affoorde 12. persons bread for 4. d. for how much shall 15 s. a quarter affoorde 12. persons bread.

Euma. I thanke you much for this good direction, for I perceiue I was going astray, if you in time had not made me to stay, but now I trust I am safely instructed for euer, and therefore with your fauour I will assume to answere this question, for doing whereof I applie the termes thereof to the figure thus.

shillings	persons
20	12
pence	shilling
4	15
persõs	
12	

Whereby I finde it to belong to the compound rule direct, and to haue the selfe same termes equal also, that the other had, wherfore omitting them I take 60. the product of 4. & 15. the 2. other lowest termes onely for diuidend, and 20. the other superior terme onely for diuisor, and so diuiding 60. by 20. the quotient bringeth 3. d. for the tayle terme or answering number, where I place it accordingly as you see.

shillings	persons
20	12
pence	shil.
4	15
persons	pence.
12	3

Thereby concluding that if 4. peniwoorth of bread serue 12. persons at a meale when wheate is sold for 20 s. a quarter, then 3. peniwoorth of bread will serue the same 12. persons at a meale, when Wheate is at 15 s. the quarter.

112. A question of the weight of a bottle of Hay.

Philo. Then answere me to this question, if when hay is solde for 20. shillings the loade, the halfepeny bottle weighe 3. pound, what shall the like bottle weighe, when the loade of hay is woorth but 13. shillings 4. pence.

Eu. Here first by your former woordes I learne that the price of the loade of Hay, is the chiefe cause of the increase or decrease of weight of the bottle, and therefore I appoynt it to the first place of the figure, saying if 20 s. affoord 3 l. weight of hay for $\frac{1}{2}$ d. what will 13 s. 4 d. affoord for $\frac{1}{2}$ pence, and so the termes being all placed in the figure as here appeareth.

shilling	weig.
20	3 l.
pence	shil.
$\frac{1}{2}$	$\frac{1}{3}$
pence	
$\frac{1}{2}$	

I finde forthwith by the 280. verse that the question is subiect to the compound rule conuerted, wherin because there are one couple of equal matchtermes,

I omit them both as though I saw them not, taking the product of the 2. uppermost extremes onely for dividend, and $\frac{1}{1}$ (the middle number remayning) onely for diuisor, the quotient of whose diuision is $4\frac{1}{2}$ pound as here appeareth.

		shilling	weig.
	20		3 l.
$\frac{x}{z}$	d		shil.
		$\frac{1}{1}$	
$\frac{x}{z}$	d		wei.
			$4\frac{1}{2}$ l

Whereby I conclude that the halfepeny bottle of hay ought to weigh so much when the loade of hay is worth but $13\frac{1}{2}$ s. which answer is iustly agreeable with the worke of the 42. question in the 10. Chapiter.

Whereupon I imagine that all the questions which are to be answered eyther by the rules of that chapiter, or of the 11. chapiter touching plurall proportion, may also be answered by the compound rule conuersed, but yet I take this chapiter to teach the easier and more speedier way.

Phi. In deede all the questions of the 10. chapiter, sauing the 3 last, may be wrought by this compound rule conuersed, and therfore as I told you there, I would not vse many examples then, because I should haue new occasion to entreate more thereof heere, and so likewise many of the questions propounded in the chapiter of plurall proportionalitie, may be answered by the compound rule either direct or conuersely, but not all.

Euma. I pray you why so.

Philo. Because the 3. last questions of the 10. chapter haue no supposition, & therfore cannot be drawne to any of these compound rules, and of the questions propounded in the 11. chapter, those only may be dissolued by the compound rules, which consist of 2. such proportionalities, as are mentioned eyther in the 273. 274. 287. or 288. verses, and none other.

Eum. So then I perceiue these rules are like officers in a noble mans house, wherin as the cooke may somtime serue in the Caters rowme, and the Butler in the Pantlers, yet no one can serue all turnes in al things, but euery man serueth best in his proper office.

Phi. You haue made a prettie similitude, but now touching the other 2 compound rules remayning, I will onely put a question or 2. answerable by them, which I will also answer my selfe, and so leaue them to your priuate practise for this time, because the vse of these rules doth more properly belong to the next booke then to this, and this is the first question.

113. A question of the diuersitie of measures.

If 52. Anudes French, make 100. sticks Flemish, & 100. sticks Flemish make 60. ells English, how many anuds is 115. ells English?

Now

Now it appeareth manifestly by the 226. maxim, that this question is subiect to the compound rule discending, not onely because it permitteth the first 4. tearmes thereof, to be placed in the figure before it begin to demaund any question, but also because it demandeth how many Anuds (which is the fyrst firname) wil make 115 ells English, which (elles) is the last firname of the supposition: therefore by the second verse of that maxim, the bias termes shall match together, and so shall the head and tayle as here the figure sheweth.

anuds	sticks
52	100
sticks	elles
100	60
elles	
115	

Wherefore by the same rule, the product of the left hand tearmes being diuided by the product of the right hand termes quoteth the aunswering number, but because one couple of the matchtearmes here are equall viz. 100. sticks and 100. sticks, therefore by the 293. verse I omit them, taking 5980. the product of the other 2. lefthand termes onely for diuidend, and 60. the other right handed terme onely for diuisor, and so diuision being performed, the quotient is 99 2/3. which I conuey to his propper place at the tayle of the figure, concluding that 115. elles English, maketh but 99 2/3. Anuds French, as heere the fabrique expresseth.

anuds	sticks
52	100
sticks	elles
100	60
elles	anuds
115	99 2/3

114. A question of exchanging.

Another question is this, a merchant of London being at Roane, deliuered in exchange to a stranger there 1000. franks (euery franke being there worth 20 soules Touruoyes) vppon condition to receiue for the same at London after the rate of 4 s. 10 d. for euery crowne of 50. soules Touruoyes. Now the question is how much sterling money will pay the billes of exchange at London.

For aunswere hereof, fyrst I frame the question in order thus, saying, if 1. franke counteruayle 20. soules, and 50. soules counteruaiue 58 d. sterling, how many pence sterling is 1000. franks, and so the termes applyed to the figure, shewe thus.

franke.	soules.
1	20.
soules	
50	58 d.
franks	
1000	

Euma. So I perceiue you reduce such tearmes as haue diuers firnames, all to the least firname, for so haue you reduced the 4. shillings 10. pence, to 58. pence, in doeing whereof I can easily iudge by the 298. verse, that this question is subiect to the compound rule ascending.

Philo.

Philo. You say truly, it is so, and therefore as that rule requireth, (the tearmes being setled in the figure as you see) I multiplie 20. 58. & 1000. together, that is the 2. right handed termes, by the lowest terme on the left hand, and they produce 1160000. which I diuide by 50. the other terme remayning (for 1. multiplieth not) and the quotient bringeth 23200. pence sterling as the the figure sheweth, which by reduction to poundes, is 96. $\frac{1}{3}$ r. that is to say, 96 r. 13 s. 4. d. sterling, whereupon I conclude that so much sterling money shall discharge the billes of exchange at London.

franke.	soules.
1	20
soules	58 d.
50	
franks	
1000	23200 d

And so hauing giuen you these 2. questions as it it were for a task, to shewe how these rules may be further app'ied and extented, then perhaps you would thinke I refer the rest that may be sayde to my third booke called Musa mercatorum.

The multiplication of a proportion by a number.

And in the meane season as I haue shewed you the multiplication of one proportion by another. So will I shewe you briefly the multiplication of any proportion by a number, which is thus

You must describe the proportion which you would multiplie in the least tearmes thereof, as often as there be vnits in the multiplier or number, whereby you would multiple it. And then multiplying all the numerators together, the product shall be newe numerator to the proportion produced, and the product of the denominators shall likewise be the new denominator thereto, which done, the multiplication is finished, as for example.

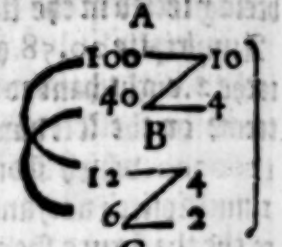
I would multiplie the proportion Sesquialtera by 3. wherefore I must describe the same proportion in the least termes thereof 3. times, for so many vnits is in 3. my multiplier, that is to say thus, as 3. is to 2. so is 3. to 2. and also 3. to 2. and now the product of the numerator is 27. and the product of the denominators is 8. wherfore I affirme that the proportion of 3. to 2. being tripled or multiplied by 3. is the proportion of 27. to 8. and so of all others.

Euma. This is easie inough to do, and is very like the reduction of particles to one simple fraction, and heere I remember you sayd in the glose of the 210. marim., that one proportionalitie may be also diuided by another, but you said, such division serued to no vse, and yet I am very desirous to see the worke of it, because I haue heard some affirme that there is no such opperation in pro-

por-

portions.

Philo. That the matter is not impossible appeareth by the 221. maxim, for if the proportionalitie A (there mentioned and here new described) multiplied by the proportionalitie B do produce the proportionalitie C as there it is affirmed and proued, then it followeth by the 94. verse, that the same product C beeing eftsoones diuided by A shall returne the proportionalitie B in the quotient, and being diuided by B shall returne the first proportionalitie A. into the quotient, but as Salignacus confesseth that he knoweth not the vse of such diuision, euen so do I, and therefore ouerpassing it as a matter more curious then necessarie, I minde here fully to close vp proportionalitie artificially compounded, and to proceede to the handling of proportionalitie artificially continued, in forme following.



Of proportionalitie artificially continued. Chap. 18.

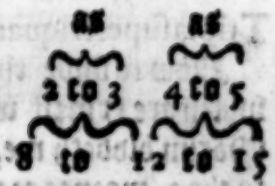
The definition of proportionalitie artificially cõtinued.

294. When any such proportions happe to fall,
Betweene 2. couple of numbers or mo,
As are not all iust alike continuall,
Yet if by art the like be found also,
In othe r termes continually still,
Then are those termes so found out especially,
Called a proportion continued by skill,
That is not by chance but artifially,
Wherein the giuen proportions must euer,
Be brought to least termes that is primes together.

This verse defineth what thing proportionalitie artificially continued is, namely that it is the artificiall finding of such proportions in certayne termes continually togeather, as is betweene 2. couple of mo giuen terms discontinually, as for example, let the 2. couples of giuen termes be 2. to 3. and 4. to 5. the proportion of whose first couple as you see and know is Subsesquialtera, and the proportion of the latter couple is Subsesquiquarta.

Now therefore to finde the same 2 proportions viz Subsesquialtera, and Subsesquiquarta in 3. termes continually proportionall, is called a proportion artifically continued or continuate, and such are these 3. termes. 8. 12. 15. for 8. to 12. is Subsesquialtera and the

the ſame 12. to 15. is Subſeſquiquarta, ſo that the explication of proportionalitie, artificially continued, appeareth in the figure here annexed, which I vtter thus, ſaying.



As 2. is to 3. ſo is 8. to 12. and as 4. is to 5. ſo is the ſame 12. to 15. and ſo the termes here, are ſayd to be continued or continuate, for though they be not continued all in one proportion (as they be in proportionalitie continuall) but in diuers, yet are they continued after the rate of the firſt proportions giuen, which I tell you of purpoſe to the end you ſhould not miſtake proportion continued, in ſteade of proportion continuall, and in this woorke the marim preſent telleth you, that the geuen proportions muſt euer be reduced to leaſt termes, that is to primes together, but here the giuen proportions 2. to 3. and 4. to 5. are in their leaſt termes or primes together alreadie, ſo that here needeth no ſuch reduction.

Euma. Very good, I vnderſtand the definition of the thing well inough, and do aſwell perceiue the diuerſitie betweene it, and proportionalitie continuall, eſpecially by reaſon of this your example and figuration thereof, which I muſt needs confeſſe to be very true, but I ſee not the artificiall way how to finde the numbers proportionall as you haue found them.

Phi. That ſhall the next rule or marim teach you, which is thus.

295. To finde the ſelfe ſame proportion or rate,
in 3. termes onely ſtill continuate,
Which is in 4. termes diſcontinuall,
Subſcribe the 2. leaſt numbers proportionall,
To both middle tearmes of thoſe 4. aſſignd,
Each to his owne terme according to kinde,
296. Which done, with a croſſe, in biaswiſe prooue,
To couple the lower to thoſe 2. aboue,
The offcome of any which couple ſo knit,
makes the midnumber which ſerues your turne fit,
297. And likewiſe th'extremes of th'vppermoſt foure,
biaſly coupled each with a lower,
Produceth the 2. extremes which you want,
On eyther ſide of the midnumber to plant.

Euclid. 4. prop. 8.

233

As for example (which in this caſe inſtructeth more then many woordes) let the proportions betweene 4. tearmes giuen be theſe, 2, to 9. and 15. to 4. that is Supquadruplaſeſquialtera, and

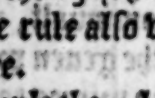
Tripla-

Triplasupertriquartas, which are in their least termes alreadie.

Now to finde the same proportions in 3. termes continually, which are expresst in the 4. former termes discontinually, this Maxim biddeth me, subscribe the 2. least numbers proportionall to the 2. meanes or middle numbers assigned, but that it may the better appeare, which termes are counted the middle termes heere, I place them all 4. in a rowe one after another, euen as they are named thus. 2. 9. 15. 4. whereby it appeareth now that 9. and 15. in respect of their places and situation are the middle termes.

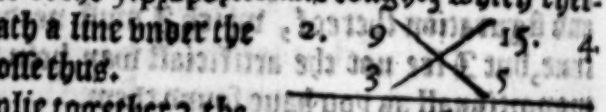
Meanes or middle termes in respect of place only.

Wherfore I subscribe vnto the same midnumbers their left proportionals, which by the 134. & 148. verses, are 3. and 5. each to his owne terme, as the rule also willeth me, and as here you see.



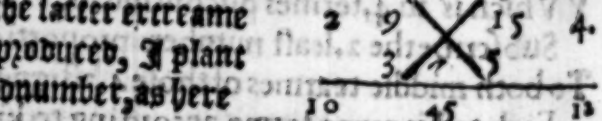
Which done, I couple the 2. lower to those 2. aboue, with a bias crosse, that is to say, 3. to 15. and 5. to 9. as appeareth.

Now sayth this maxim, that the product of any such couple so knit, maketh the midnumber sought, so that whether I take the product of 3. times 15. or of 5. times 9. both wayes there resulteth 45. for the midnumber of the 3. proportionals sought, which therfore I subscribe beneath a line vnder the middest of the bias crosse thus.



Extremes by place onely and not by quantitie.

Then do I multiplie together 2. the fyrst extreame by 5. the lower number standing bially, whereby I produce 10. for the first extreame, finally I likewise multiplie 4. the latter extreame bially, with 3. the other lower number, whereby I produce 12. for the latter extreame which 2. numbers so produced, I plant on each side of the midnumber, as here you see.



And as this present Maxim importeth. Wherupon I conclude that as 2 is to 9. so is 10. to 45. and as 15. is to 4 so is 45. to 12. the demonstration whereof is manifest.

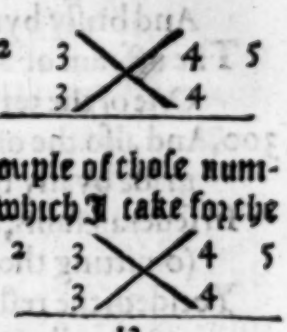
Euma. We thinke the working and finding of the numbers proportionall, as this Maxim requireth, is very easie, and therefore to trye how I can doe the like, I will attempt by this rule to finde the 3. numbers which you produced in the example of the fyrst verse of this chapter, wherfore taking the proportions there given, which were 2. to 3. and 4 to 5. being primes together already, I place them as you did in a ranke orderly thus. 2. 3. 4. 5.

And now would I take the least proportionalls to 3. and 4. the middle

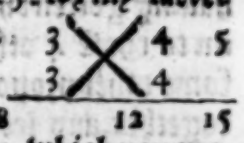
middle numbers, but they haue none lesser then themselues.

Philo. Then must you in such cases subscribe themselues vnder themselues, vsing them in all things as if they were lesser proportionalls.

Emma. That do I then in this forme, coupling them also with a byas crosse as the rule intendeth, and now drawing a line vnder all, as appeareth. I multiply any couple of those numbers biasly coupled, and their product is 12. which I take for the middle number which I seeke, subscribing it therefore vnder the middle of the crosse beneath the line thus.



Then do I also multiply 2. the first extreame, by 4. the lower bias number, and it produceth 8. for my first new extreame, which I set forthwith beneath the line vnder the same 2. as my first number desired. And finally I multiply 5. the last extreame, by 3. the other lower byas number, whereby I produce 15. for my latter extreame, which I forthwith place directly vnder the same 5. beneath the line, and so the numbers stand in this forme.



And thus haue I found the selfe same numbers, which you produced before, wherefore I may conclude, that as 2. is to 3 so is 8. to 12. And as 4. is to 5. so is the same 12. to 15. But if I would finde the like proportionalls continually to 3. or 4. couple of numbers, or moe, I see not how it should bee doone by this probleame.

Philo. And no marualle, for euen as you learned by the 226. verse, to finde first, 3. the least numbers continually, in such proportion as you would request. And then afterward by the 199. Maxim, you learned further, how to finde moe, and yet but one onely at once. Euen so heere, now you haue first learned to finde 3. termes continually, so shall you haue another rule, teaching you to finde the 4. and so as many moe as you list, and yet but one at once, or at one operation, and the rule is thus.

298. And hauing once gotten three termes in like rate,
To foure other in proportion seperate:
If too a new couple, or moe now you minde,
Continually like numbers to finde.
Note downe that couple at once, and no more,
Not far from the third, so found out before.

Euclid. 4. prop. 8. Tonstall fol. 128. 230.

299. Then take the leaſt proportionals againe,
To laſt of the three and firſt of the twaine,
Subſcribing them downe each in his place due,
And biaſly bynde them, with + made of new.
The offcum of any, which numbers makes ſtill,
One of the termes ſought, vſd which way you will,
300. And alſo the offcums of thoſe two belowe,
made byaſly by the vppermoſt rowe,
In ſeuerall ſums, and not in one groſſe,
(omitting thoſe twaine, ſo tyed to the croſſe,)
Yeeldeth the reſt of the numbers required,
continually in, proportion deſired. 234

As for example, it hath beene proued already, that as 2. is to 9. ſo is 10. to 45. and that as 15. is to 4 ſo is the ſame 45. to 12. in which example the giuen proportions were 2. to 9. and 15. to 4. and the proportions found were theſe 10. 45. 12. wherefore now hauing gotten theſe 3. termes continually in like rate of proportion to thoſe 4 other diſcontinually, let vs adde one couple more in ſome other proportion, which let be 18 to 29. which are alſo primes together, and let them be ſet not far from the ſayd 10. 45. and 12. (the 3. termes already found) but all diſtantlike in a ranke thus.

10. 45. 12. 18. 29.

Then take I (as this maxim biddeth me) the leaſt proportionals agayne to 12. which is the laſt of the 3. & to 18. which is the firſt of the twayne, ſubſcribing them in their due places (that is each one to his owne terme) and bynding them with a new bias cros thus.

10. 45 12. 18. 29.
2 3

Which done, I draw a lyne vnder all, as in multiplycation behoueth, and now whether I take the offcome of 12. by 3. or of 18. by 2. both wayes there amounteth 36. for one of the new termes ſought, which I therefore note vnder the middle of the ſame croſſe beneath the lyne thus.

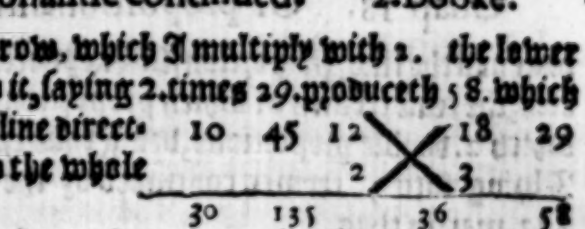
10. 45. 12. 18. 29.
2 3
36.

This being done I take the product of the 2. lower numbers, made biaſly by euery one of the vppermoſt rowe (excepting thoſe which are tied or coupled to the croſſe) ſaying firſt, 3. times 10. is 30. which I ſubſcribe vnder the line directly with the ſame 10.

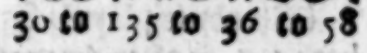
And ſecondly 3. tymes 45. is 135. which I ſubſcribe beneath the lyne directly vnder the ſame 45. and ſo being come to the termes tied or coupled to the croſſe, I ommit them, going to 29. the laſt

terme

terme of the vppermoſt row, which I multiply with 2. the lower terme ſtanding bialy to it, ſaying 2. times 29. produceth 58. which I ſubſcribe beneath the line directly with the ſame 29. & ſo the whole fabrique ſtandeth thus.



Whereby now I conclude, that among the giuen proportions, as 2. is to 9. ſo is 30. to 135. and as 15. is to 4. ſo is the ſame 135. to 36. and as 18. is to 29. ſo is the ſame 36. to 58. as this figuration preſent expreſſeth.



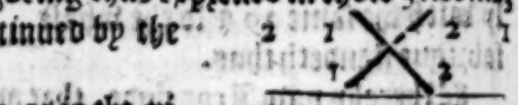
Eu. All this I vnderſtand reaſonably well, and do conſider that by like operation iterated, the proportion betweene another couple of terms, and ſo of another and another, may be continued infinitly, which in my mind is not much vnlike in nature to the 159. maxim touching finding of termes continually proportionall in any proportion aſſigned, although in worke it ſomewhat differeth, but to what purpoſe ſerueth this kinde of inuention or continuation.

Phi That ſhall you now ſee, and for the experience thereof I propound this queſtion, a man lying in his deathbed whoſe wife was great with child, bequethed his goods, which amounted to 1560. pounds in this ſort. If my wife quoth he, be deliuered of a ſonne, I will ỳ he ſhall haue $\frac{2}{3}$. of my goods, & my wife $\frac{1}{3}$ but if ſhe bring forth a daughter then I will that ſhe ſhall haue but $\frac{1}{3}$. of my goods, and my wife the other $\frac{2}{3}$. Now after the teſtators death, it chanced that the woman brought forth 2. ſonnes & a daughter: wherfore the queſtion is how the goods ſhalbe diuided according to ỳ teſtators intent. 115. A queſtion touching legacies.

Euma. This ſeemeth to me to be one of the ſtrangeſt and abſurdeſt queſtions in all this booke, for I cannot ſee how the intent of the teſtor can be knowne or vnderſtood further then by his owne words, wherein he hath made no more proviſion but for 2. perſons, to wit, for his wife and one childe, whether it be ſonne or daughter. And therefore how you can geſſe his intent in this caſe, paſſeth my cunning, wherefore I pray you worke the diſſolution of this queſtion your ſelfe for my learning.

Phi. Contented, and firſt although this queſtion ſeeme ſo ſtrange & ſo abſurd to you, yet you cannot deny but it appeareth manifeſtly by the teſtators words, that his will is, that the portion of his ſonne ſhould be double to the mothers, & that the portion of the mother ſhould be double to his daughters: wherby it is to be ſuppoſed ỳ if he had knowen his wife wᵗ child wᵗ 2. ſons & a daughter, yet he would

haue had the like proportion still kept betweene them in his legacies, the least termes of which proportions are 2. to 1. and 2. to 1. which 2. double proportions, being thus expressed in those 3. terms, I bring into 3. termes continued by the 233. maxim thus.



Wherin as the first 2. to 1. as in the giuen proportions so is 4 to 2. in the proportions continued, and as the second 2. to 1. is in the giuen proportions, so is 2. to 1. in the proportions continued.

Whereby is playnly signified, that as often as any of the sonnes hath 4. r. so often the mother ought to haue 2. r. and the daughter 1. pound.

Wherfore doubling 4. the first terme (because there are 2. sonnes) the question is concluded by the golden rule, as in the first rule of partnership, by putting 11. the totall of the said termes, for the first or generall antecedent, and the totall masse or sum to be diuided for generall consequent therto, and each perticuler terme for perticuler antecedent, so that I say thus.

If 4 + 4 + 2 + 1. that is 11. haue 1560. r. what shall 4. haue? and what 2? and what 1? and so, working by the golden rule thrice onely. I finde all 4. portions to be such as heere insueth.

Whereby appeareth, that eache of the Sonnes	Eache sonne.	The mother.	The daughter.
	11 — 1560	11 — 1560	11 — 1560
	4 — 567 $\frac{3}{11}$	2 — 283 $\frac{7}{11}$	1 — 141 $\frac{9}{11}$

ought to haue 567 $\frac{3}{11}$. The mother 283 $\frac{7}{11}$ r. and the daughter 141 $\frac{9}{11}$ r. all which latter consequents added together, amount iust equall to the generall consequent wherof they are parcels, wherby I affirme that the partition is iustly made, & the conclusion true.

Euma. Surely this diuision contayneth great equitie, for it is apparant that each sonnes portion is double to his mothers, and the mothers is double to her daughters, which seemeth to be the very intent of the testator or proposed by his testament.

Philo. No doubt it is so, and now will I put one question more, which I will also answer my selfe, and then leaue the rest to your practise: my question is thus.

116. A question of a boo-tie gotten by soldi-ers.

A B C. and D. being 4. soldiers, but not of like estate, had a bootie of 1000. crownes to diuide betweene them, wherefore in respect of their degrees, it was agreed betweene them, that as often as A. had 10. crownes, B. should haue but 8. and for euery 10. that B. had C. should haue but 6. and for euery 10. that C. had, D. should haue

but

but 4. Now the question is, how many crownes euery of them ought to haue?

Here the proportions giuen or named, are 10. to 8. 10. to 6. and 10. to 4. which termes because they are compounds together, must by the first verse of this chapter be brought to pnnes together, so that instead of the giuen termes or proportions, I must take their least proportionals, which are these, 5 to 4. 5 to 3. 5 to 2. 5 to 4. 5 to 3. & 5. to 2. which done, I continue them by the 233. and 234. Maxims to 4. termes, as heere the fabricke annexed expresseth.

5 to 4 × 5 to 3 5 to 2

4 × 5

25. 20. 12 × 5. 2

12 × 5

125. 100. 60. 24.

Whereby I finde that as 5. is to 4. so is 125. to 100. and as 5. is to 3. so is 100. to 60. and as 5 is to 2. so is 60. to 24.

Wherefore now according to the first rule of partnership 125. + 100 + 60 + 24. that is to say 309. being the totall of these termes continued, shalbe the first and generall antecedent, in the golden rule, and the 1000. crowns being the totall gaynes or masse to be diuided, shalbe the generall consequent, and so finally euery particuler terme found being seuerally taken and vsed for later antecedent the question is concluded thus, if 309. haue 1000. ∇. what shall 125. haue? and what 100? and what 60? and what 24?

And the golden rule answereth, that A. 125. shall haue 404 $\frac{16}{103}$. ∇. and that B. 100. shall haue 323. $\frac{121}{309}$. ∇. and that C. 60. shall haue 194. $\frac{76}{103}$. and D. 24. shall haue 77. $\frac{187}{309}$. crownes as heere the fabriks of the seuerall operations manyfestly declare.

A: 309 ⟋ 1000. crow. / 125 ⟋ 404. $\frac{16}{103}$.

B: 309 ⟋ 1000. crow. / 100 ⟋ 323. $\frac{121}{309}$.

C: 309 ⟋ 1000. crow. / 60 ⟋ 194. $\frac{76}{103}$.

D: 309 ⟋ 1000. crow. / 24 ⟋ 77. $\frac{187}{309}$.

The totall of all which latter consequents are iust equall to the generall consequent whereof they are percells, whereby I affirme the distribution to be good and the aunswer true.

117. A question of partition betweene brothers.

And now to proue how diligently you haue marked this kinde of operation by proportion continued, I put you this question.

There brothers A. B. and C. haue 100. r. to share betweene them so that as often as A. hath 5. r. so often B. must haue 6. and as often as B. hath 7. r. so often C. must haue 9 r. wherfore I would see you tell me, what euery mans portion is.

Euma. First the giuen proportions here are 5. to 6. and 7 to 9. which being already primes together, are in their least termes, and haue no lesser proportionals, wherefore I indeuor imediatly to continue them to 3. termes, by the 233. maxim, thus.

5 to 6 × 7 to 9
6 × 7
35 42 54

Which done, I take the totall of 35 + 42 + 54 which are the termes found by this continuation, that is to say 131. for generall antecedents in the golden rule, and 100 l. the totall masse or matter diuidable, for generall consequent thereto. And then taking euery seuerall terme found for particuler antecedent, euen as in the first rule of parternership, I cõclude euery mans share of this 100 pound by the seuerall operations of the same, as heere the letters A. B. C. insuing declare.

	A	B	C
	131 ∠ 100. l.	131 ∠ 100 l.	131 ∠ 100 l.
	35 ∠ 26 $\frac{2}{131}$	42 ∠ 32 $\frac{1}{131}$	54 ∠ 41 $\frac{32}{131}$

Wherein because the totall of the latter consequents makes iustly 100 l. equal to the generall consequent. Therefore I affirme, that euery of their portions are such as in the former fabriques is expressed.

Phi. You haue performed all operations belonging to this question very well and orderly, but yet for your further exercise I say thus. There is 500. l. bequeathed to A. B. C. and D. in such sorte that when A hath taken 2 l. B shall take 4 l. and when B. hath taken 6. l. C shall take 8. l. and when C hath taken 10. l. Then D. shall take 12. Now declare euery mans portion.

118. A question of partitiõ betweene legataries

Euma. To do that, first I survey the proportions giuen, which are 2. to 4. 6. to 8. and 10. to 12 whose termes I finde to be compounds together, which therefore by the first maxim of this chapter must be reduced to primes together, before I can proceede, and therefore in their steades, I take 1 to 2. 3. to 4. and 5. to 6. being their least proportionals.

1. to 2 × 3 to 4. 5 to 6.
2 × 3
3 6 8 × 5 6
8 × 5
15 . 30 . 40 . 48.

Which done, I continue them by the 233. and 234. Maxims, whose fabrique sheweth in this forme.

So haue I found 4. termes in such proportion continually, as the giuen proportions are discontinually, for as 1. is to 2. so is 15. to 30. And as 3. is to 4. so is the same 30. to 40. and as 5. is to 6. so is the same 40. to 48.

Whereby (as you said) is signified, that when A hath 15. pound, B. must haue 30 l. C 40. l. and D 48. l. So that if the whole sum

to

to be diuided, were but 133. l. then were these portions their very true shares or parts.

Philo. You haue hit the nayle on the very head, wherefore now as the same 133. l. is in proportion to the true totall sum to be diuided, so is euery mans particuler share thereof already found, to his true share yet vnknowne.

Emma. Therefore I take the same 133. being the totall of 15 + 30 + 40 + 48 for generall antecedent in the golden rule, and so taking euery parcell thereof in his proper place, for the latter antecedent as before. I follow the order of the first rule of partnership, comming now to the golden rule, and saying.

If 133. take 500 l. what shall 15. take ? and what 30 ? and what 40 ? and what 48 ? The seuerall operations whereof aunswereth, as here appeareth in the fabriks.

A		B	
133	500 l.	133	500 l.
15	56 $\frac{35}{133}$	30	112 $\frac{105}{133}$

C		D	
133	500 l.	133	500 l.
40	150 $\frac{70}{133}$	48	180 $\frac{84}{133}$

And because I haue tried that al these latter consequents are precisely equall to the first generall consequent. Therefore I conclude the answer to be true.

Philo. It is so, and hereby I hope you are so well instructed, in these and such like questions, that I need not to stay any longer vppon this kinde of proportionalitie, but may go in hand with Alligation.

Of the rule of Alligation. Chap. 19.

301. Alligation is a rule of such deuice,
As by certayne things giuen and expreſt,
Doth ballance the totall valure or price,
Of an whole masse or mixture increast,
To the totall price of those simple things,
Wherof the same masse or mixture so springs.

The definition of Alligatiō.

302. VVherein all measures or quantities,
Truly, must be of like name, and also the price,
Of euery measure of mixture as duly,
Must be some meane valure prepard by aduice,
Lesse then the greater, and more then the least,
Of th'extreme prices to simples assignde,

The properties of Alligatiō.

Which done, each greater extreme so exprest,
Must linke with a lesser, by nature and kinde,

Ramus libr 2. ca. 10. Ex relatione vrstic j libr 2. ca.

Although as Ramus saith, Alligation of it selfe is no proportion, and Saligniacus after him affirmeth that it ought not to be taught as a part of Arithmetick, because saith he, the cannons rules and precepts of Arithmetick ought to be of numbers abstract, which he saith in Alligation is not so, wherein as I grant that the rules and precepts of Alligation consist of numbers contract, yet because the same is not concluded, but by the helpe of Addition, Subtraction, Multiplication, Division, yea and of the rule of proportion or golden rule it selfe. Therefore I say, I haue thought it nothing amisse nor no offence, to insert it heere as a parcel of the arte, and as a conclusion properly proceeding and deriued from proportion discontinuall, and so doth Vrstitius accept it also.

And not without reason in my minde, for if there were not such a proportion, betweene 3. termes giuen and found in this rule, as by their application to the golden rule would produce the fourth proportionall sought, all thereof of the rule and worke thereof were in vayne and to no purpose.

And now to the explycation of this present maxim, whereof the first part containeth the definition of alligation, shewing you that it is a rule, which in certayne things giuen and exprest, doth ballance or equific the total valure or price of an whole masse or mixture encrease, to the totall price or valure of all the simples whereof the same mixture springeth or is so increased and compounded, as for example.

20. Ounces of base siluer worth but 3 s. the ounce is to be molten and mixed with finer siluer, worth 5 s. the ounce, in such sorte, that the total valure or price of the whole masse or mixture increase by this composition, may neither be more nor lesse, then the totall valure of both the simples or seuerall sorts of siluer, whereof it was compounded.

Note heere the 20. ounces of base siluer, and the 2. seuerall prises of 3. s. and 5. s. are the things giuen or exprest, wherefore the totall which by the same things so giuen or exprest equalleth her ballanceth. Such kinde of totalls is called alligation.

So likewise there is 10. bushells of good wheate worth 3. s. a bushell, to be mixed, with 18. bushels of worse wheate worth but 2. s. a bushels, how may 4. bushell of this mussling or mixture be sold, so that the totall price thereof may rise equall to the price

or

or valure of both the simples:

Here both the quantities of the seuerall simples & also their prices are giuen or exprest to wit 10. bushels at 3. s and 18. bushels at 2. s. And there is some parte (at the least) of the mixture giuen also, as namely 4 bushels. Wherefore the rule which by these things thus geuen ballaunceth or equiffieth their totalls propounded, is called the rule of Aligation, and this by the first verse here present is the very definition of the rule.

Now the second verse thereof sheweth the generall propertie incidently belonging to Alligation, as namely fyrst that all the measures or quantities therein, ought euer to be of like name, as thus, 2. bushells of wheate of 24. d. the bushell is to be mixt with 2. bushels of Rie of 18 d. the bushel. Now by the same verse it is required that these measures or giuen quantities be of like name as here they be viz. bushels.

Euma. I vnderstand you thus, that if it were sayd, that 2. bush. of wheate, of 2. s. the bush. should be mixt with 16. gallons of Rie, of 18. d, the bush. Then were not the measures or quantities giuen of like name, sith the one is bushels and the other gallons, and also becaule the price of the one is shilling, and the other pence, wherfore I take it that both your intent, and also the meaning of the verse is, that all the bushels in such cases should be reduced to gallons, and both prices to pence, or els that all the gallons should be reduced to bushels, and both prices to shillings before the Alligation can proceede.

Philo. You take it well, and euen so if 1. ounce of fine siluer, should be mixed with 1. ounce of base siluer, these quantities being not of like name, must be both reduced to like name by conforming reduction, so will they be $\frac{1}{2}$ and $\frac{1}{2}$.

But now further by the same verse, the propertie of Alligation also is such, that the price of euery measure or quantity of the whole mixture alwayes requireth duly, as it were of dutie to bee some meane price or valute, lesser then the greater sorte of the extreame prices of the simples, and more then the lesser sorte for otherwise ỳ conclusion sought for and desired is impossible, as thus:

In the former example of wheate, the price of the simple measurs was 24 d a bushell and 18 d. a bushell, wherefore the price of euery bushell of the mixture must necessarily be some such meane or middle price, as is lesse then 24 d. the greater extreame price exprest, and more then 18 d. the lesser extreame price.

Euma. So I vnderſtand you very well, for I perceiue that the price of euery buſhell of the mixture may be any price aboue 18 ð. and vnder 24 ð. but it may not exceede the greater in greatneſſe, nor the leſſer in ſmalnes, neither yet may it be equal to any of them both. And further I perceiue that becauſe the mixture of both muſt alwayes be a meane or middle price, betweene the prices of the ſimples, therefore the price of the mixture is not onely called the meane, but alſo the meane price is alwayes accounted to be the price of the mixture.

Meane price what

Extreame prices which.

And contrarily as the extremes do alwayes repreſent the prices of the ſimples, So are the prices of the ſimples alwayes called and accounted the extremes, whereof ſuch as exceede the meane, are named the greater extreames, and ſuch as are leſſe then the meane you call the leſſer extremes.

These extreames are ſo called in reſpect of quantitie and not of place, becauſe they all are eyther greater or leſſer then the meane price. Ergo, the meane price is alſo called the meane in reſpect of quantitie onely. Ramus lib 2. cap. 10.

Philo. You conceiue all things very rightly, wherefore now it followeth by the ſame verſe, that euery ſuch greater extreme muſt euen as it were naturally, be linked with a leſſer, which is the thing whereof the whole rule doth chiefely take the name, for the very name of Alligation ſignifieth nothing els, but a certayne linking, yoking, coupling, or tying together of things, and of this rule there are 2. kindes, as by the Axim following appeareth.

303. There are 2. ſorts or kinde of Alligation,
That is to ſay, alternate and mediall,
Alternate doth alter, and change the location,
Of ſuch exceſſes, as commonly fall,
Betweene the meane price, and both th'extremes,
VVhereby forthwith, the proportion is known,
Of euery ſimple, and ſo by that meanes,
Queſtions are anſwerd and ſoone ouerthrown,
304. And Alligation alternate at leaſt hath 2. kindes, 236
whereof the firſt doth decree,
By meaſures of ſome one ſimple expreſt,
to finde all the reſt by the rule of 3.
Placing that ſimple for laſt antecedent,
For matchterme whereof it takes the exces,
Aſcribed to the price of that ſimple preſent,
VVhoſe meaſures giuen, are known ſo expreſſe,
For conſequent laſt each exceſſe alone,
Is taken by turne, vntill all be gone.

As

As the fyrſt verſe of this maxim affirmeth, that there are 2. kindes of Alligation, to wit alternate and mediall, ſo it ſheweth you that the fyrſt kinde is called alternate, becauſe it doth alter and change the logation of the exceſſes or differences, falling out betweene the meane price of both the extremes.

Which is almuch to ſay, as that it aſcribeth that the exceſſe to the greater extreme, which proceede from the leſſer, whereunto it is linked or alligated, and that to the leſſer which came from the greater, for playne example whereof take this.

119. A queſtion of mixing corne.

A man intending to mixe 10. buſhels of wheat of 20 ſ. the buſhell, with Rie of 16 ſ. the buſhell, Barly of 8 ſ. the buſhell, and Oates of 6. ſ. the buſhel, would willingly make the mixture ſo, as he might affoorde euery buſhel thereof for 10 ſ. wherefore it is demaunded how much Rie, Barly and Oates, he ſhall commix with the ſayd 10. buſhell of wheate, ſo that the price of the mixture may iuſtly counterualue the prices of the ſimples.

Here 20. and 6. are yokefellowes, and ſo are 16, and 8.

Now to prepare the way to the anſwere hereof, the rule requireth that fyrſt the meane price be noted downe, with the extremes at the right ſide thereof in this forme.

10 ſ. { 20 / 16 / 8 / 6

Where as you ſee, 10. the meane price is fyrſt ſet, and then al the extremes at the right hand thereof each vnder other, ſo that the greateſt extremes are vppermoſt, and the leſſer beneath them, with a prettie diſtance betweene them for a diſtinction.

Then by the 3or. verſe, each of the greater extremes muſt be linked or poked to one of the leſſer, which is ordinarily done with a little crooked line in ſuch forme as here you ſee, where you may ſee that 20. is linked with 6. and 16. with 8.

10 ſ. { 20 — 6 / 16 — 8 }

This done, I take 10. the exceſſe or difference betweene 20. one of the greater extreames and the meane, and aſcribe it to 6. the leſſer extreme his yokefellow, noting it directly with the ſame 6. as if it came from him, then do I likewiſe take 4. the exceſſe or difference betweene the meane and the ſame 6. and aſcribe it to 20. the greater extreme, placing it directly with the ſame 20. as if it proceeded from it.

After that I alſo take 6. the difference or exceſſe betweene 10. the meane price, and 16. the ſecond greater extreme, and aſcribe it to 8. the leſſer extreame his yokefellow, and the exceſſe betweene the meane and the ſame 8. being 2. I aſcribe to 16. the

greater

Euma. So I vnderſtand you very well, for I perceiue that the price of euery buſhell of the mixture may be any price aboue 18 d. and vnder 24 d. but it may not exceede the greater in greatneſſe, nor the leſſer in ſmalnes, neither yet may it be equal to any of them both. And further I perceiue that becauſe the mixture of dutie muſt alwayes be a meane or middle price, betweene the prices of the ſimples, therefore the price of the mixture is not onely called the meane, but alſo the meane price is alwayes accounted to be the price of the mixture.

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By meaſures of ſome one ſimple expreſt,
to finde all the reſt by the rule of 3.
Placing that ſimple for laſt antecedent,
For matchterme whereof it takes the exces,
Aſcribed to the price of that ſimple preſent,
VVhoſe meaſures giuen, are known ſo expreſſe,
For conſequent laſt each exceſſe alone,
Is taken by turne, vntill all be gone.

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Which is aſmuch to ſay, as that it aſcribeth that the exceſſe to the greater extreme, which proceede from the leſſer, whereunto it is linked or alligated, and that to the leſſer which came from the greater, for playne example whereof take this.

A man intending to mixe 10. buſhels of wheat of 20 ſ. the buſhell, with Rie of 16 ſ. the buſhell, Barly of 8 ſ. the buſhell, and Oates of 6. ſ. the buſhel, would willingly make the mixture ſo, as he might affoorde euery buſhel thereof for 10 ſ. wherefore it is demaunded how much Rie, Barly and Oates, he ſhall commix with the ſayd 10. buſhell of wheate, ſo that the price of the mixture may iuſtly counterualue the prices of the ſimples. 119. A queſtion of mixing corne.

Now to prepare the way to the aunſwere hereof, the rule requireth that fyrſt the meane price be noted downe, with the extremes at the right ſide thereof in this forme.

10 ſ. { 20 / 16 / 8 / 6 } Here 20. and 6. are yokefellowes, and ſo are 15, and 8.

Where as you ſee, 10. the meane price is fyrſt ſet, and then al the extremes at the right hand thereof each vnder other, ſo that the greateſt extremes are vppermoſt, and the leſſer beneath them, with a prettie diſtance betweene them for a diſtinction.

Then by the 3or. verſe, each of the greater extremes muſt be linked or poked to one of the leſſer, which is ordinarily done with a little crooked line in ſuch forme as here you ſee, where you may ſee that 20. is linked with 6. and 16. with 8.

10 ſ. { 20 — 6 / 16 — 8 }

This done, I take 10. the exceſſe or difference betweene 20. one of the greater extreames and the meane, and aſcribe it to 6. the leſſer extreme his yokefellow, noting it directly with the ſame 6. as if it came from him, then do I likewiſe take 4. the exceſſe or difference betweene the meane and the ſame 6. and aſcribe it to 20. the greater extreme, placing it directly with the ſame 20. as if it proceeded from it.

After that I alſo take 6. the difference or exceſſe betweene 10. the meane price, and 16. the ſecond greater extreme, and aſcribe it to 8. the leſſer extreame his yokefellow, and the exceſſe betweene the meane and the ſame 8. being 2. I aſcribe to 16. the greater

greater extreame wherwith it is linked, and so the excesses or differences are placed and set lon as here you see.

20 } 4
16 } 2
10 b. 8 } 6
6 } 10

Alternation what it is.

And this altering or changing of the situation of the excesses, wherby the excesse of the one extreme is so transferred ascribed and attributed to the other, and placed directly with it, as it is called alternatiō, & the aligation therby is named alternate.

Eu. Very good, all this is calle enough both to understand and to doe.

Philo. Then are you further to understand also by the same verse, that the sayd excesses so alternated and interchangeably situated, do forthwith notifie and declare the true proportions of euery simple, which is to enter into the mixture, so that here the excesse 4. standing against 20 b. the price of the wheat, and 2. against 16 b. the price of the Rie, and 6. against 8 b. the price of the Barly, and 10. against 6 b. the price of the Dates do manifestly expresse & notifie vnto you, that for euery 4. bushels of wheate, employd in this mixture there must be taken 2. bushels of Rie, 6. bushels of Barly, and 10. bushell of Dates.

20 } 4
16 } 2
10 b. 8 } 6
6 } 10

Eumathes. This is a very good rule, but I see not the reason of it.

Phi. The reason and whole demonstration thereof dependeth on the 55. and 57. verses as shall appeare hereafter.

But in the meane season it followeth by the second part of this Martin, that Aligation alternate, hath 2. kindes, whereof the first ordinarily, by the measures or quantities of some one simple giuen or expresst, decreeth to finde out the rest, by the golden rule, wherein it vseth the application of such termes as in the sayd verse is named and declared.

Euma. Then as I take it, the question which we haue in hand, is of the first kinde of Aligation alternate, because that herein the measures of the wheate (which is one of the simples) is playnely giuen or expresst, to wit 10. bushels, and so is not the measures or quantities of any of the other graynes.

And therfore as the same verse and your woordes importe, this aligation present, by these 10. bushels, and by the helpe of the golden rule, findeth out the measures of all the other simples proportionally, wherein it placeth the same 10. bush. for all antecedent, & then for matcheterme thereto, it taketh 4. which is the excesse ascribed

to

to 20.ð. the price of the same wheat, and lastly for consequent, it taketh each of the other excesses alone by turne one after another, til it hath passed and ouergon them all.

So that the first excesse now to be made consequent in the golden rule, is 2. and at the next operation 6. and lastly 10. but the antecedents continue vnchangable in euery operation, wherby I perceiue that at 3. operations of the golden rule, the quantities or measures of the Rie, Barley, and Otes (being the 3. simples vnknown) which are to enter into the mixture required, shalbe found out and made knowne.

Philo. You haue sayd very well and truly, and therefore if here the quantitie of the wheate, (which is the simple whose measures are exprest) had beene but 4. bushels, then had all the other quantities signified by the excesses of this Alligation, been the true quantities of the other simples sought for.

But forasmuch as our present question is not, how much Rye, Barley and Otes, is to be mixed with 4. bushels of wheate, but with 10. bushels of wheate, therefore now seeing the quantities of euery simple to be mixt with 4. bushell of wheate, is knowne already by the Alligation; yet resteth for the golden rule to finde out, what quantities of the same simples are to bee mixed proportionally which 10. bushels of wheate, saying thus. If 4. bushels of wheate require 2. bushells of Rye, what will 10 bushells of wheate require?

Secondly, if 4. bush. of wheate require 6. bushels of Barly, what will 10. bushels of wheate require? And lastly if 4. bushell of wheat require 10. bush. of otes, what will 10. bush. of wheate require, and so working by the rule, you shall finde that 10. bushels of wheate requireth 5. bushels of Rie, and 15. bushels of Barely, & 25. bushels of oates to be mixt with it:

wheate B Rie	wheate C barly	wheate D Otes.
4 bush. — 2 bush.	4 bush. — 6 bush.	4. bush — 10 bush.
10 — 5	10 — 15.	10 — 25.

as here the seuerall fabriques testifie, & for triall therof I say thus.

The price of { 10. bush. wheate at 20. ð. the bush. is — 16 s. 8 ð. / 5. bush. Rie at 16. pence the bush. is — 6 s. 8 ð. / 15. bush. Barly at 8. ð. the bush. is — 10 s. / 25. bush. Otes at 6. ð. the bush. is — 12 s. 6 ð. } tot. 45 s. 10 ð. The Proofe.

The price of 55. bush. of mistling, at 10 ð. the bushell is 45 s. 10. ð. wherein you see the totall valure or price of the mixture, is iustly

equall

equall to the totall value or prices of the simples, as the generall propertie of Alegation requireth.

Euma. This is so playne and manifest that it cannot be denied, and I vnderstand the maner of the worke very well.

Philo. Then must I shewe you 2. speciall properties incident to this kinde of Aligation, whereof the first is thus.

Diuersitie of answers

305. When both the greater and lesser extremes,
Are plurall, then may they be linked and tied,
So diuersly and oft, that by that meanes,
Sundrye excesses for one is applyed,
So that one question of such alternation,
May haue diuers answeres by such alligation. } 237

As for example, by our late question, the fabrique of whose alligation was in this forme.

But now by this verse, sith that both the greater extremes, and also the lesser are plurall in number, therefore they may be linked or tyed together diuersly, for if I linke 20 to 8. & 16. to 6. yet do I obserue the entent of the verse, because I linke each of the greater extremes with a lesser, as here appeareth.

10 d. { 20 4 / 16 2 / 8 6 / 6 10 }

10 d. { 20 2 / 16 4 / 8 10 / 6 6 }

Where though the excesses are the same they were before, yet are they not now ascribed to those extremes which they were before in the former fabrique.

Euma. That I perceiue well, but will the answere that shalbe concluded now by the application of those excesses to the golden rule after the former fashion be true.

Phi. That would I haue you to proue.

Euma. Then for triall thereof I take the 10. bushels of wheate (bring the measures of the simple expres) for the third terme or latter antecedent in the golden rule, and for the mathterme thereto I take 2. the excesse now ascribed to 20. the price of the Wheate, and for the second terme or consequent, I take 4. the next excesse, and so working by the rule I finde the answering number to be 20. bush. of Rie as here appeareth. Secondly keeping the same antecedents but now placing 10. the next excesse for the consequent, and working by the rule agayne as before, I finde the answering terme to be 50. bushels, that is 50. bushels of barly, as here appeareth.

	wheat.	Rie.
bus.	2	4. bush
	10	20.

	Wheat.	Barly
bush.	2	10. bu.
	10	50.

Lastly

Lastly retayning still the former antecedents, and placing now 6. the last excesse for the consequent. The operation of the rule deliuereth 30. bushels of Oats for the answer, as here appeareth.

Wheat.	Oates.
2. bush.	6. bush.
10	30

And so (by this Alligation) he must take 20. bushells of Rye, 50. bushells of Barley, and 30. bushells of Otes to mixe with his 10. bushells of Wheate.

Phi. You haue done well, and you did as well to say (by this alligation) he must take so much, for otherwise you see now it is not of necessitie, that he must take so much onely of euery kinde of grayne, sith you know that by our former worke, other quantities serued aswell, and therefore euer, when both sorts of the extreames are plurall, it is better to say, that so much may be taken, then absolutely that so much must be taken. But now proceede, for you haue not done the triall of the worke, till you finde the totall value of the mixture, equall to the totall value of the simples.

Euma. That is very true, wherefore to proceede, first I finde. The Proofe.

That the price of
10. bush. Wheat, at 20 d. the bu. is 16 s. 8 d.
20. bush. Rie at 16 d. the bush. is 26 s. 8 d.
50. bush. Barly at 8 d. the bush is 33 s. 4 d.
30. bush. Oates at 6 d. the bush is 15 s.
tot 4 l. 11 s. 8 d.

And that 110. bush. of milling at 10 d the bush. amounteth to 4 l. 11 s. 8 d. Whereby I am assured the worke is good, and the conclusion true, which thing nevertheless seemeth marueilous strange to me, for now the number of all the measures, aswell of the mixture, as of the simples (sauing onely of the Wheate) is wholy changed.

Philo. You will meruayle more then, if you should yet see the same question concluded another way, as it may be, by the proport of this verse, for as this verse affirmeth that the extremes may be tied or alligate diuerslie, so it sayth also that they may be tied often, as for example, by the same question whose extreames now I linke and Alligate together thus.

	20	2. 4	
pence	16		
10.	8		10
	6		10

Where 20. now is linked both with 8. and 6. and therefore 10 the excesse of 30. about the meanes ascribed both to 8. and 6. as you see, and 2. and 4. being the excesses both of 8. and 6. are therefore also ascribed to the same 20. their yokefellow, as appeareth aboue.

Agayne:

Agayne here you see that 16. is tied both to 8. and 6. wherefore 6. the excesse of 16. aboue the meane are ascribed both to 8. and 6 and therefore both 2. and 4. being the differences of 8. and 6. from the meane, are ascribed to the same 16. their yoke fellow, wherefore by this meanes as the verse also saith, sundry excesses are applyed for one, and indeede now all the excesses which are ascribed to one extreame, and stand directly with it, are accepted but for one, and is so applied to the golden rule each in his due place.

10 d.	20	2.4	16
	16	2.4	16
	8	10.6	16
	6	10.6	16

And therefore, as many excesses as are so ascribed to one extreme, must be added together as here you see they are.

And now to see what quantities this alegation will produce, I applie the termes due to the golden rule after our former fashion, taking still the 10. bushels of Wheate (being the measures of the simple giuen or expresst) for the third tearme, and 6. being now the excesse ascribed to the price thereof, for match tearme thereto, and so euery other excesse seuerally one after another for the consequent.

Which done, and the operation of the rule being but twice performed, although for playneneſſe sake it be three times here described, the answering numbers are, 10. bush. of Rie, 26 $\frac{2}{3}$ bushels of Barly, and 26 $\frac{2}{3}$ bushels of Dates, as here the seuerall fabriques expresse.

Wheate	Rie	Wheate	Barly	Wheate	Dates
6 bush	6 bush.	6 bush.	16. bush.	6 bush.	16. bush.
10	10	10	26 $\frac{2}{3}$	10	26 $\frac{2}{3}$

And for triall how the totall price of this mixture will agree with the totall valure of the simples, I say thus:

The Proofe.

10. bush. wheate at 20 d. the bush. is 16 s. 8 d.

The price of { 10 bush. Rie at 10 d. the bush. is 13. s. 4 d. / 26 $\frac{2}{3}$ bush. barly at 8 d. the bush. is 17 s. 9 d. $\frac{1}{3}$ / 26 $\frac{2}{3}$ bush. Dates at 6 d. the bush. is 13 s. 4 d. } 3 l. 1 s. 1 d. $\frac{1}{3}$

The price of 73 $\frac{1}{3}$. bushels of the mistling, at 10. pence a bushell is 3. pound, 1 shilling 1. peny. $\frac{1}{3}$.

Euma. Truly I finde the present verse, and euery woorde therof exactly verified in these woorkes.

Philo. It is so, and as you see what diuersities of Aligation may

may be made by these 4. simples and their differences or excesses, so if the simples were mo in number, then might there be moe diuersities of alligation and thereby more diuersities of answer.

And yet I assure you this selfe same question may haue diuers other answers also, by reason of such other varieties of alligation as may be vsed: but these few may suffice for examples, and for explanation of the present verse. Wherefore leauing the rest to be excogitated by you at your further leasure, I goe forth with the next propertie, which is thus.

306. But if the greater extreame or the least,
be single alone, the rest being plurall,
Then shall it soly be linkt to the rest,
of contrary sorte, I say to them all,
And haue his excesse so oft alternated,
as it appeareth to be alligated. 238

The sence of this verse is thus. That if any sort of the extremes, whether it be of the greater sorte or of the lesser, be single, that is to say, if there be but one alone, the other extreames being plurall or many, then such single extreame shall be linked and alligated, with all the rest of the contrarye sort, which are plurall, and shall haue the excesse therof so often alternated and ascribed, as it appeareth to be linked and alligated.

120. A question of mixing Corne.

As for example. A man intending to mixe 12. bushels of Rye of 14. ô the bushell, with Wheate of 18. ô, Barley of 11. ô & Oates of 9. ô. the bushell, desireth to make the mixture so, as he may afford euery bushell thereof for 10. ô. Therefore how much Wheate, Barley and Oates, may he take to mingle with the said 12. bushels of Rye, that the totall price of the mixture, may ballance the totall price of the simples?

First here the simple, whose measures are giuen or expresst, is the 12. bushels of Rye, whose price is 14. ô. and the meane price is 10. ô. wherefore describing the same meane with his extremes, after the former order prescribed, as heere appeareth.

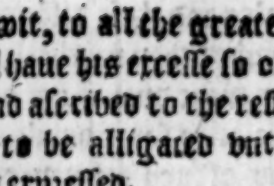
		14
pence	{	18
10	{	11
		9

You see that the greater extreames are plurall in number.

Euma. I see that well, for there are 3. of them, but the lesser extreame is but one singly alone.

Philo. You say truth, wherefore by this verse, the same extreme so standing singly alone, shall be linked to all the extreames of the

contrary sort to wit, to all the greater extremes, & shall haue his excesse so often alternated and ascribed to the rest, as it appeareth to be alligated vnto them, as heere is expressed.



Where 9. the lesser extreame being single alone, is as you see, linked and alligate with euery of the 3. greater extreames, and 1. being the difference or excesse betweene it and the meane, is therefore thrice alternated and ascribed, that is, to euery of the greater extreames once, and by the same reason, 4. 8. and 1 being the excesses of the 3. greater extreames aboue the meane, is ascribed to 9. the lesser extreame onely.

Luma. This I perceiue well, and therefore I remember that all these 3. excesses ascribed to the lesser extreame, must be added together, and be accepted and applyed in worke for one onely excesse, so that the excesse ascribed to 9 is 13.

Philo. It is well remembred, wherefore now to answer the question, I take 12. bushels, the measures of the simple expresse, for the third terme, and 1. the excesse belonging to the price of the same simple for the first antecedent, and then 1. 1. and 13. the other seuerall excesses remaining successiuely for consequent in the golden rule, by whose operation onely twice executed, I finde the quantities produced to be 12. bushels of Wheate, 12 bushels of Barley, and 156 bushels of Oates.

For seeing the excesses ascribed to 18. and 11. the prises of the Wheate and Barley, are both equall and one. Therefore one execution of the rule serueth thē both, as heere appeareth.

Rye.	whe. & bar.	Rye.	oates.
bu. 1.	1 bus.	bu. 1	13 b
12	12	12	156

Wherefore I conclude, that so much wheate, barley, and otes, must be mixed with the 12. bushels of Rye, to accomplish the intent of the myxer.

And heere I say not as before, that so much of euery graine may be taken, but absolutely that so much must bee taken, for when any sorte of the extreames is so singuler, then can there be no varietie of Alligation, and consequently no varyetie of answere.

Luma. That seemeth very reasonable, for seeing the greater extreames may not be linked but to a lesser, and that there is but one lesser, it must needs be that the same extreames cannot be otherwise linked nor alligated, but to that one lesser, and the like I consider

ster happeth, when both sorts of the excesses are singuler. But now will I be so bolde, as to make a triall of your worke, vsing your former order thus.

The price of { 12.bush Rye at 14 ô.the bushel, is 14.s. / 12.bush wheat, at 18.ô the bush. is 18.s. / 12.bush.barly at 11 ô.the bushel, is 11.s. / 156.bush.oats at 9 ô the bush is 5.l.17.s } Tot.8.l. The triall.

And the price of 192.bushels of the mixture, at 10.ô. the bushell, is also 8.l. which totals agree in so iust equalitie (as appeareth) that the worke cannot be amended.

Philo. Well said. Now for the demonstration of this rule, suppose that there are 2.kindes of wine, one of 14 ô. the Gallon, and the other of 9 ô. and let 12.ô. be a meane price betweene the same 14.ô. and 9 and let the odds or excesse of the same 14. and 9. from the meane, be 2. and 3. as heere you see in the fabrique annexed. So is the totall of both the same excesses 5. Demonstration.

12. ô. { 14) 3 / 9) 2 — 5

And now multiply 12. the meane by this totall 5. and you shall produce 60. which is as much to say, as that the price of 5. gallons of 12. ô. the gallon is equall to 60. ô. and by the 57. verse the totall of the products of each extreame by his fellowes excesse, that is, the product of 14. by 3. and of 9. by 2. is also 60. For heere 9. 12. and 14. are 3. vnequall numbers, naturally endued with the propriete there rehearsed. wherefore by the 55. verse, if 12. be multiplyed by 2 and by 3. the totall of both products, to wit, 24 + 36. shall be equall to the product of 12. by 5.

But it is affirmed, that 12. multiplyed by 5. produceth 60. wherefore the totall of the products of 12. by 2. and by 3. (the parts of 5.) shall also be 60.

But now these 3. numbers, 9. 12. and 14. being vnequall together. It hath beene already declared, that the odds or excesse of 9. and 14. the 2. extreames, from 12. the meane number or middle price, is 2 and 3. So that 9. the lesser extreame, is 12 —— 3. and 14. the greater extreame is 12 + 2. wherfore by the 57. verse. If 12 —— 3. be multiplyed by 2. and 12 + 2, be multiplyed by 3. that is if 9. be multiplyed by 2. and 14. by 3. the totall of both their products shall be equall to the totall of the products of 12. by 3. and 2. but the totall of the poducts of 12. by 3. and by 2. is 60. as appeareth before, Ergo, the totall of the products of 14. by 3. and of 9. by 2. shalbe 60. which was to be proued.

Wherefore this is the reason and cause of such alternation or chaunging of the excesses or differences, as is vsed in this rule of Alligation, whereby the totalls are brought to the equalytie desired.

Eu. O sir, now I begin to perceiue part of the vse of that mistical propertie of 3. vnequall numbers mentioned in the 57. verse, and to know the importance thereof, for I see playnely now that it is the chiefe platforme and foundation whereon this rule of alligation is builded.

Phi. I told you so then, but you could not then vnderstand it, and now touching the second kinde of this alternate alligation, the rules and precepts thereof are thus.

307. The last sort of alternate alligation,
By totall of the mixed measures knowne,
Seekes the simple measures iust in such fashion,
As in the first rule of partners is shownne,
For heere the totall of all the excesses,
Is first antecedent generally,
Next whome the totall mixture expresses,
The consequent due, wherefore by and by,
Each single excesse by reason must be,
last antecedent in the rule of three. 239

As the first sort of Alligation alternate, by the measures of some one of the simples giuen and expreſt, sought out the rest of the simples by the rule of 3. So here it is said that this latter sorte of Alternate alligation, by the totall measures of the mixture knowne or expreſt, seeketh out the measures of euery simple, by helpe of the same rule of 3. euen in such fashion as you learned in the first rule of partnership.

So that like as there the totall of all the antecedents, made the first and generall antecedent, and the totall gaines or losse made the generall consequent, and euery partners particuler stock made the last particuler antecedent, whereby euery mans portion of gaines or losse was found and discouered. So in this latter sort of Alligation, the totall of all the excesses is the first or generall antecedent, and the totall masse or mixture is the general consequent, and euery single excesse alone is particuler antecedent in the golden rule, whereby the measures of euery of the simples are found out and concluded by proportion compounded with addition.

111. A question of the mixture of Wine.

As for example. A certaine Vintner hauing 4. sorts of Wine,

to

to wit, some of 12. d. some of 10. d. some of 9. d. and some of 7. d. the Gallon, would mingle 300. gallons together, taking of euery sorte so much as he might affoord to sell the mixture for 11. d. the gallon. Now the question is, how much of euery sort of these wines he may take to serue his turne.

For answere whereto. First I write downe the meane price, which here is 11. d. with the extreame prices at the right hand thereof, as I shewed you before, and then I alligate or linke a greater with a lesser, in such order as the same extreames require, which by the 306. verse is thus.



For sithe the lesser extreames are plurall, and the greater is but one singly alone, therefore by the 306. verse, I alligate euery of the lesser extreames to that one greater, and their excesses or differences from the meane, I ascribe to the same greater, their sole yoake-fellow. And contrarilye I ascribe the excesses of that one greater, to euery one of the lesser, and so collecting the 3. uppermost excesses into one, and then adding all the excesses together, the fabrique standeth thus.

	11. d.			
12		1. 2. 4		7
10		1		1
9		1		1
7		1		1
			Totall	10

Now, sithe that by the contents of this present Maxim, the simple measures are to be sought in such fashion, as the latter consequents are sought in the first rule of partnership. Therefore (euen as I did in the same rule) so heere I take 7 + 1 + 1 + 1. (that is 10.) the totall of all the excesses, for first antecedent generally in the golden rule, and 300. gallons, which is the knowne totall of the mixture, for generall consequent thereto, and then euery seuerall excesse for latter antecedent, and so I search out the quantities of the simples by the same rule, saying thus.

If 10. gallons yeeld 300. what will 7. gallons yeeld? and what 1. 1. and 1. gallon? and the golden rule at 2. opperations, sheweth the quantities of all the 4. simples, for seeing that 3. of the differences or excesses are equall, it argueth that all those 3. quantities, to wit, of 10. d. 9. d. and 7. d. the gallon, shall be equall, wherefore I conclude that the Vintner must take 210. gallons of 12. d. and of euery other price 30. gallons, as by the fabriques above described appeareth.

Gallons.	gallons.	Gallons.	gallons.
10	300	10	300
7	210. of 12. d	1	30. of 10. 9. & 7. d.

And for confirmation hereof, I say that the price

The proofe.

Or { 210 gallons, at 12 d. the gallon, is 10. d. 10. s. / 30. gallons at 10. d. the gallon, is 1. l. 5. s. } Tot. 13. l. / 30. gallons at 9. d. the gallon, is 1. l. 2. s. 6 d. } 15. s. / 30. gallons at 7. d. the gallon, is 17. s. 6. d.

And 300. gallons of ỹ mixture at 11. d ỹ gallõ is also 13. l. 15. s.

Eu. Why I am fully perswaded that I can do this already, and therefore I pray you propound some other question, whereby you may trie my cunning therein.

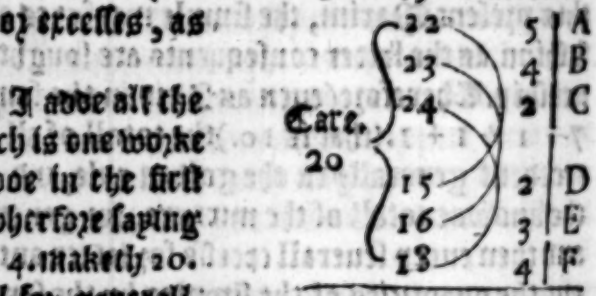
122. A question of the mixture of sundry sorts of golde.

Phi. That will I do, if it shalbe this, a mintmaister having sundry sorts of golde, viz. some of 22. carects fine, some of 23. some of 24. and againe, some other of 15. some of 16. and some of 18. carects, would melt of all these sorts so much together, as might make 100. ounces of 20. Carects fine. Now how much of euery sorte may he take?

Euma. I vnderstand not what this word Carect signifieth.

Philo. That is no matter, proceede you with the question according to the rule prescribed, which you say you can do already, and when you haue done, I will shew you what a Carect meaneth.

Euma. Then to begin, first I describe the meane value, with all the extreames giuen in the accustomed order, alligating or tying them togethers, as the rule ordinarilie requireth, and alternating their differences or excesses, as heere appeareth.



That being done, I adde all the excesses together, which is one worke more then I had to doe in the first kinde of Alligation, wherfore saying 5 + 4 + 2 + 2 + 3 + 4. maketh 20. I take the same totall for generall antecedent in the Golden rule, making 100. ounces the knowne totall of the mixture, generall consequent thereto, and euery seuerall excesse, last antecedent.

And so seeking out all the measures or quantities of the simples by that rule I say thus. If

ounces. A	ounces	ounce. B. F	ounce.
20	100	20	100
5	25	4	20
ounc. C. D.	ounce.	ounces. E.	ounces
20	100	20	100
2	10	3	15.

20. require 100. ounces, what will 5. require? and what 4? and what 2. and 2 and 3. and 4? and the rule at 4. seuerall operations,

answereth, that the Wintmaister may take 25. ounces of 22. Carects, 20. ounces of 23. carects, 20. ounces of 18. carects, 10. ounces of 24. carects and 10. ounces of 15. carects, and 15 ounces of 16. carects, as the fabriques before declare, where the second and third figures expresse the quantities of 4. sorts of the golde.

For of such simples as haue equall excesses ascribed, there shall be equall portions taken, by your former speech.

And here, as I learned of you, I say not, that he must take iust so much, but that he may take so much: for seeing that both sorts of the extreames are plurall. I perceiue by the 305. verse, that this question may haue great varitie of Alligation, and likewise sundry solutions.

Philo. You say very true, and it is well considred of you.

Euma. Then finally for proofe of my worke, I say that

25. ounces at 22 carects, commeth to 550. carects.
20. ounces at 23. carects, amounteth to 460. carects.
10. ounces at 24. carects riseth to 240. carects.
10. ounces at 15. carects, yeeldeth 150. carects.
15. ounces at 16. carects, mounteth to 240. carects.
20. ounces at 18. carects, commeth to 360. carects.

} total. 2000 carects, &

100. ounces of mixture at 20. carects, yeeldeth also 2000 carects.

The triall.

Phi. You haue done well, but you haue taken a great deale more paynes then you needed to haue done, in making this triall by the carects, for in this case it had bin sufficient to haue proued that all the latter perticuler consequents, are equall to the generall consequent, or that the totall ounces of the simples is equall to the totall ounces of the mixture, both which is one.

But I know you tooke this course, by reason you were ignorant what a carect is, which now I will tell you.

And it is thus, among Wintmaisters and goldsmiths, 1. ounce of the purest and finest gold without any mixture is imagined to be 24. carects fine, that is to say, such gold, as being tried in the fire will loose no part of his weight or quantitie, for if it loose $\frac{1}{24}$. thereof or thereabout, then it is sayd to be gold of 23. carects fine.

So likewise, such gold as by triall in the furnace loseth or wasteth $\frac{1}{12}$. of the former weight, is called gold of 18. carects, and such as wasteth $\frac{1}{6}$. is named gold of 20 carects fine, and so forth of others, wherefore a carect is $\frac{1}{24}$ of an ounce.

Now when an ounce of gold so put to the triall, loseth any such

part of his former weight, it is supposed that the substance, which made it weight before the triall, was eyther siluer, or copper, or both, or some other base metall, which superfluous substance or base metall is by Minters and goldsmithes generally called Alloye, euen as among marchants, when the weight Sugar, spices, Figs, Reasons, Pepper, Aloum, or any other marchandise, the Chests, Caskes, Frailes, Baggs and such like are commonly called Tare.

Moreouer fine and tried siluer, is not reckoned by the carect as gold is, but by the ounce, in so much that a pound weight of pure siluer without any Alloye, is said to be 12. ounces fine, that is to say altogether fine, for there is but 12. ounces in the pound, and therefore if 1. pound of siluer tryed in the fire waste $\frac{1}{12}$. thereof, then it is said to be siluer of 11. ounces fine. If it waste $\frac{2}{12}$. then of 10. ounces fine. If $\frac{1}{3}$ then but of 8. ounces fine, and so forth of others.

Eu. I thanke you much for this instruction, for I see it wilbe very necessary for diuers matters.

Phi. Now one question or 2. mo of this rule, and so will we passe ouer to alligation mediall, the first is thus.

223. A question of mixing Alloy with Golde,

A Mint maister hath gold of 20. carects, of 22. carects, and of 24. carects fine, which he would so temper with Alloy, that 100. ounces of the mixture, might beare 18. carects fine, the question is, what quantitie of Alloye he may take to performe his desire.

Eu. Why by your owne words in the glosse of the 235. maxim, this is impossible, for here 18. carects which representeth the mean, is no meane number between the extremes, but is lesser then any of them all, which is contrary to the 302. verse.

Phi. Stay your selfe, and be not so hastie, do you thinke that these 3. seuerall sorts of gold can be abased, from 20. 22. and 24. carects fine, to 18. carects without mixture of some other metall with it?

Eum. No I thinke not so, for the forme of the question argueth, that the mintmaster would put some Alloye to it.

Phi. And what is that Alloye?

Eu. You said euen now, that it is siluer or copper, or both, or els some other base metall.

Phi. Very good, and of how many carects fine is the gold that is in an ounce of such Alloye.

Euma.

Euma. Why, there is no gold at all in it of any fyneneſſe.

Philo. You ſay well, and yet the ſame Alloye is one of the ſimples that muſt enter into the melting pot, to helpe make the mixed maſſe required.

Euma. No doubt it is ſo,

Philo. Then touching our queſtion, I ſet downe 18. the meane, with his extremes, as here you ſee them, amongſt whom I haue placed a Cipher for the Alloy, becauſe it is 0. carects fine.



And now I truſt you ſee that here is one extreme leſſer then the meane, and 3. greater then it, which being alligated orderly, and the odds or exceſſe betweene them and the meane alternated and ſummed as you ſee, now I ſay by the golden rule, if 66. require 100. ounces, what will 18. 18. and 18. require, and what will 12. require, and the rule at 2. opperations aunſwereth that there muſt be taken 27 $\frac{3}{11}$ ounces of euery ſorte of golde.

ABC ounces		D ounces
66	100	66
18	27 $\frac{3}{11}$	12
		100
		18 $\frac{2}{11}$

And 18 $\frac{2}{11}$ ounces of the Alloy, as here appeareth by the letters A.B.C. and D. annexed to the alligation and fabrique.

Euma. Truly this queſtion would haue graueled me much, as perfect as I thought my ſelfe to be, but I truſt it ſhalbe a good paterne for me in the like hereafter.

Philo. Then anſwere me to this queſtion, the ſame mintmaſter hath certayne ſiluer Bullion of 9 $\frac{1}{2}$ ounces fyne, whereof he would make money that ſhould be but of 6. ounces fyne, wherefore it followeth, that he muſt put ſome copper, or other baſe mettall to it, to abaſe it, now I would know how much of his Bullion, and how much copper Alloy he muſt mixe together to make 100 l. weight of the fineneſſe required.

124. A queſtion touching the abaſing of ſiluer.

Euma. Here it is manifeſt that the meane value is the 6. ounces. and the greater extreme is the 9 $\frac{1}{2}$ ounces, wherefore the leſſer muſt needs be the Alloy, which ſith it hath no portion of fyne ſiluer in it, I perceiue muſt be repreſented by a cipher as you did before, wherfore I ſettle the meane & both the extremes in order linking them as behoueth. Then I alternate their exceſſes, which finally I adde together, and ſo their totall is 9 $\frac{1}{2}$ as here the fabrique ſheweth.

$$6 \begin{cases} 9\frac{1}{2} \\ 0 \end{cases} \Big) \begin{matrix} 6. \\ 3\frac{1}{2} \end{matrix}$$

$$9\frac{1}{2}$$

Wherefore

Wherfore now by the golden rule, as in the first rule of partnership I say if $9\frac{1}{2}$ require 100 l. weight, what doth 6. require, and what doth $3\frac{1}{2}$ require, and the operation thereof aunswereth, that he must take $63\frac{3}{19}$ pound of siluer and $36\frac{16}{19}$ pound of Copper, as here the fabriques declare.

siluer. $\frac{19}{2}$ 100 l. / 6 siluer. / $63\frac{3}{19}$

copper $\frac{19}{2}$ 100 l. / $\frac{7}{2}$ copper / $36\frac{16}{19}$ l.

Philo. It is euen so, and here if the totall of the mixture had not beene knowne or exprest, but the question had bene put so, that the mintmaster having siluer Bullion of $9\frac{1}{2}$ ounces fine, which he purposeth to abase to 6. ounces fyne, would therefore know how much siluer, and how much alloy, he should melt or mixe together to make the mixture of that fynesse. Then I say by the 303. verse that the very excesses themselues here, without any further worke, doth shewe the true portions or rather proportions of each mettall to be taken.

Euma. So then your meaning is, that for euery 6. pound of siluer which he taketh here, he must take $3\frac{1}{2}$ pounds of Copper to melt therewith.

Philo. Yea, and that for euery 6. ounces of siluer (if the question were of ounces) he must melt $3\frac{1}{2}$ ounces of Copper.

But generally the totall of the mixture is giuen and knowne, for what is he that determining to make a mixture or masse of any thing, doth not determine before hand also, the quantitie certayne which he purposeth to haue thereof.

Euma. Indeede it is reason he should do so, least when he knoweth the quantitie of his simples which he ought to vse, he hath them not in his possession to vse.

Philo. You haue sayd well, and yet I will propound one question more to you after that fashion, for the better prompting of your answere, if any such questions be demaunded of you.

125. A question of mixing diuers sorts of siluer bullions.

And it is thus, a mintmaster hath 4. sundry sortes of siluer Bullion, to wit, some of 3. ounces fine, some of 5. ounces, some of 8. ounces, and some of 10. ounces fyne, of which 4. sortes he would compound another sort that should be iust 6. ounces fine, now what quantitie of euery sort of the sayd Bullions is he to take.

For answere whereto, the meane value which is 6. ounces fyne, being set downe with the extremes in order alligated and their seuerall excesses from the meane alternated, as here you see.

3		4. A
5	6	2. B
8		1. C
10		3. D

I conclude by the 303. verse and glosse thereof, that for euery 4. pound of siluer of 3. ounces fine which he taketh, he is to take 2 r. of 5. ounces fyne, 1 r. of 8. ounces fyne, & 3 r. of 10. ounces fine.

But now if the totall masse or quantitie certayne that he would haue, be giuen and expresst, then am I to adde all the excesses, and so to worke by the golden rule after the order of the first rule of partners as the 264. verse teacheth, as for example.

But the case he would haue 500 r. weight in all, then say I by the rule of partnership, 4 + 2 + 1 + 3. make 10. wherefore if 10. require 500 r. what wil 4. require, and what 2. and what 1. and what 3. and the golden rule at 4. seuerall operations answereth as here you see.

A. weight.	B. weight.
10 Z 500 r.	10 Z 500 r.
4 Z 200	2 Z 100

Wherein sith the totall of all the last perticuler consequents is equall to the fyrst generall consequent, therefore I conclude the answere to be true.

C. weight	D weight.
10 Z 500 r.	10 Z 500 r.
1 Z 50	3 Z 150

126. A question touching the abasing of gold bullion.

Agayne a mintmaster hauing 1. pound of gold full 24. carects fyne, would abase the same to 22. carects fyne, wherefore it is demaunded, how much alloy he must mixe therewithall to make it of the baseneſſe required.

Now here 22. carects is the meane value betweene the gold of 24. carrects fyne, and the alloy of 0 carrects fyne, wherefore the numbers being placed, the simples alligated, and their excesses alternated, as here you see.

$$22 \begin{cases} 24 \\ 0 \end{cases} \begin{matrix} 22 \\ 2 \end{matrix}$$

The same excesses by the same 303 verse doth foorthwith plainly declare, that for euery 22 pound weight or 22. ounces weight of fyne gold, he must mixe 2. pounds or 2. ounces of alloy, whereupon hauing recourse to the golden rule I say thus, if 22 r. of fyne gold require 2 r. of alloy what doth 1 r. of fyne gold require, and it answereth $\frac{2}{22}$ r. or in lesser termes $\frac{1}{11}$ r as here the figure thereof sheweth.

22 Z 2 r.
1 Z $\frac{2}{22}$ r.

Wherfore I conclude that to 1 r of such fine gold, there must be taken $\frac{1}{11}$ r. of alloy, to make the mixture 22. carrects fyne.

The like is to be considered in all other questions of this rule, whether they extend to the abasing, or to the fining of the mixture, as for example.

127. A question of base siluer made e finer.

A Goldsmith hauing 1 r. weight of siluer bullion of 7. ounces fyne, would melt it with such quantitie of fyne siluer, as the same mixture might beare 10. ounces fyne, wherfore the question is, how much fyne siluer he must melt with it.

For

For answer whereof, seeing fine siluer, as I sayd before, is that which is 12. ounces fine, I describe the meane, with the extremes alligated, & their excesses alternated thus. Where the excesses forthwith notifie vnto me, that for euery 2 l. of 7. ounces fine, he must take 3. pounds of 12. ounces fine.

10 { 7 } 2
{ 12 } 3

Whereupon making accesse to the golden rule, I say thus. If 2 l. of 7. ounces fine require 3 l. of 12. ounces fine, what will 1 l. require, and it answereth $1\frac{1}{2}$ l. as the fabrique annexed sheweth. Wherefore I conclude that the goldsmith must take $1\frac{1}{2}$ l. of fine siluer, of 12. ounces fine, to melt with 1. pound of 7. ounces fyne, that so the masse growing therof, may be 10. ounces fyne, as was required.

2 l. — 3 l.
1 l. — $1\frac{1}{2}$

Euma. All this is no hard matter for him to do, that hath had so many examples as you haue shewed me, wherefore I pray you when you thinke good proceede to alligation mediall.

Philc. With a good will, and le the rule thereof is thus.

308. Aligation mediall, seeks out the meane,
By aid of proportion doubly compounded,
For sithe the measures of euery extreame,
With all their prises are playnely propounded,
And sith that some part of the mixture at least,
In this Aligation is also expresst,
309. Therefore the totall, simple measures lade,
The golden rule first, as first terme that rises,
Next whom the totall of the products made,
Betweene those mesures and their own prises,
Craues to be second and then at a woord,
The giuen mixture doth make vp the thurd.

240.

The first verse or foreparte of this maxim sheweth you, that Aligation mediall seeketh out the meane price (which as your selfe well noted, in our speach vpon the 235. maxim, alwayes signifieth the price of the mixture) by proportion doubly compounded, that is to say, both by addition and multiplication.

And in this Aligation, aswell the measures or quantities, as the prices of euery of the simples (or extreames) are not onely giuen & playnly expresst, but also there is alwayes some part (at the least) of the mixture giuen and expressly named.

Wherefore in this alligation, no alternation of excesses or differences is to be vsed, but by the second verse of this maxim, the totall rising of the simple measures serueth to saue the figure of

of the Golden rule first, as the first terme thereof, and next that the totall of all the products made betweene the same simple measures and their proper prices serueth for the second terme, and lastly the giuen mixture (how much or how little thereof soeuer it be) maketh the third.

123. A question of mistling or corne mixed together.

As for example, 10. bushels of Wheate of 16. pence the bushell, is to be mingled with 18. bushells of Rye of 10. pence the bushell, and I am demaunded what one bushell of this mystlyng shalbe worth.

Heere the question is of the mixture, which is the meane price (and not of the simples or extremes) for both the measures or quantytes of the simples, and also their seuerall prices, are playnely giuen and expresst already in the question, to wit 10. bushell of Wheate at 16. pence. and 18. bushells of Rye at 10. pence, and besides that, a parte of the mixture is also expresst viz. 1. bushell.

Wherefore now to finde the meane price, or valure of the same 1. bush. of the mixture, I must haue recourse to the golden rule, applying thereto the termes by this maxim appointed, which termes I dispose in this manner.

First I write downe the giuen quantities of the simples one by another with some conuenient distance betweene them & their own prices directly vnderneath them, and not far from thence at the right hand I set the figure of the Golden rule, readily prepared thus.

	bushels	10.	18.
	pence	16.	10.

Then making a + betweene the 10. bushels and the 18. bushels signifieng therby that they must be added together, I say 10 + 18 bushels make 28 bushels, which totall I foorthwith applie to the fyrst place of the sayd figure as the first terme proportionall thus.

	bushels	10. + 18,	28	bush.
	pence	16.	10.	

Then do I multiplie each of the vpper number as they stand by his owne inferiore, that is the 10. bushels by 16. pence their owne price, and the 18. bushels by 10. pence their owne price, the 2. products whereof is 160. and 180. as here appeareth.

	bushels	10. + 18		bush.
	pence	16.	10	28
		160	180	

Betweene whom foorthwith I likewise make a + signifying thereby their Addition also, which beeing performed, I finde their totall

totall to be 340. as here appeareth.

Which I imediately take and vse for second terme in the golden rule.

And sith the giuen part of the mixture is but 1. bushell I take that for the third terme as this present Waruin willeth me, and so working by the golden rule, the aunswering terme is 12. $\frac{1}{7}$ d. whereby I conclude that so much 1. bushell of the mixture is worth.

bushels	10 +	18	bush. pence
pence	16	10	28 — 340
	160 +	180	1 — 12 $\frac{1}{7}$
	180		
total	340		

Iuma. I vnderstand the order of the worke in this Alligation very well by this your so playne example, and doubt not but I can do the like, but yet I perceiue not the reason whereby it is so brought to passe.

Philo. That will I quickly impart vnto you by so playne a demonstration that you shall easily conceiue it, and it is thus.

Demonstration.

First when the 10. bush. are added to the 18. bush. they make 28. bushels, which you know is the whole quantitie of the mixture. Agayne, when 10. bush. is multiplied by 16 d. the product is the whole price or value of the same 10. bush. for the price of the same 10. bush. at 16 d. the bushell, is 10. times 16 d that is to say 3 s. 4 d. or 160 d. and likewise when 18. bushels is multiplied by 10 d. the product is the whole value or price of the same 18. bush. (for the price of 18 bushell at 10 d. a bushell is 18. times 10 d. that is 15 s. or 180 d. wherefore when both these products are added together, the totall being 340. pence, must needes be the totall price of the seuerall simples, but by the 3. vers the propertie of euery Alligation, is to equifie the totall price of all the simples to the totall price or value of the mixture, wherefore I conclude by the Golden rule, that if 1. bushell of the mixture be worth 12 $\frac{1}{7}$ pence, then 28. bushels being the whole summe of the mixture is worth 340 d. as here the fabrique annexed witnesseth.

bush.	pence
28	340
1	12 $\frac{1}{7}$

And so you see, that the totall value of the mixture, is equall to the totall value of the simples, wherefore the price of 1. bushell of this mixture shalbe 12 $\frac{1}{7}$ d. which is the thing that was to be proued.

Iuma. Truly this proofe is so sensible that nothing can be plainer, wherefore now I pray you propound some more questions, aunswerable by this kinde of alligation.

Philo.

Philo. With a good will, and let this be the first.

129. A question of a dredge poulder.

An Apothecarie making a certayne dredge poulder occupieth therto 6. pound of Sugercandie of 2 s the pound, 2 r. of Liquirice of 6 d the r. 3. pounes of Annis seedes of 12 d. the pound and 1. pound of fennell seede of 6 d. now the question is, what 3 r. of this mixture is woorth to be sold.

Suma. For aunswere hereunto, first I write downe all the seuerall simples with their seuerall prices directly vnder euery of them as you did, which being done, foorthwith I couple all the giuen measures or quantities together with crosses, to signifie that they must be added, and their totall taken for first tearme in the golden rule, which I haue set in areadinesse for that purpose thus.

pounds weight	6	+ 2	+ 3	+ 1
pence	36	6	10	6

12 Z

After this I multiplie euery of the vpper numbers euen as they stand by his inferior, that is, by his proper price, coupling all their products with crosses also for a memoriall that they are to be added, and their totall taken for the second tearme, which being done accordingly and setled in due place standeth thus.

pound weight	6	+ 2	+ 3	+ 1.
pence	36	6	10	6.
	216	+ 12	+ 30	+ 6

12

30	pound.	pence.
6	12 Z	264.

So that now taking 3 pound the giuen part of the mixture for the third terme, I conclude that if 12. pound of the mixture cost, or be woorth 264 pence, then 3. pound thereof shalbe woorth to sell 66 d. as here appeareth.

The totall. 264

pound.	pence.
12 Z	264
3 Z	66

Which is after 22 d. the pound, and for triall thereof, I vse the former accustomed fashion saying thus.

The total price of the simples as apeareth before is 264 d. or 22 s. and 12 r. weight of the mixture, at 22 s the r. is also 264 d or 22 s

proofe.

Philo. Well done, you haue wrought very orderly, now what say you to this, a merchant hath 27 r. of large cloues of 6 s the r. 15 r. of the middle sort, of 3 s the pound, and 10 r. of fast of 2 s 2 d. the pound, now when all the same is mixt together, I would know how much 1. pound of the mixture is woorth

130. A question of diuers sorts of cloues mingled together.

Suma. For aunswere, before I begin I reduce the prices of all the simples to like surname, and then I proceede as before, first

writing

writing downe euery of the same simples, with their seuerall prices directly vnder them, and then foorthwith coupling them with crosses I take their totall for fyrst tearme in the Golden rule, and so the woorke standeth thus.

pound weight	27 + 15 + 10	
pence.	72	36 26
		52 Z

This being past, I multiplie euery superior number by his inferior, that is euery simple by his propper price, and then adding all their products fyrst, with crosses onely for memorie, and afterwarde by the 23. Parim, Finally applie their totall for second terme in the Golden rule and so the fabrique appeareth in this forme.

pounds	27 +	15 +	10.
pence.	72	36	26.
	1944	+ 540	+ 260
	540		
	260	52 Z	2744
tot.	2744		

Wherefore now taking 1. pound the giuen part of the mixture, for the third terme proportionall, I conclude by the operation of the same rule that 1. pound of this mixture is woorth $52\frac{10}{13}$ pence as here the fabrique declareth.

pounds	pence
52 Z	2744
1 Z	$52\frac{10}{13}$

The proofe whereof is manifest. for the totall value of the simples is found already by the former fabrique to be 2744. d. and so much also, is the totall value of the mixture.

For 52 l. weight of the mixture at $52\frac{10}{13}$ d. the pound amounteth iust to 2744. pence.

131. A question of Holland cloth.

Philo. Item a merchant hauing sundry sortes of fine Holland, to wit, 30. elles of 5. shillings the ell, 35. elles of 7. shillings 6 pence the ell, 40. elles of 9. shillings the ell, and 60. elles of 12. s. 6 pence the ell, is bid set the price of an ell certayne, how he will sell all, one with another, I demaund therefore what price he ought to set, to be neyther looser nor gayner aboue the former prices.

Eumathes. To answere you, fyrst I reduce the prices to like denominations or coynes, whereof because the least heere named is 6. pence, which in knowne coyne is a Testern, I see that I needs not bring them all to pence, but into Testerns onely.

So for 5 shillings, I put 10 Testerns, for 7 shillings 6. pence I vse 15. Testerns, for 9. shillings I take 18. Testerns, and in steade of 12 shillings 6. pence. I take 25. Testerns, and so placing euery of the simple measures or quantities giuen, with their prices directly vnder them. And vsing all the circumstances and operations

rations before taught, according to the rule of this alligation, as heere appeareth by the Fabrique.

Elles.	30	+ 35	+ 40	+ 60
Testerns.	10	15	18	25
	300	+ 525	+ 720	+ 1500

525
720
1500
Tot. 3045

Elles. testerns.
165 Z 3045.
1 Z 18 5/11.

Finally I finde by the Golden rule, that the price of the elle of these Hollands one with another, ought to be 18 5/11 testerns, that is 9.s.2.d. 5/11 d. whose proofe is apparant, either by the fabrique after the order last vsed, or else by repeating the prices of the simples thus.

The price of { 30. elles at 5.s. the ell, is 7.l. 10.s. / 35. ells at 7.s. 6.d. is 13.l. 2.s. 6.d. / 40. ells at 9.s. the ell, is 18 l. / 60. ells at 12.s 6.d is 37.l. 10 s. } total. 76.l. 2.s. 6.d.

And the price of { 165. elles, at 9 s. 2. d 5/11 d. the ell one with another, is all so in } Totall. 76.l. 2.s. 6.d.

Philo. You haue done very well. Now how answer you this question. A Goldsmith hauing 8 l. weight of siluer bullyon of 7. ounces fine, 15 l. of 8 1/2 ounces fine, and 13 l. weight of 10. ounces fine, would melt altogether, and would know of what fineneſſe a pound weight of the masse shalbe.

132. A question o the fineues of a mixed masse.

Euma. To answere this, I describe the given quantities, with their seuerall prices or valures of their fineneſſe, right vnder them thus.

Pounds.	8	+ 15	+ 13
Ounces fine.	7	8 1/2	10
	56	+ 127 1/2	+ 130

127 1/2
130
tot. 313 1/2.

pound. ounces
1/2 Z 8 5/23

And so adding and multiplying, and then adding againe, as the prescript of this Alligation requireth. Finally by application of the due termes to the Golden rule, and by the operation thereof, I conclude, that 1. pound of this mixed masse is 8 5/23. Ounces fine, as heere the Fabrique aboue expresseth. Whose triall I make thus.

The valure of { 8.l. of bullion of 7. ounces fine, is 56. ounces. / 15.l. at 8 1/2 ounces fine is 127 1/2. ounces. / 13 l. at 10. ounces fine, is 130. ounces. } tot 313 1/2 ounces.

And the valure of 36. pound, of 8 5/23. ounces fine, is also 313. 1/2. ounces.

Phi. All this is well done, now a goldsmith hath melted 12 l. 5. ounces weight of gold bullio of 18 carects fine with 4 l. 4 ounces $\frac{1}{2}$ of 21 carects fine, wherefore I desire to know of how many carects fine 1 pound of the masse is?

Eu. Here I see presently that the given measures or quantities are not all of like name (as alligation naturally requireth) wherefore they must be all reduced to like denominatiō before I can proceede, for performance whereof, I first reduce the 12 l. 5. ounces into the least sirname expresst in the question, viz. into $\frac{1}{12}$, so are they $\frac{245}{12}$ l. and next I reduce the 4 l. 4 ounces, and $\frac{1}{2}$, vnto the like sirname also, and so are they $\frac{105}{12}$. l.

This being done, forasmuch as their basses now are all like, I reiect them, vsing onely their numerators in the steede of the whole fractions, which I write downe with their values subscribed directly, and adding them together, I assigne their totall for first terme in the Golden rule. Then I multiply each of the vpper numbers by his inferior, to wit, each given quantitie by his proper price or value of fineneſſe, taking the totall of their products for the second terme. Finally taking 1 l. being the given part of the masse or mixture for the third. The golden rule executed, declareth that 1. pound of this masse is $18\frac{115}{409}$. Carects fine, as the Fabrique aboue sheweth.

Pound weight.	245	+ 105	
Carects fine.	18	21	
	5364	+ 2205	
	2205		
tot.	7569	403	7569
		1	$18\frac{115}{409}$

Phi. It is euen so, for triall whereof (to be the lesse troubled with fractions) you may (if you will) suppose euery carect to be worthe some price certaine, proceeding so to examine the agreement of the totall values together after that price, as for example.

I will value euery carect here at 12. d. so euery ounce of 18. carects fine, is esteemed worth 18. s. and euery ounce of 21. carects fine, is accompted worth 21. s. Wherefore the 12. pound 5. ounces, reduced to ounces, make 149. ounces, and the 4 l. 4 $\frac{1}{2}$. ounces, reduced to ounces, make 52 $\frac{1}{2}$ ounces. Now

The price of { 149 ounces at 18 s. the ounce, is 134 l. 2 s. } tot. 189 l.
{ 52 $\frac{1}{2}$ ounces at 21. s. the ounce, is 55 l. 2. s. 6. d } 4. s. 6. d.
And the price of { 201 $\frac{1}{2}$ ounces being the totall of the mixed masse, at $18\frac{115}{409}$ s. the ounce, is also } 189 l. 4 s. 6. d.

Euna.

Foma. I like this triall very well, for it is as certaine, as that which is made by multiplication of the Carects, and yet is nothing so tedious.

Philo. Now will I put two or three questions more, which I will answere my selfe, to the end you may the better direct your worke by my examples when you shal happen on the like hereafter, and so we will make an end of Alligation. The first is thus.

A Goldsmith hath three sundry sorts of siluer bullion, that is to say, 5. l. 7 $\frac{1}{2}$. ounces of 7 $\frac{1}{2}$ ounces fine, 12. l. 3. ounces of 6. ounces $\frac{1}{2}$. fine, and 4. l. of 9. ounces fyne, all which he purposeth to melt together. Wherefore it is required to know, of what fynenesse a pound of the masse shalbe? 133. A question of the fineness of siluer.

Here (as you well noted before) the giuen quantities are of diuers names, for some are pounds, some ounces, some $\frac{1}{2}$ ounces, which diuersitie Alligation cannot abide, wherfore they must fyrst be all reduced to like denomination, so instead of 5. l. 7 $\frac{1}{2}$ ounces, I must take $\frac{135}{24}$ l. and for 12 l. 3 ounces, I must take $\frac{294}{24}$ l. and for 4. l. I take $\frac{96}{24}$ l. So now their basses being all like, I reiect them, vsing onely their numerators 135. l. 294 l. and 96. l. as meere Integers.

This done, I reduce the mixt numbers, to wit, 7 $\frac{1}{2}$ ounces fyne, and the 6 $\frac{1}{2}$. ounces fyne into simples, and so instead of the one, I haue $\frac{15}{2}$ ounces, and for the other, I haue $\frac{13}{2}$ ounces, and as for 9. the third needeth no reduction.

After this, I adde the giuen quantities together, and their totall is 525. which I make fyrst terme in the Golden rule, then 135. multiplyed by $\frac{15}{2}$ maketh $\frac{2025}{2}$, and 294. multiplyed by $\frac{13}{2}$ produceth $\frac{3822}{2}$. And lastlye, 96. multiplyed by 9. produceth $\frac{864}{1}$, all which products added together, make $\frac{6711}{2}$, which (as the rule requireth) I set for the second terme, taking 1. l. the giuen quantitie of the mixture for the third, and so vsing the rule, the answering terme is 7. ounces $\frac{251}{525}$.

Which fraction reduced to peny waights, maketh 2 pence, and $\frac{101}{175}$ of a peny weight, and the same $\frac{202}{175}$ d. eftsoones conuerted into graines, yeelderh 10 $\frac{2}{35}$ graines.

So that euery pound weight of this masse, containeth 7. ounces, 2. peny weight, and 10 $\frac{2}{35}$ graines fyne, and that you may the better conceiue the order of the worke herein, beholde I haue annexed the Fabrique of all the chiefe operations vsed in the question.

Pounds weight.	135 + 294 + 96.	The addition of the numeratoꝛs. 6075 11172 5184
Ounces fine.	$\frac{15}{2}$ $\frac{19}{3}$ $\frac{8}{1}$	
The Pꝛoducts.	$\frac{2025}{2} + \frac{5586}{3} + \frac{864}{1}$	22431
Conꝛoꝛming reduction.	6075 + 11172 + 5184 / 6	The totall of the numeratoꝛs, which with the denominatoꝛ 6. is applied to the Golden rule.

Euma. This is a very busie question, by reason of the fractions, which by the 155. verse must needs be reduced by conꝛoꝛming reduction, beꝛoꝛe the totall of their pꝛoducts can be known, but this description of the operations helpeth memoꝛy well, and maketh the matter plaine inough.

525 ⟌ 22431/6
1 ⟌ 7 311/315

134. A question whether any Alloye be needfull in the mixture or not.

Phi. Another question is this, a mint maister hath 100. ℔. weight of golde of 22. carects fine, and 20. ℔. weight of 19. carects fine, which he would so mixe together, that euery pound weight of the whole masse might be 20. carects fine, the question is, whether he need to mix any alloy with it foꝛ that purpoꝛse oꝛ not, & how much?

Foꝛ answer herein, first it is to be considered and learned of what fynenes 1. pound of the masse wilbe, when both soꝛts of golde are molten together, wheꝛefoꝛe, woꝛking by this pꝛesent rule of alligation after my foꝛmer oꝛder, I describe the given quantities with their values, as here you see. And so using the accustomed operations, I finde that a pound weight of the masse will be of 21 1/5 Carects fine, as here appeareth.

pounds weight.	100 + 20	
Carects fine.	22 19	
	2200 + 380	
	380	
Totall.	2580	120 ⟌ 2580 / 1 ⟌ 21 1/5.

Wheꝛby it is euident that there must some Alloye be put to it, to abasse it from 21 1/5. carects to 20. but how much that Alloye must be, is to be found and learned by the fyꝛst alligation alternate, wheꝛefoꝛe now by the 236. maxim, I place the same 20. carects as the meane valure between this golde of 21. 1/5. carects fine, & the Alloye of 0. carects fyne, and so alligating the extremes and alternating their excesses, as that alligation requireth, and as here appeareth.

20 { 21 1/5 ; 0 } 20 ; 1 1/5

The same excesses pꝛesently declare, that foꝛ euery 20. pound weight of golde, he must take 1 1/5. ℔. of Alloye, wheꝛefoꝛe accoꝛding to the oꝛder of the fiꝛst alligation alternate, I conclude by the gol-

den

ven rule, that if 20. r. weight of golde bullion, require $\frac{1}{2}$ r. of alloy, Then both this Mintmaisters sortes of golde, conteyning in the whole 120. pound weight, will require 9. r. of Alloye, as the fabrique heere sheweth.

20 r. — $\frac{1}{2}$ r.
120 — 9

And therfore, so much Alloy must be melt with the 120 r weight of golde bullion, to make the masse beare iust 20. carects fine.

135. Another question whether any Alloye be needfull or not to the mixture.

My third question is thus. A Goldsmith hauing diuers sortes of siluer bullyon, to wit, 24. r. weight of 7. ounces fine, 30. pound weight of 9. ounces fine, and 16. r. weight of 12. ounces fine, would melt all three sortes together so, as a pound weight of the masse might beare 10. ounces fine, wherefore it is demaunded whether he ought to melt any Alloye withall, or not, and how much?

Here first the valure of the meane, that is to say, of 1. pound of the mixture made of all these three sortes of siluer is to be sought and found by this Alligation Mediall, by whose operation, and by the golden rule, 1. pound of the masse is found to be 9. ounces fine, as here the fabrique expresseth.

But by the tenor of the question, the Goldsmith would haue it 10. ounces fine: wherefore sithe this masse or mixture is not yet so fyne (by 1. vunce in euery pound weight) as he would haue it. It followeth necessarily by common reason, that there is no Alloye to be incorporated with it, but rather contrarily, that the Goldsmith must yet put too more fine siluer.

Pounds weight.	24	+ 30 +	16
Ounces fyne.	7	9	12
	168	+ 270 +	192
	270		
	192	70 —	630
Sum totalis.	630.	1 —	9

Wherefore to finde out how much fine siluer he must melt withall to bring it to 10. ounces fine, is the office of the first Alligation alternate, which I execute thus. First I take 10. ounces the meane valure, which I write downe, with 9. & 12. the 2. extremes, at the right side thereof, and so alligating them together, and alternating their excesses, according to ouer, the fabrique standeth thus.

10 { 9) 2
{ 12) 1

Whereby is signified that for euery 2. pound weight of these 3. mixed bullions, there must be taken 1. pound of fine siluer.

Wherefore I conclude proportionally by the golden rule, that if 2. r. thereof require 1. r. of fyne siluer, then 70. r. the totall masse will require 35. r. of fine siluer, as the figure of the rule here declareth.

2. r. — 1. r.
70 — 35

And therefore I conclude that so much fine siluer must be incorporated with the 3. first giuen quantities, to make the 1. weight therof 10. ounces fine, whose proofe may be made thus.

First 70 l. weight of 9 ounces fine contains 630. oun. } tot. 1050
And 35. l. weight of 12. ounc. fine, conteins 420. oun. } ounces.

And so 105. l. of the mixture at 10. ounces fine, containes in total. 1050. ounces.

Or els you may proue it thus.

24 l. weight of 7. ounces fine, conteineth 168 ounces.
30. l. weight of 9. ounces fine, conteineth 270. ounces. } tot. 1050
16. l. weight of 12. ounces fine, conteineth 192. oūces. } ounces.
35. l. more of 12. ounces fine added, conteins 420. oūce.
105. l. weight of the mixture of 10. ounces fine, containes in total 1050. ounces.

How say you, do you vnderstand these examples?

Euma. Yea, I trust so.

136. A third question whether Alloye be needful or not.

Phi. Then take you this one question more, and so an end. The same Goldsmith hauing 36. l. of siluer bullion of 9. ounces fine the pound, and 24. l. of other siluer bullyon of 6. ounces fine, would melt them both together, so that 1. pound of the mixture might containe 8. ounces fine. Wherfore the question is, whether he must melt any Alloy with these bullyons, or els must adde any fine siluer with them, to accomplish his pretended purpose.

Euma. This question will I assume to answer with your fauor for performance whereof, first because it is vncertaine of what fineness 1. l. weight of the mixture will be, when both these simples are mixed in one masse, therefore I perceiue the same price (being the price or valure of the meane) must be sought out by this Alligation Mediall, by whose maxim I worke, as heere you see, til I haue found the terms proper to the goldē rule, which being knowne, I apply to the same rule, and vsing it, do conclude thereby, that 1. pound of the mixture shalbe 7 ½. ounces fine, as the figure thereof expresseth.

Pound weight.	36	+ 24
Ounces fyne.	9	6
	324	+ 144
	144	
		60 / 468
Total.	468	1 / 7½.

Which is not so fyne yet as is required by ½ ounce in the pound weight, for (you say) he would haue it 8. ounces fyne.

Wherefore it is manifest, that the putting of Alloye to this mixture, will make it baser, and therefore in reason there must no

Alloye

Alloy be molten with it, but there muſt be ſome fyne ſiluer added to it, to make it holde the fyneſſe required. The quantitie wherof I ſeeke by the fyrſt Alligation alternate. Wherein I ſet 8. ounces the meane value deſired, with $7\frac{1}{2}$. and 12. ounces the values of the 2. extremes, thus as here you ſee. $8\left\{\begin{matrix}7\frac{1}{2}\\12\end{matrix}\right.\Big)\begin{matrix}4\\\frac{1}{2}\end{matrix}$

And ſo alligating the extreames, and alternating their differences, the ſame differences declare that euery 4. l. of this mixed maſſe requireth $\frac{1}{2}$ l. of fyne ſiluer to be mixed therewith, to the end y euery pound of the ſame maſſe may iuſtly conteine 8. ounces fine.

Whereupon I conclude by the golden rule, that if 4. l. weight of the fyrſt mixture, require $\frac{1}{2}$ l. of fyne ſiluer to be molten withall. then 60. l. the whole mixture, will require 3. l. of fyne ſiluer, as heere the fygure importeth. 4. l. $\frown$ $\frac{1}{2}$ l. / 60 $\frown$ 3

Whoſe triall I make thus. The triall.

36. l. of 9. ounces fyne, containes 324. ounces.
24 l. of 6. ounces fyne, containes 144. ounces. } tot. 504. ounc.
3. l. of 12. ounces fyne, contaynes 36. ounces.

And 63 l. of the mixture of 8. oun. fyne, conteines in tot. 504. oun.

Phi. You haue wrought the queſtion well, and confyrmed your worke by a perfect triall, wherefore here we will end Alligation.

Of the rule of falſe poſitions, otherwiſe named the rule of Helcatayme, *and firſt as it is ſingle by one poſition onely.* Chapter. 20.

310. There are 2. kindes of rules of falſe poſition,
The firſt is ſingle, the ſecond is double,
Both which, diſſolue doubts with ſuch expedition,
That to the worker, it ſeemeth no trouble. 241
311. The ſingle rule by one ſole poſition,
And by the aide of golden rule withall,
Serues beſt for queſtions wherein ſome pertition,
Is made by numbers in parts proportionall.

Theſe verſes are ſo plaine, that they neede no interpretation, but for an example take this.

Three perſons A. B. and C. conſent together to buye a certaine Manor, which will coſt them 2700. l. for payment whereof they agree that B. ſhall pay double to that which A. payeth, and C. ſhall pay triple to that which B. payeth. Wherefore the queſtion is, how much euery of them ought to paye? 137. A queſtion touching the buying of a Manor.

Now here appeareth a partition to be made in numbers by parts

proportionall, to wit, in proportion Dupla & Tripla, for this question requireth nothing els in effect, but to diuide 2700. into 3. such parts, that the second may be double to the first, and the third triple to the second.

Which that I may do, I put some number at all aduentures for the portion of A which I apply by doubling and tripling, according to the forme of the question, vntill I conclude eyther truly or falsely, if the conclusion fall out truly, and agreeable to the proport of the question, then haue I the number sought for without further labour.

But if it misse, eyther by excesse or by want, yet by the ayde of the golden rule, such false conclusion shalbe a meanes to finde the very truth and number desired, for looke what proportion such false conclusion beareth to the false position whereof it came, such proportion doth the giuen nũber beare to the true position vnknowne, or number sought for.

As thus, at aduentures I put 6 l. for the portion which A. ought to pay, now this number so put instead of the true number vnknowne, is that which is called the position, wherefore to applie it according to the forme and order of the question, I say thus to my selfe.

If A. pay 6. pound, then B. must pay 12. pound, (for 12. is double to 6.) and then C. must paye 36. pound (for 36. is triple to 12.) but now 6. pound + 12. pound + 36. pound, make but 54. pound, which by the intent of this question ought to be 2700 l. wherefore it is apparant, that I haue missed the truth very much in my position, for this conclusion yeelding but 54 l. wanteth a great deale of 2700. pound.

Neuertheless this false conclusion by the helpe of the golden rule, will deliuer me the truth, whereupon I repayre thereunto, saying thus, if 54. pound come of my position 6. of what position commmeth 2700. pound, and the operation ther- pound pound pound
of answereth of 300 l. as here appeareth by the 54 Z 6
fabrique. 2700 300

Wherefore so much must A. pay, whereby it followeth that B. must pay 600 l. and C. 1800 l. which numbers are in the proportion required, and should haue beene found by such application and operation as I haue vsed, although I had put any other number for my position.

Eu. This rule is so easie, that me thinkes I can do it presently.

Philo.

Philo. Then answere me to this, a certayne man hauing a bag of money in his hand, was demanded how much money he had in his bagge, whereunto he answered that he knew not readily, but this quoth he I know, and am sure of, that $\frac{1}{3}$ and $\frac{1}{4}$ and $\frac{1}{5}$ thereof maketh iust 94 r. wherefore tell me now how much money had he in his bagge.

138. A question of money knitvp in a bagge.

Euma. I put at aduentures 20 r. for the money that he had.

Phi. Although you may attayne the truth as certaynely by this your position as by any other number, yet for the auoyding of fractions, and to proceede with the more facilitie, it is alwayes best to chuse such a number for your position, as hath the partes expressed in the question, which here is $\frac{1}{3}$ $\frac{1}{4}$ and $\frac{1}{5}$. and such choyse is easily made by the 150. and 151. verses, but 20. hath not those partes, for although $\frac{1}{4}$ thereof be 5. and $\frac{1}{5}$ 4. yet hath it not $\frac{1}{3}$ in Integers, but falleth into a mixt number, to wit, $6\frac{2}{3}$.

Euma. Very good, I perceiue that to be very reasonable, wherfore now I put 60 r. for my position (which number hath all the parts expressed in the question) whereof $\frac{1}{3}$ is 20 r. and $\frac{1}{4}$ is 15 r. and $\frac{1}{5}$ is 12 r. all which added together make but 47 r. but they should make 94 r. wherefore making recourse to the golden rule, I say thus, if 47. come of my position 60. of whom commeth 94 r. & the rule answereth of 120 r. as here appeareth by the figure.

pound.	pound.
47	60
94	120

Whercupon I conclude that he had 120 r. in his bagge, whereof $\frac{1}{3}$ is 40 r. $\frac{1}{4}$ is 30 r. and $\frac{1}{5}$ is 24 r. all which added together make iust 94 r. according to the purport of your question.

Philo. You haue done well, & here you may see that the effect of this question is no more but to finde a number whose $\frac{1}{3} + \frac{1}{4} + \frac{1}{5}$ added together make iust 94.

Agayne a certayne sheepheard being demanded how many sheep he had in his flocke, answered that if he had as many mo as he hath, and $\frac{1}{2}$ and $\frac{1}{3}$ and $\frac{1}{4}$ of those that he hath with 1. ouerplus he should haue iust 630. sheepe, now how many sheepe had he.

139. A question of sheepe.

But here I must tell you that this question cannot be answered by this rule except the 1. odde sheepe or vnit ouerplus be first subtracted from the giuen number, because the quantitie of the same 1. sheep, is permanent, and doth not rise and fall proportionally as do the fractions, and the proportion Dupla named in the question, which thing the last verse of this rule requireth.

The reason why this question cannot be answered but by omission & restauration of the one sheepe.

But the same 1. being subtracted from 630. the number propo-

sed

ser, then may it be easily aunswered by this rule, for then is there nothing els sought here in effect, but to finde such a number as being taken twice together with $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ thereof, shall make 629. whereunto if the vnit first subtracted be at the length restored, the totall shalbe 630.

Wherefore to proceede in this question, I put 12. for the number of his sheepe (because it contayneth such partes as the fractions expresse) whereto if (as the question importeth) I put as many mo, then is it 24. agayne if I put thereto $\frac{1}{2}$ $\frac{1}{3}$ and $\frac{1}{4}$ of 12. that is to say 6.4. and 3. then will the totall be 37. but it ought to be 629. wherefore comming to the Golden rule, I say, if 37. come of my position 12. of whom commeth 629. And following the rule it aunsewereth me of 204. as the fabrique sheweth, wherefore I conclude that so many sheepe were in the flock, wherto if I adde as many more, & $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ of as many, that is to say, 204 + 102 + 68 + 51. the totall will be 629. as here the Addition it selfe annexed declareth.

37 \ 12
629 \ 204

204
204
102
68
51
629

Unto the which now at the last restoring the vnit first subtracted, there amounteth iust 630. agreable with the sheepheardes aunswere, and the tenor of the question.

The like is to be obserued in all other like questions wherein any one number constant or permanent is giuen, as for example.

140. A question touching the crownes in a mans purse.

One being asked how many crownes he had in his purse, aunswered, that if he had $\frac{1}{2}$ as many as he hath, and $\frac{1}{3} + \frac{1}{4}$ of so many, together with 100. crownes more, he should haue iust 300. crownes, whereupon it is demaunded how many crownes he had in his purse.

Here I say the 100. crownes are first to be subtracted from the giuen sum of 300. crownes (for a while) and lastly to be restored agayne, because the same number is permanent, and doth not rise and fal proportionably as the fractions do, so that the effect of this question is but to finde such a number, as with $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ thereof first added together may make 200. and then afterwardes hauing 100. lastly added thereto, may amount to 300. according to the contentes of this question.

Wherefore seeing 12. contayneth all the partes of the giuen fractions, I put the same here also for my position, as the number of this mans crownes, vnto the which I adde $\frac{1}{2} + \frac{1}{3} + \frac{1}{4}$ of 12. that is 6.4. and 3. so is the totall 25. but by the order of the question it ought

ought to be 200. wherefore I consult with the golden rule saying, if 25. come of 12. of whom commeth 200. and it aunswereth me of 96. as here appeareth by the fabrique, wherefore I conclude that he had so many crownes in his purse, whereof $\frac{1}{3}$ is 48. $\frac{1}{5}$ is 32. and $\frac{1}{2}$ is 24. all which partes added together make 104. now finally adding the 100. crownes first subtracted to 96 + 104. the totall is 300. as the question requireth.

25 Z 12
200 Z 96

Euma. All this, hetherto I understand very well, and perceiue the reason thereof withall.

Philo. Then I trust you can aunswere this question, a certayne young gentleman which had traueiled out of England to Venice and so to Rome, at his returne finding that he had but 36. crownes left in his purse, perceiued that he had consumed and spent in his voyage $\frac{2}{3}$ + $\frac{1}{5}$ of all his stocke, wherefore I demaund what his whole stocke was. 141. A question of a trauilers stocke

Euma. I perceiue that this question doth but seeke such a number as hauing $\frac{2}{3}$ + $\frac{1}{5}$ thereof abated from it, leaueth 36. remayning.

Philo. You haue sayd very truly, wherefore go forwards.

Euma. Then to proceede I put 15. crowns for his entire stock, whereof $\frac{2}{3}$ is 10. and $\frac{1}{5}$ is 3. now 15 —— 10 —— 3 is but 2. which by the intent of the question ought to be 36. wherefore vsing the golden rule I say thus, if 2. come of my position 15. of whom commeth 36. and it aunswereth of 270. as here appeareth by the figure.

2 Z 15
36 Z 270

And therefore I affirme that his stocke was so many crownes whereof $\frac{2}{3}$ is 180. crownes, and $\frac{1}{5}$ is 54 crownes, now finally 270. crowns —— 180 —— 54. leaueth 36. crownes, euen as the question willeth.

Philo. Well sayd, and here you shall note, that when the fractions or parts expresst in any like question to be subtracted, exceede an vnit or one integer, so that they cannot by reason be subtracted from the position or number put, then is that question vtterly absurd and impossible.

As if I should bid you giue me a number whose $\frac{2}{3}$ + $\frac{4}{5}$ thereof being abated shall leaue 36. remayning, you might aunswere me that my request is meerely impossible, for $\frac{2}{3}$ + $\frac{4}{5}$ of any thing, maketh $\frac{22}{15}$ which is more then the entire thing whatsoeuer it be, and therefore if this question had required $\frac{2}{3}$ + $\frac{4}{5}$ to be abated, as it did $\frac{2}{3}$ + $\frac{1}{5}$, then had it bene vtterly impossible, for $\frac{2}{3}$ of 15 my position is 10.

and

and ⅕. thereof is 9. both which coniojned make 19. now how is it possible to subtract 19. from 15.

Euma. All this I conceyue very well by the 26. verse and glose thereof, and also by the glose of the 217 maxim.

142. A question of a mans age.

Philo. Then answer me to this, a certaine olde man being asked how many yeeres olde he was, said that if to the yeares of his age were added ⅓. so many mo, and from that totall ¼. abated, he should be 99. yeeres olde, now I demaund how olde he was.

Eu. I put 40. yeeres for his age, whereto adding 20. (which is ½. thereof) the sum is 60. from which totall I subtract 15. (which is ¼. thereof) as the question requireth, and so remayneth 45. but this remayner by the fourme of the question should be 99. wherefore making accesse to the golden rule I say, if 45. come of my position 40. of whome commeth 99. and it answereth me of 88. as here appeareth.

45 — 40
99 — 88

Whereby I conclude, that he was so many yeres olde, for if 44. which is ⅑ thereof be added to 88. the totall will be 132, and then 132 — 33. leaueth the remainer 99. so satisfying the question.

143. A question of the price of Scarlet & other cloth.

Philo. Againe, a gentleman bought 30. yardes of Violet in graine, and 40. yardes of Scarlet, which cost him 330. ▽. But euery yard of Scarlet, cost double so much as a yard of the Violet. Now the question is, what 1. yarde of the Scarlet coste?

Eu. I put 4 crownes for the price of 1. yarde of Violet in graine, So shall the price of 1. yarde of Scarlet be 8. ▽. (for that is double to 4) whereby it followeth, that the price of the 30. yardes of Violet, groweth to 120. ▽. and the price of the 40. yardes of Scarlet, amounteth to 320. ▽. Now 120 ▽. + 320 ▽. make 440 ▽. which should be but 330. Wherefore comming to the golden rule, I say. If 440 ▽. come of my position 4. of whom commeth 330? and the rule answereth of 3. as the fabrique here expresseth.

440 — 4
330 — 3

Wherefore I conclude, that euery yarde of the Violet did coste 3 ▽. and euery yarde of the Scarlet 6. crownes, for so the 30. yardes of Violet rise to 90. crownes, and the 40. yardes of Scarlet to 240. crownes, whose totall is 330. crownes, agreeable with the proposit of the question.

144. A questiō of fractions touching parts of a number.

Philo. You haue done well: wherefore tell me now what two numbers are they, whose ⅔ of the first is ¾ of the second?

Euma. I pray you answere this question your selfe, for I see not how to do it by this rule.

Philo.

Phile. Then will I teach you, wherfore for the first I take some number hauing parts like the fyrst fraction, as namely 12. whose $\frac{2}{3}$ is 8. now to know of what number 8. is to $\frac{4}{5}$, I put some such number for my position, as hath $\frac{4}{5}$ in it, namely 20. now $\frac{4}{5}$ of 20. is 15. but the number which I seeke is 8.

Wherefore comming to the golden rule I say; if 15. come of my position 20. of whom cometh 8. and it answereth of $10\frac{2}{3}$ as the figure here declareth.

15 Z 20
8 Z $10\frac{2}{3}$

Wherefore I conclude, that 12. my fyrst number taken and $10\frac{2}{3}$ now last found, are the 2. numbers required, for as 8. is $\frac{2}{3}$ of 12. so is 8. likewise $\frac{4}{5}$ of $10\frac{2}{3}$.

145. Another q-estiō touching the parts of a number in fractions.

And so likewise if I were demaunded what 2. numbers they are whereof $\frac{1}{2} + \frac{1}{3}$ of the one is equal to $\frac{1}{4} + \frac{1}{5}$ of the other, foorthwith I would take some such number for the fyrst, as hath parts like the fractions of ỹ first Integer. As namly here I take 24. whose $\frac{1}{2} + \frac{1}{3}$ is 20. then I put 40. for the second, whose $\frac{1}{4} + \frac{1}{5}$ is 18. but the number that I seeke is 20. wherefore resorting to the golden rule as to a common refuge I say, if 18. come of my position. 40. of whom commeth 20. and it answereth of $44\frac{4}{9}$ as here the fabrique expresseth.

18 Z 40
20 Z $44\frac{4}{9}$

Wherefore I conclude that 24. my fyrst number taken, and $44\frac{4}{9}$ this last number found by the rule, are the 2. numbers required. For $\frac{1}{2} + \frac{1}{3}$ of 24. is 20. and so much is $\frac{1}{4} + \frac{1}{5}$ of $44\frac{4}{9}$.

Euma. All this I vnderstand well now, and can do the worke aswell, but me thinks you make 2. positions in each of these 2. last questions, for in the one, first you tooke 12. and secondly you tooke 20. for your position, and in the other, fyrst you tooke 24. and then secondly 40. so that I thinke these questions are wrought by the rule of double positions mentioned in this marum.

Phi. You are much deceiued, for both 12. and 24. being the numbers which I take first in each question, continue & remaine absolutely one of the true numbers in deede that serueth to answer the questions, without any false position or alteration, but neyther is 20. nor 40. which I take for my positions, any of the true numbers required, but serue onely to induce vs to the other of the true numbers vnknown.

Wherefore assure your self, here is but one position vsed in each of these questions, as your selfe wil better perceiue anon, when we come to the double rule in deede.

And

And thus although many mo questions answerable by this rule, might here be propounded, yet not doubting but the examples giuen are sufficient for any meane capacitie, to extend the vse of them further, I will here make an end, and will now handle the rule of double positions, for there is no question aunswerable by this rule, but may be aunswered by that, but not contrarily.

Of the second part of the rule of faulse positions, otherwise named the double rule of Helcataym *or the rule of double positions, the*

21. Chapter.

As the rule of double faulse positions is generally the last of all ordinarie rules deriued from Geometricall proportion discontinuall (as appeareth by the table) so is it commonly the last rule that is taught in vulgar Arithmeticke.

Not that it is inferior to any, but rather superior to all, and a rule of so high nature, that it approcheth very nere to Algebra it selfe, wherefore because it is not onely of such excellencie and dignitie, but is also the very conclusion, and as it were the vttermost bounds of all the rules of proportion, I will inforce my selfe a little the more to vtter both it and the questions therein propounded, somewhat according to the worthynesse thereof, and fyrst, the rule is comprehended in these foure verses following.

312. As fortune faules so make some supposition,
which at left head of byas crosse expresse:
Then prooving how you hit or misse the cushion,
You shall finde fault by want or by excesse,
Which fault is called first error I confesse.
That error at the crosse left foote entitle.
VVith his due signe of too much, or too little,

313. Secondly gesse some other number newly,
which to the right head of your crosse assigne,
Then once agayne try if it happen truly.
If falsly still to error it incline.
That error at the crosse right foote combine.
VVith the due signe as late you did before.
For lesse vse a line, and a crosse for more.

314. Thirdly infold, in crosse or bias wise,
Each position by contrarie error.
VVhose products noted, if signes alike rise,
Bate the lesse product out of the greater,
The rest doth yeelde the diuidend the better,
So the lesse error from greater abated,
Leaues a diuisor of purpose created.
315. And lastly if signes contrarily come,
Those ofcums added for diuidend take,
Also both errors vnit in one summe,
Taking their totall diuisor to make,
VVhich vsed as it ought, the question doth quake,
And lets go the truth, in spite of his hart,
VVhich quotient doth catch, and to thee impart.

242

Now the best and briefest way to expound the sence of these verses (in my minde) is by a present example, for which cause I put this question following.

141. A question touching the partition of gaynes.

There are 3. merchants, to wit, A. B. C.
VVho parting an 100. crownes do agree,
That B. shall haue mo crownes then A. by three,
And that C. shall haue 4. crownes mo then B.
Now how many crownes each Merchant must haue,
Is all the question and thing that I craue.

For answere whereof (as the first verse of this rule biddeth me) I suppose euen as it falleth in my minde by fortune, or at all aduentures, that A. ought to haue 33. crownes for his share, which number being called the first position, I forthwith expresse at the left head of such a bias crosse thus.

33

Then proouing how I haue hit or mist the cushion (that is the true number whereat I aymed) the sequell sheweth, that I haue vtterly missed it, and gone quite beside the cushion, for if A. haue 33. cro. for his share, then B. must haue 36. and C. must haue 40.

For the question sayth, that B. shall haue 3. crownes mo then A. and that C. shal haue 4. mo then B. but the totall of 33 + 36 + 40 is 109. which should be but 100. and so it is 9. more then it ought to be, wherefore as the first verse here sayth, that I shall finde fault (that is to say a variance or error from the truth) eyther by want or by excesse, so heere it is manifest that the faule found consisteth not in want, but in an excesse or ouerplus of 9. too much

which

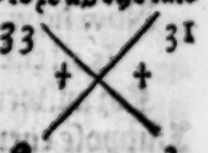
which fault is called the fyrſt erroꝛ, & I am bidden by the verſe to note oꝛ intitle it, at the left foote of the croſſe with the due ſigne of + foꝛ too much, oꝛ of — foꝛ too little, as the caſe requireth, ſo that becauſe this erroꝛ is too much, I note it at the left foote of the croſſe with the ſigne + thus.



And now comming to the ſecond verſe of this rule, the ſame biddeth me to geſſe, ſome other number newly foꝛ my 2. poſition, and to aſſigne it to the right head of the croſſe, wherefoꝛe to doe accoꝛdingly, euen at aduentures agayne as beforꝛe, I ſuppoſe now, that A. ought to haue, but 31. crownes foꝛ his ſhare, which poſition I preſently aſſigne to the right head of the croſſe thus.



And then appoynting the ſhares of the other 2 accoꝛding to the propoꝛt of the queſtion, it followeth thereby that B. muſt haue 34. crownes, and C. 38. whereupon comming agayne to triall how truly theſe numbers agree with the queſtion. I finde the concluſion ſtill falſe and enclyned to erroꝛ, foꝛ that the totall of 31 + 34 + 38. is 103. which ſhould be but 100. and ſo the fault of this poſition conſiſteth in an exceſſe alſo of 3. too much, which fault oꝛ exceſſe is called the ſecond erroꝛ, wherfoꝛe as the ſaid ſecond verſe willeth me, I combine oꝛ couple this ſecond erroꝛ to the right foote of the croſſe, with like ſigne thus.



Euma. So then I ſee well that as each of the poſitions is taken in ſteade of the very true number ſought foꝛ, ſo each of them is to be vſed, applyed, proued, and examined in all things, accoꝛding to the propoꝛte and conditions of the queſtion, as if they were the very true numbers indeede, in which application and examination, if the ſequell and concluſion of any poſition fall out agreeable to the queſtion, then is that poſition the true number ſought foꝛ, without further labꝛ, but if the concluſion fall out contrarily, then is the fault thereof eyther in that the poſition taken, was too little oꝛ too great, whereby inſueth an erroꝛ accoꝛdingly, eyther of want oꝛ of exceſſe.

Philo. You vnderſtand the matter very directly, and I aſſure you that to vſe, conferre, and apply the poſitions rightly, accoꝛding as the propoꝛt and conditions of the queſtions require, is harder to doe, then to execute all the reſt of the rule beſides, and yet is that no matter of great difficultie to do, as you haue ſeene by this example.

Wherefoꝛe

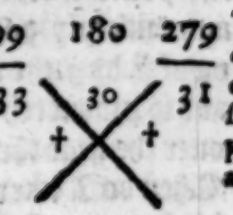
Wherefore now to come to the third verse, and to proceede with my worke as the same biddeth me, I infolde in crosse or bias wise, each position by contrary error, that is to say, 33 the first position by 3. the second error, and 31. the second position by 9. the first error. So are the severall products 99. and 279. which I note downe severally, each ouer the head of his owne position, as aptest place for that purpose. And because the signes of the figure are like. to wit + and +, therefore I abate 99. the lesser product, from 279. the greater, so shall the rest, (viz. 180.) serue for my dividend, which I place in the midst betweene the products, at the head of the bias crosse as heere you see.



And so likewise, abating 3. the lesser error, from 9. the greater, the remainder is 6. the diuisor purposely created, to dissolue the question, and deliuer the true number sought for, which diuisor is vsually noted amidst the 2. errors at the foot of the crosse, where you may also see that I haue placed it accordingly.

Euma. Then your meaning is, that 180. being deuided by 6. quoteth the answering number, or true number of crownes due to A. for his share, which is the number presently sought for.

Philo. My meaning is euen so: wherefore diuiding 180. by 6. there issueth 30. in the quotient, the true number of crownes which A. ought to haue, which aunswering number is to be placed in the vpper space of the crosse, neere to the center thus.



The middest of the crosse assigned to place the answer.

Whereupon it followeth by the tenor of the question, that B. hauing 3. crownes moe, shall haue 33. and that C. hauing 4. crownes more then B. shall haue 37. and so finally, 30 + 33 + 37. make iust 100. crownes, as the question requireth.

And if in this case, the positions, (which the consequence of the positions assigned to A.) allotted to B. or C. had beene also multiplyed byasly by the contrarye errors, then should the operation of the rule, haue brought foorth their shares in the quotient at severall times.

As for example.

I will set the first positions of them all three, at the left head of the crosse: and their second positions at the other head, with their

former errors (which are common to them all) in their due places thus.



For you know that when I put 33. crownes for the first position of A. Then (not I, but) the forme of the question it selfe allotteth 36. for the first position of B. and 40 for the first position of C. and likewise when I put 31 for the second position of A. then not I, but the order of the question it selfe appointeth 34. crownes for the second position of B. and 28. for C.

Eu. Yea I know and perceiue that well inough.

Phi. So then although there seeme to be 6. positions in the question, yet in truth there are but 2. onely put, or chosen by me, nor but one figure occupied about the whole opperation and dissolution of the question, and here I call the great bias crosse the figure properly partayning to this rule, euen as I call this figure the figure of the golden rule.

Figure pertayning to this rule.



Eu. All this I vnderstand well.

Phi. Very good, then to confirme my former saying, I now omit the position of A. heere, sith his share is found and knowne already, and multiply 36. the first position of B. by 3. the contrary error, and it produceth 108. as you see ouer the head thereof. Then do I likewise infolde 34. his second position by 9 the contrarye error, and it produceth 306. as also appeareth ouer the head thereof. And because the signes are like, I subtract 108. the lesser product from 306. the greater, so resteth 198. my dividend, which diuided by 6. the diuisor common to all, quoteth 33 the nomber of crownes due to B.



Likewise for C. I infolde biasly 40. his first position in 3. and 38. his second position in 9 the contrary errors, and their products are 120 and 342. whereof (because the signes are like) I abate the lesser from the greater, and so remayneth 222 for dividend, which being diuided by 6. the former common diuisor quoteth 37. the true share due to C. which shares I haue placed at the right hand

of

of the letters, because they could not conueniently stand all at once in the center.

But such perticuler operations, for euery seuerall number, (for the most part) needeth not, because that the first number being once found and knowne, all the rest are most commonly found by the forme, purpose, and consequence of the question.

For if the share of A. be once knowne to be 30. crownes, then sith B. by order of the question must haue 3. mo it followeth of necessitie that his share shalbe 33. crownes, and agayne sith C. shall haue 4. mo then B. it foloweth necessarily that his share be 37. crownes as I said before, but yet some times a question may be so intricate, that the seuerall numbers required, shalbe more aptly found by such perticuler operations then by consequence of the question propounded.

Eu. Truly as I must needes confesse that this rule is of an excellent inuention, so now I see and acknowledge that the order and processe in the working thereof, is farre vnlike to that of the former rule, I meane of single position.

But although it seemeth you haue fully answered this question yet me thinks you haue not fully expounded all the rule, for touching the last verse thereof you haue said nothing.

Philo. As concerning that verse you shall vnderstand, that if by my choise of positions both the errors insuing thereuppon had happned to haue offended or missed the truth by want (as you see here they did by excesse) yet had the operation of the rule proceeded in such manner as I did now worke it, because the third verse heere saith, If signes alike rise, bate the lesser product out of the greater, &c.

But if by choise of my position it had happned, that the one had offended or erred from the truth by excesse, and the other by want, then must the operation of the rule proceede, according to that fourth or last verse.

For then (by that verse) the signes comming contrarilie, that is one + and the other —. requireth the diuidend to be made, by addition of both the byas products, and the diuisor by addition of both the errors, which thing forasmuch as it happened not in my former forme of worke, therfore I had no occasion as yet to speake of that verse.

Eumathes. Surely my minde giueth mee, that I vnderstand both the forme of woorke which you haue alreadye wrought,

and also the meaning of the last verse, and for triall thereof I pray you giue me leaue to proue how I can answer the selfe same question by the latter verse.

Phi. With a good will.

Euma. Then at all aduentures I put 34. crownes for the share of A. which position I immediately ascribe to the left heade of the figure thus.

34

Then accompting what B. and C. ought to haue after that rate, by the purporte of the question, it insueth that if A. haue 34. crownes, then B. must haue 34 + 3. that is 37. and C. must haue 37 + 4. that is 41. but the totall of 34 + 37 + 41. is 112. which should be but 100. wherefore it is manyfest, that this my first position hath missed the truth, exceeding the same by an error of 12. too much, wherefore I annere the same error 12. to the left foote of the crosse, with this signe + thus.

34 + 12

Now for my second position, because I wil be sure not to misse againe by excesse, I put but 25. crownes for A. supposing that to be his share, which being forthwith ascribed to the right head of the figure thus.

34 25 + 12

I begin to apply it, prouing how it concorde with the tenor of the question, saying. If A. haue 25. crownes, then B. must haue 25 + 3. that is 28. and C. must haue 28 + 4. that is 32. But now the totall of 25 + 28 + 32. is but 85. crownes, which ought to be 100. so that this my second position hath erred by want or lacke of 15. to little, which second error therefore (as the saide verse biddeth me,) I couple to the right foote of the crosse, with a lyne, for the signe of want thus.

510 300

34 25 + − 12 15

Thirdlye, I infolde biasly each position by his contrarye error, that is 34. by 15. and 25. by 12. and the products are 510. and 300. which I place ouer the heads of the positions as you did.

And now (if I vnderstand the last verse rightlye) because the signes heere come contrarilie or be dislike, that is to saye, because the one is + and the other ——, therefore nowe I must not abate, but must adde both those products or offcums together, to wit,

wit 510. and 300. placing their totall which is 810. in the middest betweene them both for my deuisend.

And then must I likewise adde together 12. and 15. the 2. errors, placing their totall, viz. 27. amidst the errors for my diuisor, which vsed as it ought, that is (as I vnderstand it) being applyed to their due vse of diuision, as the same last verse intendeth, quoteth 30. for the share of A. euen as before, as the figure and fabrique of the worke here declareth.

510 810 300
34 30 25
\+ −
12 27 15

Whereby it followeth of necessitie that B. shall haue 33. crownes and C. 37. all which 3. percells make iust 100. crownes, as the question requireth.

And thus it seemeth, that whatsoeuer numbers, be taken or put for the positions, whether they be both to great or both to little, or one to great and the other to little, yet by this rule the very truth) which lieth as it were lurking among the errors, not willing to be knowne) is at the last forced to issue and to shew it selfe in the quotient.

Phi. You haue said very truly, and haue conceiued the meaning of the last verse as well, which variety of workes if you do aswell remember, you can neuer misse, for in all other things, the processe and operations, as you see are vniforme.

The reason and ground of the rule of double positions demonstrated.

And now to make you conceiue the reason of this rule, you shall vnderstand that the demonstration thereof dependeth partly vppon the difference, aswell betweene the positions, as betweene the errors, and partlye vppon the first and second propositions of the second booke of Euclid. For example whereof, I will renew the first fabrike & opperation of this question, which with the positions, errors, and signes, was such as heere appeareth.

99 279
33 2 31
\+ +
9 6 3

Where the first position being 33. produced the greater error 9. and the second position being 31. produced the lesser error 3. Now the difference betweene 33. and 31. (which are the 2. positions) is 2, and the difference betweene 9. and 3. (which are the 2. errors) is 6. Which 6. is ingendred of the same 2. the difference betweene the positions: for euen as the first position, breedeth the first error, and the second position the second error, euen so the

difference of the positions must necessaryly breed and cause a difference betweene the errors, which difference in all the operations of this rule is generally the diuisor, whether the signes be like or vnlike, as I will shew you anone, and in the meane season I subscribe both the difference of the positions, and also of the errors, one directly vnder the other, in the midst of the vacant space of the figure, as appeareth aboue.

Now as it is euident by the signes, that both positions haue erred from the truth by excesse or ouerplus, so it is apparant that the first position, erred more from the truth then the second, because the first offended by 9. too much, and the second offended but by 3. too much, so that if the second position had yet beene lesser, then had the error thereby ingendred been the lesser also, and so consequently had approached neerer to the truth.

Neuertheless it is euident, that the difference of the positions is such a meane betweene them both, that if it be added to the lesser position, it maketh it equall to the greater, or if it be abated from the greater, it leaueth the same equall to the lesser.

Euma. That I perceiue very well, for 31 + 2. is equall to 33. and 33 — 2. is equall to 31.

Philo. You say well. Againe the difference betweene the errors is such a meane, and so indifferent to them both, that if the same be added to the lesser error, it maketh it equall to the greater, or if it be abated from the greater, it leaueth the rest equall to the lesser.

Euma. That is apparantly true also, for 3 + 6. is 9. and 9 — 6. is 3.

Philo. Very good, then marke what insueth thereby, which is this: First as 31 + 2. (that is 31. the second position, and 2. the difference of the positions) are parcels of 33. the first position, so 3 + 6 (that is 3. the second error, and 6. the difference of the errors) are parcels of 9. the first error.

Euma. That appeareth plaine inough.

Philo. Then secondly it followeth further thereby, that heere are 3. proportionals giuen or knowne, whereby the 4. may easilye be found, of which proportionals the difference of the errors is alwayes the first, the difference betweene the positions the second, and each of the errors seuerally the third.

For as 6. the difference of the errors, is to 2, his owne progenitor the difference of the positions (whereof it was bred,) so is 9.

the

the greater error, to the vnknowne excesse (or ouerplus aboue the truth) contayned in his owne position whereof it was ingendered, and so is 3. the lesser error to the vnknowne excesse (or ouerplus aboue the truth) contayned in his owne position from whom it descended.

And alternately as the same 6. the difference of the errors is to 9. the greater error, so is 2. the difference of the positions, to the vnknowne ouerplus of the truth contayned in 33. the position which bred that error.

And againe, as the same 6. is to 3. the lesser error, so is the same 2 the difference of the positions, to the vnknowne ouerplus of the truth contained in 31. the position ingendring that error, which proportions is the very reason, ground, and cause whereby this rule is inabled to dissolue such hard and intricate questions as otherwise, without such proportionalitie could neuer be answered thereby.

Now for almuch as by the tenor of this rule the difference of the errors, is alwayes the onely diuisor, (euen as the first terme proportionall is in the golden rule direct (by whose operation the true answering number, is quoted and deliuered.

And sith also by the 94. verse, the true and lawfull diuidend is the product of the same difference by the true answering number: It must necessarily follow, by the same verse, that if the same difference multiply the true answering number, then the product therof shalbe the true and lawfull diuidend.

But when 31. the second position (being it which here approcheth nerest to the truth) is multiplied by (9) the contrary error, then is not 30. the true number onely multiplied by 6. (the difference of the errors and very true diuisor) but 30. + 1. is multiplied by 6 + 3. whereby ariseth a product of 99. too many, for the product of 30. by 6. is but 180. but the product of 31. by 9. is 279.

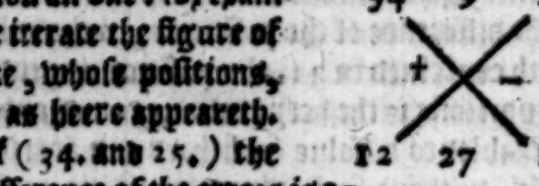
Wherefore when (33) the first position, beeing multiplied by (3) the contrarye error, produceth 99. also, and when the same 99. beeing the lesser product, is (by order of this rule) abated from 279. the greater, then I say by such abatement, the superfluous ouerplus of 99. which are there too many, being iustly corrected and removed, the remaynder must needes be the true diuidend, viz. 180. equall to the product of (30) the true number, by (6.) the difference of the errors. Wherefore the same remaynder

being diuided by the same 6. quoteth 30. the true answering number, which was to be demonstrated.

The selfe same reason serueth aswell when both positions, are defective, or offend by want, sauing that then the numbers here referred to the vnknowne ouerplus, aboue the truth contayned in the positions, are there to be referred to the vnknowne want, lesse then the truth lacking in the same positions.

Eum. That is easie inough to be coniectured by reason, but what if one of the positions be excessiue and the other defective?

Philo. Yet is the reason all one: for example whereof, I will heere iterate the figure of your owne former worke, whose positions, errors and signes, were as heere appeareth. Where the difference of (34. and 25.) the positions is 9. and the difference of the errors is 27.



Eu. Why sir you mistake much, for the difference of 12. and 15. is but 3. but 27. is the totall of both the errors and so the rule it selfe nameth it.

Phi. And yet I mistake not a whit, for that which this rule in that place calleth the totall of the errors, and which seemeth to you so to be, is nothing els indeede but their very difference, for let A.B.C. be 3. numbers, whereof let B. the middlemost represent 30. the true numbers, and let A. conteine 30 + 12. and C. 30 —— 15. So is A. 42. in all, and C. is but 15. in all. Now see you not plainely, that the difference betweene 42. and 15. is iust 27?

Euma. Yes truly, I see it very well, but I see no reason why it should be so.

Philo. The reason is onely the diuersitie and vnlikenesse of the signes + and ——. For if the signes were eyther both + or both ——, then should their difference be but 3. as you saide before, for the difference betweene 30 + 12, and 30 + 15. is but 3. and so is the difference likewise betweene 30 —— 12. and 30 —— 15.

Euma. So now I vnderstand the matter, and see withall, that you haue verified your former speeche, to wit, that in all operations of this rule generally, whether the signes be like or vnlike, yet alwayes the difference of the errors is the onely Diuisor.

Philo. Well then, as before, so now heere againe it followeth, that

that as this 27. (whether you call it the difference or totall, it skileth not so you vnderstand it) is to 9. the difference of the positions which begot it, so is 12. the first error to the vnknowne ouerplus of the truth contayned in 34. his owne excelliue position, and so is 15. the second error, to the vnknowne want of the truth lacking in 25. his owne defectiue position.

And alternately as the same 27. is to (12) the first error so is the same 9. to the vnknowne ouerplus of the truth contayned in (34) the fyrst position, and agayne as the same 27. is to (15.) the second error, so is (9) the difference of the positions to the vnknowne want of the truth lacking in (25.) his owne defectiue position.

Wherfore for as much as by the last verse of this rule, the totall of both the errors (which as I sayd before, is but their difference) is the ordinarie diuisor, who by diuiding the lawfull diuidend, quoteth the true number required, it followeth by the 94. verse, that when the same totall multiplieth the true number required, then the procuct shalbe the true diuidend.

But when (34) the first position, being multiplied by (15) the contrarie error produceth 510. and (25.) the second position being multiplied by (12) the contrarie error produceth 300. as here you see.



And both these products added together make 810. then haue we found the selfe same number that should be produced, if 30. the true number were multiplied by 27. the totall of the errors, wherfore when the same number, to wit, 810. is diuided by 27. the quotient by the forsayd 94. verse must needes be 30. the true answering number, which was to be prooued.

And thus hauing shewed you the reason of this rule so farre as you can vnderstand it without knowledge of Algebra from whence in truth it is deriued and cannot be perfectly demonstrated, but Algebraically I will now proceede to propound me questions for your exercise, whereof this is one.

147. A question touching a prisoner taken in warres.

A. and B. taking a prisoner in warres,
had for his ransome an 100. crownes iust,
In parting whereof, they fell at such iarres,
that each refused the other to trust.
Snatching and catching each one what he might,
till after awhile (their heates being laide,)

A. said

A. ſaid vnto B. giue me here in fight,
$\frac{1}{5}$. of the crownes which thou haſt conueyed.
And I will giue thee $\frac{1}{3}$ of mine,
Contented quoth B. which done as they ſayd,
Each of them had iuſt 56. crownes in fine.
Now would I fayne here him tell it that durſt,
How many crownes each of them
Snatched vp at firſt.

Euma. That with your fauour dare I to doe, or at the leaſt wiſe to endeauor it, wherefore I begin to ſuppoſe that A. ſnatched vp 40. crownes of the 100.

Philo. I told you in the former rule of ſingle poſition, that although touching the veritie and certaintie of the rule, it ſkilled not what numbers you put for your poſitions, yet for eaſe in worke, and auoyding of fractions, it ſhould be alwayes moſt commodious to chuſe ſuch numbers as are moſt like for the purpoſe, and readieſt to be diuided into ſuch partes as are expreſſed in the queſtion, which admonition you may thinke reaſonable to be obſerued here alſo, and yet now here, where A. is ſayd to yeelde his $\frac{1}{3}$ to B. you put ſuch a poſition for A. as hath no $\frac{1}{3}$ in whole number.

Euma. Indeede I was too raſh, but I will correct my fault out of hand, ſuppoſing that A. caught vp 30. crownes onely which beeing placed in the figure, I begin to examine the ſequell therof, according to the purport of the queſtion, ſaying thus.

If A. caught vp 30. crownes, then B. caught all the reſt, that is 70. wherof when he giueth to A. $\frac{1}{3}$ that is 14. then hath A. 30 + 14. that is 44. and ſo hath B. but 56. left, vnto the which when A. hath reſtored $\frac{1}{3}$ of his 30. that is to ſay 10. then hath B. 56 + 10. that is 66. ſo that now A. hath but 34. left, who ought to haue 50.

Whereby it appeareth that this poſition hath miſſed of the truth, by an errror of 16. too little, wherefore I annexe the ſame error to the left foote of the figure, with the ſigne of want thus.

30
16

And ſo taking a new poſition I ſuppoſe now ſecondly that A. caught 45. crownes, which I addreſſe forthwith to the right hand of the figure, then it followeth that B. caught all the reſt, that is to ſay 55. crownes, whereof when he giueth 11. (that is his $\frac{1}{5}$) to A. then hath A. 45 + 11. that is 56. and ſo B. hath but 44 left, but

when A. giueth to him backe agayne 15. which is $\frac{1}{3}$ of his 45. then hath B. 44 + 15. that is 59. and so hath A. but 41. left, who ought to haue iust 50.

Wherefore this position concludeth falsely also by an erronious want of 9. too little, which erro^r I annexe to his propper place at the right foote of the crosse with the due signe thereto pertayning.



This done, I enfold biasly each position by contrarie erro^r, so are the products 270. and 720. whereof as the third verse of this rule biddeth me, (because the signes are like) I abate the lesser from the greater, and so remayneth 450 for my diuidend, then do I also abate 9. the lesser erro^r from 16. the greater, whereby remayneth 7. for my diuisor.

Finally 450. being diuided by 7 quoteth $64\frac{2}{7}$ the true number of crownes which A. had snatcht vp, as appeareth here in the center of the fabrique.



Whereby it followeth that B. hauing the rest, had $35\frac{5}{7}$, whereof when he gaue $\frac{1}{3}$ to A. that is $7\frac{1}{7}$ crowns then had A $71\frac{1}{7}$. And so B. had left himselfe but $28\frac{4}{7}$. But when A returned back to B. $21\frac{3}{7}$, (which is $\frac{1}{3}$ of the $64\frac{2}{7}$ that he had caught vp,) then did he leaue himselfe but iust 50. crowns, Ergo, B. had the other 50. which appeareth manifestly, for by adding the $21\frac{3}{7}$ which he receiued of A. to $28\frac{4}{7}$ his former remaynder, the totall is 50. crownes, and so their shares were equall, and the conditions of the question satisfied.

Philo. Your willing endeauor hath had good successe, for you haue both made good and apt choyse of positions, and also answered the question very well and orderly, wherefore for your well doing you shall haue a merry question, which is this.

148. A question of Apples stolne.

A likerish theefe which loued apples well,
Brake into an orchard of purpose to steale.
Where when he had fild his panch without measure,
He pocketed vpp a number to serue him at pleasure,
Where with as he was fast packing away,
He met a stranger which made him to stay,
Quoth he halfe these apples I giue to thee here.
So thou without stay wilt let me go cleere.

Content

Content quoth the ſtranger and tooke it in briefe,
Returning a doſen agayne to the theefe,
VVhereof he was glad, and paſt foorth apace,
But loe another ſtayd him in like caſe.
To whom ſtraight he offered one halfe of his fruite,
To let him eſcape and make no purſuite,
who taking his offer and ſeeing his greefe,
Gaue backe agayne 7. foorthwith to the theefe,
whereof he reioycing marcht forward amayne,
Vntill a third man ſtraight ſtayd him agayne,
To whom he did offer as erſt he did eſt,
One halfe of the apples which then he had left,
which he foorthwith tooke as they did before,
And of the halfe taken, gaue backe agayne foure,
which done, yet this theefe perceiued he had plentie,
For when he had told them he found them iuſt twentie.
Now of this queſtion the hardeſt and woorſt,
Is how many Apples he pocked vp at firſt.

Euma. Indeede this is ſuch a prettie queſtion as allureth me to attempt the anſwere, and for that purpoſe I put 40. apples at the left head of the figure for my firſt poſition, ſuppoſing that he had pocketed vp ſo many, whereof when he gaue the firſt man halfe, he left him ſelfe but 20. but then that man giuing him 12. backe agayne, made his number 32. of which the ſecond man hauing $\frac{1}{2}$ alſo, left the theefe but 16. which with the 7. that the ſame man gaue to him agayne, made him to haue 23. whereof when the third man had taken $\frac{1}{2}$ there remayned no more to the Theefe but 11 $\frac{1}{2}$, ſauing that when he receyued backe 4. Apples of the third man, it made his number 15 $\frac{1}{2}$, but by the order of the queſtion he ſhould haue iuſt twentie.

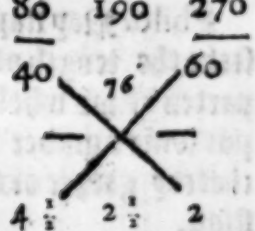
40
$4\frac{1}{2}$

Wherfore this firſt poſition miſſeth the cuſhion, concluding vntruly by an error of 4 $\frac{1}{2}$ too little, which I annexe to the left foote of the figure with the ſigne of want thus.

Secondly I put 60. at the right head of the figure for my ſecond poſition, ſuppoſing now that he had ſtolen ſo many Apples, whereof when he gaue the firſt man $\frac{1}{2}$ then had he him ſelfe 30. remayning, which with the 12. that the ſame man gaue him backe againe, make 42. of which when hee gaue to the ſecond man $\frac{1}{2}$

then

then had he 21. left for himselfe, which with 7. that the same man restored to him agayne make 28. whereof when he eftsoones gaue to the third man ½, then had he 14. left, which with the 4. Apples redeliuered by the same man, made his whole number 18. apples, but by the prescript of this question it should be 20. so that this position hath also missed the marke, erring likwise from the truth by want of 2. too little, which error I place at the right foote of the figure, with the proper signe appertayning.



This done I multiplie each position byasly by his fellowes error, & their products are 80. and 270. as appeareth ouer their heads, whereof (because the signes are both like) I subtract the lesser from the greater, and so remayneth 190. for diuidend.

Then do I likewise foorthwith abate 2. the lesse error, from 4 ½ the greater, and so resteth 2 ½ for diuisor, both which appeare also in their proper places of the figure.

Finally diuiding 190. by 2 ½ the true answering number 76. issueth in the quotient, and so many apples had he stolne to carrie away, which number I finde by examination satisfieth the conditions of the question precisely, & therefore I haue set it in his proper place, to wit, in the center of the figure as you see.

Philo. You haue done well, and because you are so readie and quicke in vnderstanding, answere me to this question.

149. A quest of a fishe.

By South sea cost, neere where I dwell,
A shallow creeke doth cut the shore,
VVherein the sea by rage doth swell,
And so brought in, of fish good store.
VVhereof one monstrous fish was found,
VVhich at retiring of the floud,
The people spied vpon the ground,
Halfe waterlesse to plunge in mudde,
VVhich Monster more by strokes then strength,
At last they slew and cut in three,
VVhose head which was 4. foote in length,
VVas cut off first for men to see.
His tayle was tried as long to be,
As halfe his body and his head,
His bodies length did iust agree,
VVith head and tayle as it lay dead.

Now

Now this is all the thing I wish,
To know the length of all this fish.

Euma. I will do my good will to tell you, and for that purpose first I suppose the length of the whole fish to be 20. foote, whereof sith the head is sayd to be 4. foote long, then must the body and the tayle be the rest, that is 16. foote.

Philo. May stay, for you are farre out of the way, for you must seeke the length of the whole fish by his giuen partes, and not his partes by his whole length, and therefore to helpe you herein, put some number for the length of his least part onely, and so thereby gather out the rest according to the formalite of the question.

Euma. Then I take his head for that purpose, because I thinke that to be shorter then his tayle.

Philo. As for the length of his head, you know it is alreadie precisely named, and affirmed for certayne to be 4. foote, so that there needeth no further search for that, for howsoeuer the length of the bodie or tayle, fall out, yet the head can be neyther more nor lesse then is appoynted, wherefore to put any number at aduentures for that which is alreadie certaynely knowne, were a meere follie and absurditie.

Euma. That is true, wherefore vsing better consideration, I will begin with the tayle, which as the length thereof is vnknowne so it by reason ought to be shorter then the bodie.

Wherefore I put 6. foote at the left heade of the figure, for the length thereof, so both the head and the tayle together make 10. foote, and so long is the bodie by the purport of the question, for the bodie is sayd to agree with the head and tayle in length, whereby it followeth, that the whole fish is 20. foote long, but now the tayle ought also to be as long as halfe the bodie and the head, but halfe the bodie is 5. foote, and the head 4. foote both which make 9. foote, which is more then my position by 3. foote and so my first position hath bredde an error of 3. to little, which error with his signe being ascribed to the due places of the accustomed figure, as here you see. I begin agayne, putting now 14. at the other side of the crosse for my second position.



So the tayle together, with the head are 18. foote long, wherefore the bodie also being equall in length to the head and tayle is 18. foote, and so the whole fish is 36. foote long.

Now

Now the tayle ought also to be as long as halfe the bodie and the head, but halfe the bodie is 9. foote, and the head 4. foote, both which make 13. foote, which by my position is 14. foote, so that the same erreth therefore by 1. too much, this erroz therefore with his signe setled in their due places, I multiplie biasly each position by contrarie erroz, so are their products 6. and 42. which (because their signes are vnlike) I adde together, taking 48. their totall foz diuidend: also the errors added together make 4. which I take foz my diuisor. Finally, division being perfozmed, there issueth 12. foote, foz the true length of the tayle, oz part sought foz, as the fabzique sheweth



Philo. Well sayd, now by knowledge of the length of the tayle, you may easily finde the length of the bodie, and consequently of the whole fish, foz seeing the bodie must be as long as both the head & tayle, it followeth that the bodie shalbe 4 + 12. foote, that is 16. foote, and so the whole fish shalbe 32 foote. Wherefore now if the tayle be as long also as halfe the bodie and the head, the question must needes be satisfied, but halfe the bodie being 8. foote, and the head 4. foote, make 12. foote iust equall to the tayle, and so all things agree, as the question requireth.

Agayne I put this question, which I will answere my selfe, foz your better instruction.

150. A question of 2. burdens

A mule with her foale which felt not as yet,
VVhat paynes her dam tooke with burdens of weight,
For that shee but newly was broke to the bit,
And had not bene loden so heauie and straight,
VVere both at once laden with great sacks of corne,
To carry to market I wot not well whither,
which weight sith the foale had neuer erst borne,
Oft made her to grone before she came thither.
Thou gronst quoth her dam without any neede,
For if 1. bushell of thy burden were,
Cast vppon my backe, then should I indeede,
Beare iust asmuch more, as thou now dost beare.
But 1. of my bushels layd vpon thine,
would make thy burden iust equall to mine.
The question is now by art to declare,
How many bushels each of these beastes bare.

For

Content quoth the ſtranger and tooke it in briefe,
Returning a doſen agayne to the theefe,
VVhereof he was glad, and paſt foorth apace.
But loe another ſtayd him in like caſe.
To whom ſtraight he offered one halfe of his fruite,
To let him eſcape and make no purſuite,
who taking his offer and ſeeing his greefe,
Gaue backe agayne 7. foorthwith to the theefe,
whereof he reioycing marcht forward amayne,
Vntill a third man ſtraight ſtayd him agayne,
To whom he did offer as erſt he did eſt,
One halfe of the apples which then he had left,
which he foorthwith tooke as they did before,
And of the halfe taken, gaue backe agayne foure,
which done, yet this theefe perceiued he had plentie,
For when he had told them he found them iuſt twentie.
Now of this queſtion the hardeſt and wooreſt,
Is how many Apples he pocked vp at firſt.

Euma. Indeede this is ſuch a prettie queſtion as allureth me to attempt the anſwere, and for that purpoſe I put 40 apples at the left head of the figure for my firſt poſition, ſuppoſing that he had pocketed vp ſo many, whereof when he gaue the firſt man halfe, he left him ſelfe but 20. but then that man giuing him 12. backe agayne, made his number 32. of which the ſecond man hauing $\frac{1}{2}$ alſo, left the theefe but 16. which with the 7. that the ſame man gaue to him agayne, made him to haue 23. whereof when the third man had taken $\frac{1}{2}$ there remayned no more to the Theefe but $11\frac{1}{2}$, ſauing that when he receyued backe 4. Apples of the third man, it made his number $15\frac{1}{2}$, but by the order of the queſtion he ſhould haue iuſt twentie.

40 ✕

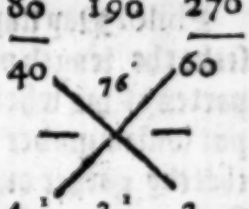
Wherfore this firſt poſition miſſeth the cuſhion, concluding vntruly by an error of $4\frac{1}{2}$ too little, which I annexe to the left foote of the figure with the ſigne of want thus.

40 ✕ $4\frac{1}{2}$

Secondly I put 60. at the right head of the figure for my ſecond poſition, ſuppoſing now that he had ſtolen ſo many Apples, whereof when he gaue the firſt man $\frac{1}{2}$ then had he him ſelfe 30. remayning, which with the 12. that the ſame man gaue him backe againe, make 42. of which when hee gaue to the ſecond man $\frac{1}{2}$

then

then had he 21. left for himselfe, which with 7. that the same man restored to him agayne make 28. whereof when he eftsoones gaue to the third man ½, then had he 14. left, which with the 4. Apples redeliuered by the same man, made his whole number 18. apples, but by the prescript of this question it should be 20. so that this position hath also missed the marke, erring likwise from the truth by want of 2. too little, which error I place at the right foote of the figure, with the proper signe appertayning.



This done I multiplie each position byasly by his fellowes error, & their products are 80. and 270. as appeareth ouer their heads, whereof (because the signes are both like) I subtract the lesser from the greater, and so remayneth 190. for diuidend.

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Finally diuiding 190. by 2 ½ the true answering number 76. issueth in the quotient, and so many apples had he stolne to carrie away, which number I finde by examination satisfieth the conditions of the question precisely, & therefore I haue set it in his proper place, to wit, in the center of the figure as you see.

Philo. You haue done well, and because you are so readie and quicke in vnderstanding, answere me to this question.

149. A question of a fishe.

By South sea coast, neere where I dwell,
A shallow creeke doth cut the shore,
VVherein the sea by rage doth swell,
And so brought in, of fish good store.
VVhereof one monstrous fish was found,
VVhich at retiring of the floud,
The people spied vpon the ground,
Halfe waterlesse to plunge in mudde,
VVhich Monster more by strokes then strength,
At last they slew and cut in three,
VVhose head which was 4. foote in length,
VVas cut off first for men to see.
His tayle was tried as long to be,
As halfe his body and his head,
His bodies length did iust agree,
VVith head and tayle as it lay dead.

Now

Now this is all the thing I wish,
To know the length of all this fish.

Euma. I will do my good will to tell you, and for that purpose first I suppose the length of the whole fish to be 20. foote, whereof sith the head is sayd to be 4. foote long, then must the body and the tayle be the rest, that is 16. foote.

Philo. Nay stay, for you are farre out of the way, for you must seeke the length of the whole fish by his giuen partes, and not his partes by his whole length, and therefore to helpe you herein, put some number for the length of his least part onely, and so thereby gather out the rest according to the formalite of the question.

Euma. Then I take his head for that purpose, because I thinke that to be shorter then his tayle.

Philo. As for the length of his head, you know it is already precisely named, and affirmed for certayne to be 4. foote, so that there needeth no further search for that, for howsoeuer the length of the bodie or tayle, fall out, yet the head can be neyther more nor lesse then is appoynted, wherefore to put any number at aduentures for that which is already certaynely knowne, were a meere follie and absurditie.

Euma. That is true, wherefore vsing better consideration, I will begin with the tayle, which as the length thereof is vnknowne so it by reason ought to be shorter then the bodie.

Wherefore I put 6. foote at the left heade of the figure, for the length thereof, so both the head and the tayle together make 10. foote, and so long is the bodie by the purport of the question, for the bodie is sayd to agree with the head and tayle in length, whereby it followeth, that the whole fish is 20. foote long, but now the tayle ought also to be as long as halfe the bodie and the head, but halfe the bodie is 5. foote, and the head 4. foote both which make 9. foote, which is more then my position by 3. foote and so my first position hath bredde an error of 3. to little, which error with his signe being ascribed to the due places of the accustomed figure, as here you see. I begin agayne, putting now 14. at the other side of the crosse for my second position.



So the tayle together, with the head are 18. foote long, wherefore the bodie also being equall in length to the head and tayle is 18. foote, and so the whole fish is 36. foote long.

Now

Now the tayle ought also to be as long as halfe the bodie and the head, but halfe the bodie is 9. foote, and the head 4. foote, both which make 13. foote, which by my position is 14. foote, so that the same erreth therefore by 1. too much, this erroz therefore with his signe setled in their due places, I multiplie biasly each position by contrarie erroz, so are their products 6. and 42. which (because their signes are vnlike) I adde together, taking 48. their totall foz diuidend: also the errozs added together make 4. which I take foz my diuisoz. Finally, diuision being perfozmed, there issueth 12. foote, foz the true length of the tayle, oz part sought foz, as the fabzique sheweth.



Philo. Well sayd, now by knowledge of the length of the tayle, you may easily finde the length of the bodie, and consequently of the whole fish, foz seeing the bodie must be as long as both the head & tayle, it followeth that the bodie shalbe 4 + 12. foote, that is 16. foote, and so the whole fish shalbe 32 foote. Wherefoze now if the tayle be as long also as halfe the bodie and the head, the question must needes be satisfied, but halfe the bodie being 8. foote, and the head 4. foote, make 12. foote iust equall to the tayle, and so all things agree, as the question requireth.

Agayne I put this question, which I will answere my selfe, foz your better instruction.

150. A question of 2. burdens

A mule with her foale which felt not as yet,
VVhat paynes her dam tooke with burdens of weight,
For that shee but newly was broke to the bit,
And had not bene loden so heauie and straight,
VVere both at once laden with great sacks of corne,
To carry to market I wot not well whither,
which weight sith the foale had neuer erst borne,
Oft made her to grone before she came thither.
Thou gronst quoth her dam without any neede,
For if 1. bushell of thy burden were,
Cast vppon my backe, then should I indeede,
Beare iust asmuch more, as thou now dost beare.
But 1. of my bushels layd vpon thine,
would make thy burden iust equall to mine.
The question is now by art to declare,
How many bushels each of these beastes bare.

Foz

For answere wherof I suppose the Mules burden to be 6 bushels, and her foles 4. bushels, now if 1. bushell be taken from the Mule and layd on the Foale, then should their burdens be equall, and so they are, to wit, 5. bushels a peece, for 6 — 1. and 4 + 1. make both wayes 5. but when 1. bushell is taken from the Foale and layde on the dammes backe, then doth the dam carry 7. bushels and the foale 3. but by the order of the question the dams burden now should be as much more as her foales. that is to say double thereto, but 7. bushels is more then the double of 3. bushels by 1. and so this position hath erred in excesse by 1. too much, which position and error with the signe pertayning, I note in their proper places of the crosse or accustomed figure.

Which done, I chuse a second position, supposing now the dams burden to be 8. bushels, and the foales 6 so when the damme hath cast 1. bushel from her to her foale their burdens ought to be equall and so they are, to wit, 7. bushels a peece, for 8 — 1. and 6 + 1. make both wayes 7.

But when the foale returneth the same bushell backe agayne to her damme, with 1. bushell of her owne burden also, then is the Mules burden 9. bushels, and the foales but 5. which 9. bushels by the order of this question, should be double to 5. the foales burden, but it is not, for 10. is the double to 5. and so this position erreth in want by 1. too little, which second position and error with the proper signe, I duly settle in the figure also thus.

Mule	6	14	8
Foale	4	7	6
		+	−
		1	2	1

But now whilest I goe about to multiplie bias by 6. by 1. & 8. by 1. as the rule willeth, I see presently þ here needeth no bias multiplication at all, because the multiplier euery way is but 1. (which as hath bene often sayd, doth neyther multiplie nor diuide) wherefore omitting multiplication, I die the rest of the rule onely, viz. because the signes are contrarie or vnlike, I adde together 6. and 8. the 2. positions referred to the Mule, and they make 14. my diuidend, likewise I ad together the 2. errors, & they make 2. my diuisor, finally diuiding 14. by 2. the quotient vttereth 7. bushels for the Mules burden.

Whereby it followeth necessarily that the foales burden was 5. bushels, which with 1. bushell of the Mules burden made each of their burdens, 6. bushels equally, but if 1. bushell of the foales 5. were layd on the Mules backe, then should her burden be 8. bushels, which is double to 4. the remaynder of the foales burden.

Also

Also the Foales burden may be found by adding 4. to 6. the positions, referred to the Foale, whereof commeth 10. the diuidend, which being diuided by 2. the common diuisor, quoteth 5. bushell for the Foales burden, agreeable with the former inuention.

Euma. All this I vnderstand very well, wherefore if there be no other doubts or varieties of workings, I trust I am sufficiently instructed in this rule.

Phi. There is neither other doubts nor other varieties of workings then you haue seene, but yet will I put one question more, which I will answer also, that thereby you may the better perceiue how to applie your numbers to the rule, and so I will end it. The question is thus.

151. A question of a fountaine

But late I sawe a fountaine faire, most stately to beholde,
At top whereof a Lyon stood, made all of molten golde,
Out of whose mouth, his eyes & foot, the christall liquor gusht,
And in a bowle of Iuorye, with siluer streames it rusht.
This fountaine was so finely fram'd, with vices for delight.
That lions mouth & eies might close, & stop their courses quite.
Now if his issues all were stopt, saue onely his right eye,
In 2. dayes it would fill the bowle, though it before were drye.
And if the left eye onely run, three dayes then serues the turne,
His foote alone 4. dayes must haue, the same thing to performe.
But if his mouth alone do spoute, but onely 6. houres space,
The water will iust fill the Bowle, and neuer misse an ace.
Now if these issues all at once, should keep their course at wil,
Then shew me in how short a time, this empty bowle should fil.

For answer hereof, sith here is mention both of daies and houres, it is requisite first to reduce, both these sirnames to the subtillest or least denomination, which is houres.

Day artificiall. Scheubelius tract. 5. in fine.

And although a day naturall containe 24. houres, yet in this question, I will vse the day artificiall, which is esteemed to be but 12. houres onely, after which rate both Scheubelius and Peucerus, do esteeme it, in their answeres of the selfe same question.

Peucerus inter regulas logisticas suas. pag. 149.

So after that rate, in steade of 2. dayes I take 24. houres and in stead of 3. dayes 36. houres, & for 4. daies 48. houres, this done, I suppose the bowle will fill in 2. houres, in which time the right eye filleth $\frac{1}{12}$, the left eye $\frac{1}{17}$, the foote $\frac{1}{24}$, and the mouth $\frac{1}{3}$.

Euma. That do not I perceiue, wherefore I pray you shew me how I shall know it.

Phi. No do? why you know that the question saith, that the right

eye, will fill the bowle in 2. dayes, which heere is esteemed to be 24. houres.

Euma. Yea I know that very well.

Philo. Why then if 24. houres fill 1. bowle, know you not how to finde what 2. houres will fill.

Eu. Yes, if I shall vse the golden rule about it, then in deede I finde that 2. houres will fill $\frac{1}{12}$. of the bowle.

Philo. And thinke you that there is any rule so single, that it nedeth not the golden rule? nay I assure you the very grand rule of commonly Equation, which is called the rule of Cosse & by some Regula Magna (wherof you shall learne hereafter) is nothing els but the golden rule, or finding of a fourth tearme proportionall, wherefore do not you disdaine to vse it when occasion requireth it.

Neuertheless you might haue considered here euen by memory, (without actuall working by the golden rule) touching the right eye, that as 2. houres is $\frac{1}{12}$. of 24. houres, so $\frac{1}{12}$ of the bowle must needs fill thereby in that time.

And for the left eye, that as 2. houres is $\frac{1}{18}$ of 36. houres, or 3. daies, so $\frac{1}{18}$. of the bowle must needes fill in that while.

And so likewise for the foote, sith 2. houres is $\frac{1}{24}$ of 48. houres or 4. dayes, so $\frac{1}{24}$ of the bowle shalbe filled thereby in that space.

And finally for the mouth, that as 2. houres is $\frac{1}{3}$. of 6. houres so $\frac{1}{3}$. of the bowle shalbe filled thereby in that time.

But now to proceede, I adde all these fractions of the bowle together, to wit, $\frac{1}{12} + \frac{1}{18} + \frac{1}{24} + \frac{1}{3}$. by the 155. and 159. verses, and their totall is $\frac{37}{72}$. as the fabrike of the reduction and addition here annexed sheweth. Whereby appeareth that in 2. houres, all the issues fil but $\frac{37}{72}$ of the bowle, which by position ought to fill $\frac{72}{72}$ thereof, that is to say, the whole bowle entirely. So that looke how much $\frac{37}{72}$ lacketh of $\frac{72}{72}$, so much lacketh my position of the truth, and that is $\frac{35}{72}$, which position 2 with the error and signe appertaining, I commit to the figure as you see, in their due places.

$$\begin{array}{cccc} & & & 37 \\ 6 & + 4 & + 3 & + 24 \\ \hline x & x & x & x \\ \hline 12 & 18 & 24 & 3 \end{array} \qquad 72$$

2 − $\frac{35}{72}$

Euma. Now I pray you let me worke forth the rest, for (by this your good example) I perswade my selfe I vnderstand all the order of the operation.

Philo. Go to, I am contented.

Euma. Then to begin where you left, I put 4. houres for the second

second position, supposing that in that space, the bowle shall fill. In which time by the purport of the question, the right eie filleth $\frac{1}{2}$, the left eye $\frac{1}{3}$, the foote $\frac{1}{12}$, and the mouth $\frac{1}{4}$. For as 4. houres is $\frac{1}{6}$. of 2. dayes or 24 houres, so is it $\frac{1}{8}$ of 3. dayes or 36. houres, and $\frac{1}{12}$. of 4. dayes or 48. houres, and so finally it is $\frac{1}{8}$ of 6. houres.

Wherefore conforming and adding together all these fractions of the bowle, I finde that their totall is $\frac{37}{36}$, which representeth the entire bowle, and $\frac{1}{36}$ ouerplus, so that this position hath erred by $\frac{1}{36}$ too much, which position with his error and signe, I commit to the figure in their due places, as heere appeareth.

$$\frac{37}{6+4+3+24}$$

$$\frac{x}{6} \quad \frac{x}{9} \quad \frac{x}{12} \quad \frac{x}{8}$$

$$36$$

And now multiplying byasly $\frac{1}{2}$ by $\frac{1}{36}$. the product is $\frac{1}{72}$ (which I change to $\frac{4}{72}$, that so the balles of both products being like, they may the sooner be added.)

Likewise I multiply byasly $\frac{4}{1}$ by $\frac{37}{72}$, and the product is $\frac{140}{72}$, which 2. products (because the signes are vnlike,) I adde together, and they make $\frac{144}{72}$ my diuidend, also I adde the errors together, and they make $\frac{37}{72}$ my diuisor.



And now (by the 173. verse) reiecting the equall balles, I diuide onely 144. by 37 and so ariseth in the quotient $3\frac{33}{37}$ houres, for the true time or nũber of houres in which the bowle shalbe iustly filled.

Philo. You haue done well. And here before I end this rule, I will shew you how to answere this question, or any like, after another forme, which is thus.

The same question otherwise answered.

First imagine the bowle to conteine some measure certaine, as for example. I will imagine it to containe 72. gallons. Now to the end you shall see the sequell of both opperations, I will take another position, supposing that the bowle shall fill in 1. houre, so in that time the right eye filleth 3. gallons, the left eye 2. gallons, the foote $1\frac{1}{2}$ gallons, and the mouth filleth 12. gallons. All which fillings added together, make but $18\frac{1}{2}$. gallons, which should be 72. gallons, so that this position erreth by $53\frac{1}{2}$ too little.

Which position and error, with the due signe, being orderly applyed to the figure, as heere you see, I begin againe.



Taking now 3. houres for my second position, supposing that the bowle shall fill in that space, after which rate, the right eye in that time filleth 9. gallons, the left eye 6. gallons, the foote $4\frac{1}{2}$. gallons, and the mouth 36. gallons: all which fillings added together make $55\frac{1}{2}$ gallons, but they ought to make 72. and so this position erreth in want also, by $16\frac{1}{2}$ gallons too little: which position and error, with their due signes, I describe at the other side of the figure in their proper places, as here appeareth.



Which done, I multiply byasly each position by contrary error, and their products are $16\frac{1}{2}$, and $160\frac{1}{2}$, as appeareth. Whereof (because the signes are like) I abate the lesser from the greater, so resteth 144. for my diuidend. Also I abate the lesser error from the greater, and so remaineth 37. for my diuisor: finally 144. diuided by 37. quoteth $3\frac{33}{37}$ houres, euen as before.

Euma. This change or varietie of operations I like well, for me thinkes this latter forme is easier then the former: but I pray you what order is there for the proofe or triall of this rule.

Phi. The proofe may be made 2. wayes: the one is by conferring the quotient or answering number, found by opperation of the rule, with the contents and conditions of the question, wherein if all things agree accordingly, as you see they do in all the former questions of this chapiter, then is there no doubt but the answer is true.

Proofe.

The second, is by chaunging of positions, as I did heere, wherein if the quotient or aunswering number of the latter worke be equall to that of the first, it cannot but be a true aunswere.

And here I am not ignorant how some rack this rule of double positions further, strayning it beyond the nature therof, to the answering of certayne very difficult questions, which in my minde are not within the compasse thereof.

Tonstallus li. 4. quest. vltima. Peletarius lib. 4. ca. 6 quest. 3. Clauius, cap. 23. quest. 5.

For if such questions, be within the compasse of 2. positions, as they pretend, then why doe they in the practise of their dissolution, teach the putting of 6. seuerall positions at the least, and do not shew how to dissolue them by 2. positions onely?

But if so many positions, be necessarily requisite thereto (as in truth there are, or els they cannot be explicated, then why in-

include they such questions, vnder the title of the rule of 2. positions: not making a further diuision of the rule of false positions, but onely the single rule of one position, and the double rule of 2. positions?

But I knowe they will alledge a distinction, saying that 2. positions onely are the principals, and the other 4. are but accessaries or accidentals.

Euma. I gesse by your speach, that the 4. positions which they distinguish from the 2. principals, by name of accedentals, are such as fall out consequently, by the forme and purport of the question, in such manner as you shewed me before in the example and worke of the first question of this chapter.

Phi. May nothing lesse, for I assure you there is not one of their 6. positions, but is specially put, named, and chosen, at the will of the workeman, and euery couple of them applyed to a seuerall figure, whereby although the question is at the last (with much a do) explicated, yet is it such a toylsome trauaile and tedious labour to bring it to passe, as being clogged with prolixitie, difficultie, and obscuritie, is able to wearye any man, and to dull a right good wit.

Euma. I pray you do but rehearse one of those questions, for my mindes sake.

Philo. I am loath to stay so long about it, but yet to satisfie your minde, loe this is one. Three merry companions, to wit, A.B.C. talking of the money in their purses, A. said to his fellowes: if you two will giue me $\frac{1}{3}$ of your money to that which I haue already, I shall haue iust 20. crownes, so shall I haue also said B. if you two will giue me but $\frac{1}{5}$ of yours. May quoth C. yet rather giue me but $\frac{1}{4}$ of both your moneys, and so shall I haue 20. ▽. whereupon they demaunde how much money, or how many ▽. each of these good fellowes had in their purses.

This question, as saith Goselinus, is propounded by Tartalea, and by Pedro de Burgo, and againe by Lucas Paccioli, all which three authors as he saith, declare the explication thereof, so confusedly, so obscurely, and with so many positions, that it seemeth to him vnpossible (specially after the handling therof by so great learned men) to dissolue it any wayes by two positions onely.

Tartalea, lib. 17. quest. 41. Petrus Burgi parte vltima tract. Lucas Paciolus tract. 2. dist. 2. Goselius lib 17. tract. 1.

Neuertheless he promiseth to doe his endeuor, to illustrate the explication thereof, and to render it cleere and easie, by 2. positions onely. And in the 17. booke of his first treatise, sheweth the

manner of his deuise, and forme of his conceipt, wherein how cleere he hath made it, I leaue to their iudgement that can skill thereof, for I can see no more clearenesse in the thing then I could before.

And as it is true, that this rule is derived and proceedeth originally frō the art of Algebra, euen like as the light of the Moone proceedeth from the sunne, so is it as true also that it can no more compare with the perfection of Algebra it selfe, then the Moone-light is to be compared to the brightnes of the Sunne-beames.

Wherefore I for my part, thinking it no orderly method to wrest this rule to the answering of such questions as are not possible to be regularly explicated by such number of positions, as the rule it selfe, in name and title comprehendeth, doe vtterly omitte them here, referring them to the rule of Cosse or Algebraicall Equation, and in the meane season, he that is desirous to see a plaine and apt method of their explication, let him resort to the 27. and 28. probleames of Diophantus his first booke, with the Scholies of Maximus Planudis and Xylander, where he shall finde the selfe same question in effect.

And yet I confesse, that this rule by the helpe of extraction of Rootes onely, stretcheth farre further, then heere is now time to declare, because you are yet ignorant of that Arte. Wherefore as this chapiter endeth all the members and branches of numbers relatiue in qualitie of proportionalitie, euen so heere will I end the Chapter also, and will go in hand now with disproportionalitie in forme following.

Of Disproportionalitie. Chap. 22.

316. A disproportion, by true definition,
is two proportions dislike in qualitie,
When neither to other agree in condition,
but by their discorde shew a newtralitie,
So that the one to the other in being,
is wholy vnlike, and quite disagreeing. } 243

Now heere, I thinke I shall not neede to make a new repetition of that which I told you in the 168. and 170. Maxims, to wit, that by likenesse or dislikenesse in qualitie is to be vnderstood, likenesse, and dislikenesse of differences onely, or of proportions onely.

Euma. No indeed, for I remember the verses very well, & do perceiue

reine by this verse present, that as 2. or mo differences of like quãtitie, or 2. or mo proportions of like quantitie, make a proportionalitie, so 2. or mo differences of diuers quantities, or 2. or mo proportions of diuers quantities, make a disproportionalitie. And therefore 2. to 3. and 4. to 7 are a disproportionalitie, whether they be taken Arithmetically or Geometrically, for neither the difference of 2. to 3. is like the difference of 4. to 7. nor yet the proportion of 2 to 3. is like the proportion of 4 to 7.

Philo. You vnderstand it very well, and of this parte, some, namely Vrsicius, saith there is no Arte. Neuertheless such principles as I haue found thereof, I will impart vnto you, which are these fewe insuing.

317. If any two numbers, which differ a sunder,
be both compared to any third number,
The rate which the greater doth beare to the third,
Is greater then that which the lesse can affoord. } 244 Euclid. 8. prop. 5.

As in these 2. numbers 6. and 4. which are vnequall and differ in their quantities, the rate or proportion of 6. the greater, to 2. a third number is Tripla, but the rate of 4. the lesser number to the same 2. is but Dupla. Wherefore of 2. vnequall numbers, that is the greatest, which hath greatest proportion to a third number.

Euma. This nedeth small proofe in my minde.

Philo. Well then, another Maxim is thus.

318. Any one number compared with two other,
which two are not both coequals together,
Makes the proportion thereof to the lesser,
greater then if it compared with the greater. } 245 Euclid. 8. prop. 5.

As for example, let 6. be compared to these two other numbers 3. and 2. which are not coequals together. Now I say (by this verse) that the proportion of 6. to 2. the lesser number, is greater then the proportion of 6. to 3. the greater number, for 6. to 3. is but Dupla, but 6. to 2. is Tripla, as heere appeareth.

6. to { 3. Dupla / 2. Tripla

Euma. This also is very manifest to common reason.

Philo. Then shall you haue this one more following, and so haue we done.

319. Wherefore if one single number compare,
with any two other, which vnequall are,
The number to whom such single terme goeth,
in greatest proportion is least of them both. } 246 Euclid. 10. prop. 5.

As in the example of the next verse before, 6. being one single number compared with 3. and 2. was but Dupla to 3. but it was Tripla to 2. wherefore by this verse, because the proportion of 6. to 2. is greater then the proportion of the same 6. to 3. therefore I say 2. is lesser number then 3.

Euma. This is such a maxim as I thinke no man doubteth of, and I take it to be the conuerse of that next before it.

Philo. It is so, and thus friend Eumathes haue we passed through all the parts, branches, & members, of vulgar Arithmetick, according to the diuisions therof prescribed in my table, wherby we are now growne to a full end, for the which let vs conclude with thankes giuing to Almightie God, the author of all good gifts, and fountaine of all wisdome and knowledge, who liuing & reigning 3. in number of persons, and one in vnitie of substance, ordering all things in measure, number & weight, and framing all his creatures in a most delectable and excellent proportion, hath vouchsafed to assist and prosper these our proceedings, from the beginning to the end. To him therefore I say, which numbereth the starres, and calleth them all by their names, be honor, glory, prayse, and thankes giuing, world without end, Amen.

Ecclesi. 1.

Wisdom 11. verse 13.

Ecclesi. 42.

Psal. 147. verse 4.

Euma. Amen and Amen.

The end of the Second Booke.



A ſhort preface or foreſpeech had betweene Philomathes *and* Eumathes the interlocutors, touching the third booke.

Eumathes.

GOod friend *Philomathes*, although I haue been often and long troubleſome vnto you already, yet ſith the olde prouerbe is, that promiſe is debt, I am bold to challenge your third booke of Arithmeticke, as a debt due by your promiſe.

Philo. The booke which I promiſed you, named *Muſa Mercatorum*, I minde God willing to performe, and that euen preſently, yet not as any ſeuerall part of the art of Arithmeticke, for I ſuppoſe the whole art of vulgar arithmeticke to be ſufficiently comprehended in my two former bookes, and therefore I communicate this third booke vnto you, but as an example of the common practiſe of Merchants, who vſing the art generally to no other purpoſe, but to caſt, or foreknow, their gaynes or loſſe, haue by their dayly practiſe deuiſed many kindes of rules and formes of application of numbers, which the learned Mathematician, neyther occupieth nor ſcarcely knoweth.

All which neuertheleſſe are generally deriued from ſome one or mo of the rules taught in my former booke, and eſpecially from the golden rule. And agayne becauſe Merchants chiefely aboue other men, haue commonly to do, not onely with the exchange and trucking or bartring of their wares, merchandies, and commodities, but alſo with the exchange of forraygne coynes, and with the reductions of the weights and meaſures, of ſundry nations, with whom they trafique, al which cheefely depend, or are deriued from the two latter kindes of the golden rule compounded, as in the 17. chapiter of my ſecond booke I haue declared.

Therefore (I ſay) I did both reſerue and referre the example

examples of many things of purpoſe there vntill now, becauſe the queſtions anſwerable by the ſaid rules, ſeeme not ſomuch appertayning to the mathematicall ſcholer as to the politique merchant.

Wherefore I muſt tell you, that you ſhall learne but little here now, that hath not eyther beene taught you alreadie in my former bookes, or at the leaſt, that may not be aſwell performed by the rules there taught, though not in the ſame forme or faſhion of woorke.

Euma. Well yet I pray you let me ſee the forme of both, that ſo I may chuſe whether I will vſe the ſcholers order or the merchants faſhion, for I haue heard ſay, that the merchants rules are merueilous ſpeedie and compendious.

Philo. Some of them are ſo indeede, whereof I will impart the beſt and moſt principall vnto you, beginning with that rule, which in an auncient written copie that I haue, well neere an hundred yeeres olde, I finde intituled *Regula Mercantiæ*, but I like better to call it *Regula Mercatorum*, that is to ſay, the merchants rule, the operation whereof, taketh place chiefely in numbers contracted to coines, of ſome ſuch ſubtill ſirnames as are fractions or parts of the coyne of a greater ſirname.

As in mites or graynes, which among merchants is eſteemed to be $\frac{1}{12}$ of a peny, and ſo they commonly call a Mite, and count it in their reckonings, and then as you know a peny is $\frac{1}{2}$ of a groate, $\frac{1}{4}$ of a Teſtern, and $\frac{1}{12}$ of a ſhilling, and a ſhilling is $\frac{1}{20}$ of a pound, and this rule is originally deriued from the Golden rule direct, and ſecondarily from the firſt propoſition of the ſecond booke of *Euclid*, contayned in the 55. verſe of my firſt booke.

¶ And is in very deede the ſelfe ſame rule, whoſe inuention the Italians chalenge to them ſelues, calling it *Praxis Italica.*

The

THE THIRD BOOKE named *Musa Mercatorum*: Comprehending sundry necessarie and profitable rules for the trade of merchandies.

Of the merchants rule, called in latine Regula Mercatorum, *or* Praxis Italica, *the first Chapiter.*

Philomathes.

1. VVhen the first terme in golden rule direct,
Is but an vnit as oftimes it is,
Then doth the merchants rule take best effect,
For speedie woorking, whose order is this,
2. Account the third terme or number demaunding,
To be eyther pounds, or shillings, or pence,
which though it seem false wil bring notwithstanding,
A very true answere by such a pretence,
3. Of which number take such alliquot part,
Eyther of peny, shilling, or pound,
As the price giuen it selfe doth impart,
So by the quotient the answere is found,
Turned *ipso facto*, to coyne of like kinde,
To the alliquot part proposed or assignd,
4. And if such diuision do leaue ought remayning,
Reduce that remaynder to shillings or pence,
Or mites if neede be, alwayes constrayning,
The first diuisor not once to go thence,
But still to diuide each coyne after other,
So long till it can diuide them no further.

Here you may perceiue by the very first verse of this rule, that the same is deriued from the Golden rule, as I sayd before, and is applied by merchants to dissolue such questions onely as haue 1. for their first antecedent, in the golden rule, for if the first terme be not an vnit this rule holdeth not.

Fuma. I pray you put an example of the practise.

Philoma. Then let the same be this, at 2 s. 6 d. an ell of Holland, what will 544. ells cost, heere you knowe that if these termes

1. A question of Holland.

termes should be applied to the Golden rule vnreduced, they would stand thus in the figure, where the first terme is but 1. as you see, the giuen price is 2 s. 6 d. which is expressly named to be the price of 1. ell, and the third terme or number demaunding the question (wherein the doubt or thing sought for, consisteth) is the 544. elles of Holland, which number among Merchants is generally called Numero, and is commonly by them expressed thus N° 544.

elles. s. d.
1 2. 6.
544

Which name aswell because it is shorter and sooner vttered, then to call it the demaunding number, I purpose to vse heere also throughout this chapiter.

Euma. Very good, all this is playne inough.

Philo. Now although indeede and truth, this present Numero, 544. signifieth and is nothing els but 544 elles of Holland, yet the second verse here biddeth me, precisely to accompt it to be eyther 544. pounds of money, or 544. shillings, or 544. pence (according as the giuen price is) and then the third verse willeth me to take such an aliquot part thereof as the giuen price containeth, eyther of peny, shilling or pound, affirming that so I shall finde the answere in the quotient immediately, turned ipso facto to such kinde of money or coyne, as the aliquot part proposed is, which is nothing like to the forme of the operation vsed in the golden rule.

Wherefore to come to the practise, I behold 2 s 6 d. the giuen price, and consider that it is too great to be the aliquot part, eyther of a peny or of a shilling, (because it containeth both many entire pence, and also many entire shillings) but I see it is an aliquot part of a pound, that is to say $\frac{1}{8}$, for 8. times 2 s. 6 d. is iust 20 shillings, or 1. pound.

Wherefore sith it is the aliquot part (not of a peny, nor of a shilling, but) of a pound therefore I say, I esteeme the demaunding number here called Numero, not to be pence, nor shillings, but pounds, according as the giuen price is, the aliquot part of 1. pound.

Of which Numero, I take onely $\frac{1}{8}$ (as the third verse here willeth me,) which as you know is soone done, by diuiding it by 8. the denominator of the same aliquot part, and so the quotient 68 r. the true aunswere to this question.

For (by the same verse) the quotient turneth ipso facto euen by the diuision onely into coyne or readie money, of like kinde and surname, to the aliquot part proposed or assigned.

But

But the aliquot part here proposed is $\frac{1}{1}$, not of a peny, nor of a shilling, but of a pound, wherefore the quotient being correspondent thereto, aunswereth 68. poundes, and so much will the 544. elles of Holland cost.

Another example is thus, at 3 ß. the pound of Currants, what will Numero 160 ℔. subtill cost: here if I accept this 3 ß. the giuen price, for the aliquot part of a pound, as I may doe, for it is $\frac{1}{1}$. of a pound, then must I likewise accept 160. which is my Numero, to be 160. poundes of money also, and so diuiding it by 80. the denominator of the same part, the quotient presently aunswereth that the 160. pound of Currants will cost 2. poundes, according to the denomination, whereof the giuen price is an aliquot part. 2. A question of Currances.

But if I esteeme the same 3 ß. to be but the aliquot part of a shilling (as I may also very truly doe, for indeede it is $\frac{1}{4}$ of a shilling) then must I likewise esteeme Numero, to be but shillings, and so diuiding it by 4. the denominator of $\frac{1}{4}$, the quotient bringeth now, not 2. pound, but 40. shillings for the answering terme, according to the coyne or money, whereof the giuen price was an aliquot part, so that the aunswere which before turned ipso facto into poundes, now here turneth ipso facto into shillings, and yet both answeres are true, for you know that 2. pound = 40. ß.

A third example is thus. At 4. Mites the yarde of Inckle, what will Numero 216. yardes cost: here you see that 4. Mites is the aliquot part of a peny, viz. $\frac{1}{6}$ for 6. times 4 is 24. and so many mites marchants assigne to 1. peny. 3. A question of Inckle.

Wherefore now for as much as the giuen price is but the aliquot part of a peny, I must esteeme Numero to be 216. pence, whereof I take $\frac{1}{6}$ by diuiding it by 6. the denominator of the same part, and the quotient now bringeth 36. (not pounds, nor shillings, but) pence, correspondent to the coyne, whereof the giuen price was an aliquot part, wherefore I conclude that the 216. yardes of Inckle will cost 36 ß. which by reduction is 3. shillings.

Euma. Indeede this rule is very briefe in operation, and by these examples I thinke I vnderstand the sence of the 3. first verses very well.

Philo. Then to the end you may vnderstand the last verse as well also, take this example. Almonds being at 10 ß. the ℔. what will Numero 1600 ℔. subtill weight cost. 4. A question of Almonds.

For answere hereof, first I consider, that 10. pence the giuen price

price is $\frac{1}{24}$ r. for which cause immediately I account my Numero to be 1600 r. of money, diuiding it forthwith by 24. the denominator for the aliquot part, and the quotient bringeth 66. pounds, and there remayneth 16. which by supposition are 16. pounds, wherefore (as the last verse of this Maxim willeth me,) I reduce them to shillings, so are they 320 s. which I constrayne 24. my former diuisor eftsoones to diuide, & the quotient is 13 s. and yet there is a remayner of 8 s left, which (as the same verse also willeth me) I reduce into pence, and so are they 96. pence, whom I eftsoones constrayne 24. my first diuisor, to diuide also, which done the quotient is 4 d. iust, and there is nothing left for my diuisor, to diuide any further, whereupon I conclude that the 1600 r. of Almonds will cost 66 r. 13 s. 4 d.

Euma. All this I vnderstand very well by sight of this your example, and presume that I can do the like, but me thinks there needed not so many diuisions and reductions herein, for when the first quotient was 66 r. and there remayned $\frac{16}{24}$ r. it had beene lesse labour (in my minde) and sooner done, to haue abridged that fraction to $\frac{2}{3}$ pound, which euery childe almost knoweth to be 13. shillings 4. pence.

Philo. You haue sayd truth, and I am glad you conceiue the practise of this rule so readily, but you must vnderstand that Marchants whose chiefe purpose and studie, tendeth to gaynes, haue diuised to breake a peny into 24. partes which they call sometimes Mites as I sayd before, and sometimes they call them graynes, whereby in some cases they do necessarily extend the operation of Diuision yet further, to wit, past pence or farthings, euen to those Mites, as you sawe partly by the question of Inckle before, which thing such generall Reduction as you vsed here, cannot so aptly at all times accomplish, as the experience thereof hereafter will teach you, and in the meane season I must let you vnderstand what exceptions are in this marchants rule, whereof one is such as followeth.

5. But if the price knowne, in no wise will make,
An ailquot part, yet this shall you speede,
Some part of the price first by it selfe take,
And then another or mo as you neede,
Diuiding *Numero* for each part alone,
And then adde their quotients all into one, } 2

As for example, suppose I haue bought Numero 147. yardes of cloth

cloath at 6 s.3 d.the yarde,and would knowe what I haue to pay for it all. 5.A question of Cloath.

Here because 6 s.3 d.the giuen price, taken wholy is no aliquot part,neyther of pound nor shilling,therefore (as this present verse biddeth me) first I take some such part thereof, as is an aliquot part eyther of a pound or shilling, and then another or moe as I need,which he y is but a beginner herein,shall the better do,if(till he be perfectly acquainted with the order)he doe set downe the giuen price all in aliquot partes of poundes or shillings with their propper fractions, vnder each of them thus.

5 s + 1 s + 3 d.
¼ l.	1/20 l.	¼ s.

Where you see that 5 s beeing ¼ l. and 1.shilling beeing 1/20 l. and 3. pence beeing ¼ s. haue each of them their propper values expressed vnder them beneathe the line in fractions, which giuen price may bee also parted into other aliquot partes thus.

4 s + 2 s + 3 d
⅕ l.	1/10 l.	¼ s

Where now you may see that 4.shillings beeing ⅕ pound, and 2. shillings being 1/10 l. and the 3. pence which continueth ¼ shillings, haue each of them their propper values expressed vnder them in fractions, so that now by which soeuer of these partitions I woorke (yet as the totall of the aliquot partes,alwayes maketh iustly,) the giuen price 6 s.3 d. so shall the totall of the quotients make the aunswering number as iustly and truly.

Euma.I vnderstand you thus,that like as 5. shillings + 1 shilling + 3.pence,make iust 6 shillings 3. pence the giuen price, so doth 4.shillings + 2.shillings + 3.pence make the same price also,wherfore if you diuide Numero first by 4.pound, 20.pound,and 4 shillings seuerally,and afterwardes by 5.pound, 10.pound, and 4.shillings seuerally,yet shall the totall of the 3. first quotients be all one with the totall of the 3.last.

Philo.You vnderstand it rightly,wherefore comming to diuide Numero by the denominators of the first 3.fractions seuerally, I beginne first with 4.vsing it for diuisor, and the quotient is 36 ¼ l. that is 36 l.15 s which I set aside by it selfe in this order.

l	s	d.
36	15	0.

Then secondly for the 1 s. I diuide Numero by 20.and the quotient is7 l.7 s. which I subscribe directly vnder the former quotient (that so it may be the readier to be added together)thus.

l.	s.	d.
36	15	0
7	7	0

Thirdly for the 3. pence although all this while hetherto

hetherto I accompted Numero to be pounds, because the parts of the giuen price hetherto, were aliquot partes of a pound, yet now sith this 3 d. is here esteemed but as the aliquot part of a shilling, therefore now I must esteeme Numero to be 147. shillings, and therefore diuiding it by 4. (because 3 d. is $\frac{1}{4}$ shilling) I finde the quotient to be 36. $\frac{3}{4}$ s. that is to say 1 l. 16 s. 9 d. which I subscribe directly vnder the former quotient thus.

l.	s.	d.
36	15	0.
7	7	0.
1	16	9.
45	18	9.

Finally adding all these quotients together, as this verse willeth me, the totall quotient or aunswering number is 45 l 18 s. 9 d. as here appeareth.

And so much haue I to pay for the 147. yardes of cloth.

Euma. I perceiue the order of this worke very well I trust, and therefore I pray you let me finde the answere to the same question by the second partition of the aliquot parts, which were these.

4 s + 2 s + 3 d.

$\frac{1}{5}$ l. $\frac{1}{10}$ l. $\frac{1}{4}$ s.

Philo. I pray you doe so.

Euma. Then first for 4 s. being $\frac{1}{5}$ l. I diuide my Numero 147. by 5. and the quotient is 29 $\frac{2}{5}$ l. that is 29 l. 8 s. which I set aside thus.

l.	s.	d.
29	8	0.

Secondly for the 2 s. being $\frac{1}{10}$ l. I diuide Numero by 10. and the quotient is 14. $\frac{7}{10}$ l. that is 14 l. 14 s. which I subscribe directly vnder the former quotient, that so it may be the more readie for Addition thus.

l.	s.	d.
29.	8.	0.
14.	14.	0.

Thirdly for 3 d. which is $\frac{1}{4}$ (not of a pound as the rest were, but) of a shilling, I diuide Numero by 4. and so it quoteth 36 $\frac{3}{4}$ s. as it did before, which I also subscribe vnder the former quotients thus.

l.	s.	d.
29	8	0.
14	14	0.
1	16	9.
45	18	9.

Finally adding all these quotientes into one summe, as the rule bidden me, the totall quotient or aunswering number is 45 l. 18 s. 9 d. euen iust as it was before by your worke.

Philo. It is so, and yet he that is skilfull and readie in these aliquot partes, may conclude the answere sooner, vsing onely but 2. operations: for the giuen price may be deuided into these 2. aliquot partes, onely here described.

5 s + 15 d.

$\frac{1}{4}$ l. $\frac{1}{12}$ l.

Whereof the first as you see is $\frac{1}{4}$ pound, and the latter $\frac{1}{12}$ l. as appeareth by the fractions subscribed.

So that if (for the 5 s.) Numero be first diuided by 4. the quotient wilbe 36 l. 15 s as it was in my first example, and then for 15 s it being diuided by 16. the quotient wilbe 9 1/2 l. that is 9 l. 3 s. 9 d. both which quotients added together make also 45 l. 18 s. 9 d. agreeable with the former operations, as here the addition it selfe sheweth.

l.	s.	d.
36	15	0.
9	3	9.
45	18	9.

Euma. I see very well the order and nature of this rule, which is such, that when it cannot answere a question at one operation, because the whole giuen price is not one iust aliquot part of pound, shilling, or peny, then it must be done at 2. or 3. operations or mo, according to the number of aliquot partes, whereunto the giuen price is diuided.

But this of all other things seemeth strange to me, that a number which indeede and veritie, is neyther pounds, nor shillings, nor pence, beeing diuided by the aliquot part of a pound, should bring true pounds, and being diuided by the aliquot part of a shilling, should bring true shillings, and being diuided by the aliquot part of a peny, should bring true pence in the quotient, and yet I see manifestly it doth so, & finde by the golden rule direct, that the answers are true euery way, but the reason thereof I can neyther perceiue nor deuise.

Philo. Although I purpose not to spend any time to demonstrate the reason of any rules in this booke, yet is the reason of this rule very apparant, euen by the 154. verse of my first booke, as for example in the first question of this booke, who knoweth not that 544. ells of holland at 2 s. 6 d. the ell, will cost 544. halfe crowns, wherefore if the same 544. halfe crownes be reduced to pounds, by the said verse, they will certaynly yeeld 68. l. iustly agreeable with the answer of that question, but the diuision of Numero by 8. the denominator of 1/4. l. is nothing els but a secret reduction of the same 544. from halfe crownes vnto pounds, whereby they presently turne vnto 68. l. the like whereof is to be vnderstoode in all other questions of this rule, wherevppon followeth (as a propertie generall incident thereunto) this maxim insuing.

A generall propertie incident to the Merchants rule.

6. Diuiding Numero, by part of a pound,
Pounds in the quotient are presently found,
And then the remaynder, if any abounde,
Must needes be accepted for parts of a pound,

7. Diuiding Numero, by part of a shilling,
Shillings come shyning the quotient fulfilling.

And if ought remayne as some may vnwilling,
Then is that remaynder still partes of a shilling.
8. Diuiding *Numero* by part of a peny,
The quotient yeeldes pence, a few or els many,
And then the remayner if there be any,
Doth alwayes expresse the partes of a peny.

The effect of which maxim appeareth manifestly by your owne woorking in the last question, for first when you there diuided Numero 147 r. by 4 s. as by ¼ r. the quotient yelded you pounds presently: For as Numero was pounds, so was the quotient, which brought foorth 29 r. and there remayned ¾ which was ¾ r. but when you afterward diuided Numero by 3 d. as by ¼ of a shilling, then came the quotient filled with shillings onely, for you had therein 36 s. and there remayned ¼, which was ¼ s. onely, or 9 d. and so likewise when Numero is diuided by any aliquot part of a peny, the quotient shalbe pence, and the remainder parts of a peny.

Euma. All this I vnderstand welinough by your former speech, and it is the onely thing whereat I meruepled.

6. A question of Sugar

Philo. So then if you aske me what Numero 80 r. of Sugar will cost, at 15 d. the r. and I for answere thereof take this 15 d. the giuen price wholy for ¼ r. as indeed it is, then by the fyrst verse of this propertie, and by that I haue sayd before, I need do no more but diuide my Numero 80. by 16. the denominator of the aliquot part, and the quotient presently bringeth 5 r. for the aunswering number, shewing that so much the 80 r. of Sugar will cost.

But if I beeing ignorant that 15 d. is such an aliquot part of a pound, do therfore expresse the giuen price, in such aliquots part as I know, whereof one peraduenture may be the aliquot part of a pound, and the other an aliquot part of a shilling, thus as here you see.

1 s. + 3 d.
¼ r. ¼ s.

Where the 1 s. is taken for ¼ r. and the 3 d. is taken but for ¼ s. as appeareth, yet doth it not hinder the veritie of the woorke, as you haue seene heretofore by former examples, but onely it breedeth somewhat more labour, because fyrst I must diuide Numero by 20. whereupon (by the first verse of this propertie) there commeth 4 r. in the quotient: and secondly I must diuide Numero by 4. whereupon (by the second verse of this propertie) there issueth 20 s. in the quotient: and thirdly I must adde both quotients together, whereby amounteth 5 r. iust equall to the former aunswere.

Againe, if I set downe the giuen price altogether in aliquot parts

of

of a shilling, as I may also be thus.

6 d + 6 d + 3 d
1/2 s. 1/2 s. 1/4 s.

Now although this may seeme a disaduantage in worcking, to make 3. aliquot parts, where I neede make but 2. yet may it sometimes bcrue (yea and very often doth) a more speede, and no lesse ease to the worker, then if there were but 2. aliquot parts. as for example, to aunswere the former question by these aliquot partes of a shilling, for the fyrst part, I diuide Numero by 2. and the quotient is 40 s. which I set aside thus.

2 l. 0 s. 0 d.

Which done, now sith the next diuisor is 2. also, as it was before, I neede make no new diuision, but subscribe ẏ selfe same quotient again vnder the former thus.

2 l 0 s 0 d
2 0 0

And for the third part sith it is 1/2 of 6. pence, the quotient thereof must needes be 1/2 of the former, wherefore without any mo diuisions, I also subscribe 1/2 thereof, to wit, 20 s. or 1 l vnder the former quotients thus.

2 l 0 s 0 d
2 0 0
1 0 0

Finally these 3. sums added together, make 5 l. also agreeable to ẏ former answers as appereth.

5 0 0

For this is generally to be noted in this rule, if the giuen price be diuided into aliquot partes, whether the same parts be but of one kinde of money or of diuers, yet looke what proportion one of those partes beareth to other in valure, such proportion shall the quotients of their diuision beare together, as for a further example take this. Note.

A Merchant deliuered French crowns Numero 3540. at London by exchange, to receiue for them in Andwerpe 7 s. 6 d. a peece Flemish, the question is how many pounds Flemish will pay the billes of exchange. For answer hereof, sith 7 s 6 d. the giuen price taken wholy, is no aliquot part of a pound, I set it out in these aliquot partes which here you see. 7. A question of Exchange.

5 s + 2 s. 6 d.
1/3 l 1/8 l.

Where it appeareth quickly, that the latter part is iust 1/8 of the first (for 2 s. 6 d. is 1/8 of 5 s. whereby it followeth that the quotient of the last part, shalbe iust 1/8 of the first, & therefore after that I haue (for the first part) diuided Numero by 4. & found the quotient to be 885 l. forthwith (without any further diuision) I subscribe 1/2 thereof for the latter part, and so adding both together, I haue 1327 l. 10 s for the true answere, without further labour as here appeareth. And so many pounds Flemish will pay the billes of exchange.

885 l. 0 s. 0 d.
442 10 0
1327 10 0

3. A question of Veluet.

So likewise at 17 s. 6 d. the yarde of Veluet, what will Numero 458. yards cost. For answere hereof I set downe the giuen price in aliquot parts thus.

10 s. + 5 s. + 2 s. 6 d.
½ r. ¼ r. ⅛ r.

Where I may soone see that the second part is ½ of the first, and the third part ½ of the second, wherefore after that by diuision of Numero by 2. I haue found the quotient of the first part to be 229 r. forthwith without further labor, I take ½ thereof for the second part, and ½ of the second for the third part, whose total is iust 400 r. 15 s. and so much will the 458. yardes of Veluet cost, as here appeareth, where euery figure of the whole worke is playnely expressed.

		r.	s.	d.
458	229	0	0	
zzz	114	10	0	
	57	5	0	
	400	15	0	

Euma. This I vnderstand well, and perceiue it to be no lesse agreeable to reason, then compendious in worcking, and by the same reason I see also, that if in this example you had begun to worcke for the least aliquot part first, diuiding Numero by 8. because it is ⅛ r. then would the quotient thereof haue bene 57 r. 10 s. which then being doubled for the second part (because the same part is noble to the least) would be 114. r. 10 s. and that, by like reason being likewise doubled for the greatest part maketh 229 r. total of which 3. sums, would likewise make 400 r. 15 s. agreeable with the former answere as heere the whole tablique thereof also declareth.

		r.	s.	d.
2	57	5	0	
458	114	10	0	
88	228	20	0	
	400	15	0	

Philo. That is very true, for the operations differ in nothing but in priorite and posteriorite of worcking, wherby although the quotients or numbers varie in situation, yet differ they nothing in values or significatiõ, but are y^self same they were before, wherby the answere must consequently be the same it was before, but the best way is to beginne alwayes with the greatest aliquot part first, as I did.

Euma. Well then I hope I vnderstand the forme and practise of this rule hereto reasonably well, but to finde readily what aliquot parts of pound, shilling, or penny, the giuen price is, and to know which of them are most apt to be applied to the present question, is it that troubleth me most, and seemeth to me to be the greatest difficultie.

Philo. And euen so it is, wherefore you shall haue especiall memoratiue rules for that purpose to helpe you therein anon, but first

first I must rehearse another exception belonging to this rule, which is thus.

9. And when the price known is iust 20. shilling,
Or any sum els exceeding a pound,
Then cannot diuision although it were willing,
Without other helpe still serue for a ground,
For euery iust pound craues multiplication,
But all vnder pounds diuision doth fashion. 4

As for example I aske this question. If 1 peece of Reisons cost 24 s. 10 d. what wil Numero 130. peeces cost? here it is manifest that the price knowen or geuen is more then 1. pound by 4 s. 10 d. And therefore being taken wholy, cannot by reason be any aliquot part of a pound wherfore by this verse, such diuision onely as hath bene vsed heretofore, will not serue here now, without other helpe, for euery iust pound, of the knowen price, craueth multiplication by the number of poundes conteyned in the same price. But all the rest vnder a pound is framed and fashioned by diuision after the former order. 9. A question of Reysons.

Wherfore to worke accordingly sith I see there is but 1 entire pound in 24 s. 10 d. the giuen price, I multiplie my Numero 130 r. by the same 1. and it continueth 130 r. still, which I set aside as a parcell of the answering number thus. 130 r. 0 s. 0 d.

Then for the 4 s. 10 d. residue of the price, (seeing it is vnder 1. pound) I vse the former order, diuiding Numero first by 5. because 4 s is $\frac{1}{5}$ r. and the quotient yeeldeth 26 r. which I subscribe vnder the former product thus.

	r.	s.	d.
	130	0	0
	26	0	0

And lastly I diuide Numero by 24. because 10 d. is $\frac{5}{12}$ r. and the quotient bringeth 5 $\frac{5}{12}$ r. this 5 r. 8 s. 4 d. which I subscribe to the former quotientes, and so finally adding aswell the product as the 2. quotients together, the totall or aunswering number is 161 r. 8 s. 4 d. and so much will the 130. peeces of reason cost as here appeareth.

	5	8	4
	161	8	4

So likewise if you should aske me, what is the price of Numero 145. hogsheads of oyle at 5 r. 12 s 6 d. the Hogshead. 10. A question of Oyles.

For answere I must consider (by this verse) that the giuen or knowne price contayneth 5. entire poundes, wherefore as the same verse importeth, I multiplie my Numero 145. by the same 5 r. and the product is 725 r. which I forthwith set a part as a par-

cell of the answering number thus. 725 l. 0 s 0 d

Then for the 12 s. 6 d. remayning, because the same is vnder a pound, I set out the aliquot parts therof thus.

10 s + 2 s. 6 d.		
½ l.	⅒ l.	

For the first part, whereof I diuide Numero by 2. and the quotient is 72 l. 10 s. which I subscribe vnder the former product thus.

725 l	0 s	0 d
72	10	0

Then for the latter part, I diuide Numero by 8. and the quotient is 18 l. 2 s. 6 d. which I subscribe vnder the former quotient, and so adding all together, aswell the product as both the quotients the totall aunswering number is 815 l. 12 s. 6 d. as here appeareth.

725 l	0 s	0 d
72	10	0
18	2	6
815	12	6

And so much will the 145. Hogsheads of Oyle cost.

Euma. By these examples I doe not onely vnderstand the meaning of the whole rule, and both the exceptions clearely inough, but also I see further, that when the giuen price is euen pounds onely, without any odde shillings or pence, then is there no diuision at al to be vsed in this merchants rule, but Numero is to be multiplied only by so many vnits as there are pounds in the giuen price.

Philo. That is very true, and so this present exception affirmeth in effect. And therefore, if 1. tunne of Iron cost 10 l. it is easie to knowe that 60. tunnes wil cost 600. l. And so likewise, if 1. barrel of Herringes cost 20 s. which is 1. euen pound, who is so ignorant of numbring or rather so voyd of reason, but can say that Numero 254 barrels will cost 254. pound.

Wherby it followeth also that when the giuen price is 1. entire peny, 1. entire shilling, or 1. entire pound iust, then is Numero it selfe the true answere to the question, being accompted eyther for pence, shillings, or pounds, as the price requireth.

Now to prepare this Merchants rule readie,
All aliquot parts of shilling and pound,
Are made here in meeter to keepe in minde steadie,
To vse assoone as occasion is found,
VVhereof no one part no sooner is named,
But straight his diuisor followes fit framed.

The partition of a pound of money into aliquot parts.

1. shil. 11 A shilling 20. serues, A grote by sixtie carues, 1. grote.
A peny 200. and 40. 1 d.

2. ſhil. 12. 2. ſhillings 10. requeſt, 2. grotes thinks 30. beſt. *2. grotes.*
2. pence, an 100. and 20. 2 d.

3. ſhil. 13. 3. ſhil. fits emptie, 3. grotes throws foorth 20. *3. grotes.*
3. pence, eate eightie vp. 3 d.

4. ſhil. 14. 4. ſhilliugs fauour 5. 4. grotes for 15. ſtriue. *4. grotes.*
4. pence, with ſixtie ſup. 4 d.

5. ſhil. 15. 5. ſhillings 4. coſen, 5. grotes a browne doſen. *5. grotes.*
5. pence felled fortie eight. 5 d.

6. ſhil. 16. 6. ſhillings ſlip aſide, 6. grotes by 10. diuide. *6. grotes.* } 6
6. pence, for fortie waite. 6 d.

8. ſhil. 17. 8. ſhillings emptie goe, 8. grotes do likewiſe ſo, *8. grotes*
8. pence thinkes 30. plentie. 8 d.

10. ſhil. 18. 10. ſhil. tender twaine, 10. grotes 6. entertaine *10. grotes.*
10. pence takes 4. and 20. 10 d.

Noble. 19. A Noble thriues with 3. a crowne & 4. agree *a Crowne*
And ⅓ a crowne with 8. *⅓ a crowne.*

15. d. 20. 15. pence at the laſt, ſeekes out 16. in haſt.
To make the pound full weight. 15 d.

The partition of a ſhilling into his aliquot parts.

21 A farthing firſt findes fortie eight,
An halfepeny hopes for twentie foure.
22 Three farthings ſeekes out 16. ſtreight,
A peny puls a dozen lower.
23 Dicke dandiprat drewe 8. out deade,
Two pence tooke 6. and went his way,
24 Tom trip and goe with 4. is fled,
But goodman grote on 3. doth ſtay.
25 A teſterne onely 2. doth take,
Moe parts a ſhilling cannot make. } 7

The partition of a peny into his aliquot parts.

26. A peny parted by this lore,
one myte deuides by twenty fower,
27. Two mytes by 12 and 12 by twayne.
Three mites by 8. and backe agayne,
28. Foure mytes by 6. and 6. by 4.
A peny hath not one part more. } 8.

In the firſt of theſe partitions, you may ſee one pound or 20 s. deuided into 19. ſeuerall aliquot parts, whereof none is equall to other.

And in the second you may see a shilling diuided into 9. seuerall aliquot parts whereof each are also vnlike to other.

And in the third partition you may see a peny diuided into 7. seuerall aliquot parts, whereof each differeth from other. In euery of which partitions you may also see, that there is no aliquot part sooner named, but by and by his diuisor followeth at his heeles.

As first in the partition of the pound, you see that 1 shilling is no sooner named, but 20. followeth imediately, seruing for the diuisor thereto, because a shilling is $\frac{1}{20}$. l. agayne a grote is no sooner named, but foorthwith 60. followeth as fit diuisor thereto, because it is $\frac{1}{60}$ l. likewise a peny is no sooner named, but imediately 240. followeth for diuisor thereto, as that which is $\frac{1}{240}$ l. and so foorth of the rest.

Euma. So then I perceiue that 2. s requeste 10. for diuisor because it is $\frac{1}{10}$ l. and 2. grotes thinke 30. best for diuisor, because it is $\frac{1}{30}$ l. and 2. pence taketh 120. for diuisor because it is $\frac{1}{120}$ l. and so of the rest, but what meane you by 3 s. sitting emptie.

Philo. The meaning thereof is nothing els, but that 3 s. being no aliquot part of a pound, hath no diuisor, but is put in onely to keepe the order of the shillings (where any other coine of that number is to be expect) as 6 s. 8 s. and 8. grotes likewise are, whereof the one is sayd to slip aside, and the other to go emptie.

And as for all the rest, as it is manifest inough of it selfe, so I perceiue you vnderstand it wellinough, the like whereof is to be vnderstoode in the partition both of the shilling, and also of the peny.

Euma. I may easily gesse so, and therefore as a farthing is $\frac{1}{48}$ s. so it first findeth 48 for diuisor, next whom a halfepeny being $\frac{1}{24}$ s. hopeth for 24 for diuisor, and so foorth of the rest, wherein I see no difficultie, for I perswade my self that the rest of the woords besides the numbers, are vsed onely but for meeters sake, that so the matter may be the more apt to be retayned in memorie.

Philo. It were folly to thinke otherwise, and therefore you must haue those verses by hart as perfectly as your Pater noster, If you minde to be prompt and readie in the merchants rule, wherfore to quicken your remembrance aswell by sight, as by hearing, I haue heere annexed a table of all the sayd aliquot partes with their diuisors in another order, for heere the greatest part is first, and so each after other in order decreasing vnto the least, which is last.

Beholde

Beholde the Table.

The Aliquot parts of a pound.				The aliquot parts of a shilling		The Aliquot partes of a peny.	
Where the denominator of euery aliquot part, is the number for the same part.	$\frac{1}{2}$ £	10.s.	$\frac{1}{10}$	12.d	$\frac{1}{2}$ s	6 d	$\frac{1}{2}$ d 12.mites
	$\frac{1}{3}$	6.s 8.d.	$\frac{1}{12}$	10.d	$\frac{1}{3}$	4 d	$\frac{1}{3}$ 8.mites
	$\frac{1}{4}$	5.s.	$\frac{1}{15}$	8 d	$\frac{1}{4}$	3 d	$\frac{1}{4}$ 6.mites
	$\frac{1}{5}$	4.s.	$\frac{1}{20}$	6.d	$\frac{1}{6}$	2 d	$\frac{1}{6}$ 4.mites
	$\frac{1}{6}$	3.s. 4.d.	$\frac{1}{25}$	5.d	$\frac{1}{8}$	1½ d	$\frac{1}{8}$ 3.mites
	$\frac{1}{8}$	2.s. 6.d.	$\frac{1}{30}$	4.d	$\frac{1}{12}$	1 d	$\frac{1}{12}$ 2.mites
	$\frac{1}{10}$	2.s.2.d ½	$\frac{1}{40}$	3.d	$\frac{1}{16}$	½ d	$\frac{1}{24}$ 1.mite.
	$\frac{1}{12}$	2.s.	$\frac{1}{50}$	2.d	$\frac{1}{24}$	½ d	
	$\frac{1}{15}$	20 d.	$\frac{1}{120}$	1.d ½	$\frac{1}{48}$	¼ d	
	$\frac{1}{16}$	16.d.	$\frac{1}{240}$	1.d			
	$\frac{1}{18}$	15.d.	$\frac{1}{480}$	¼ d			

Euma. This table maketh all things exceeding plaine, and in it I finde mo aliquot parts of the pound then were rehearsed in the 6 Marim: but yet I purpose to occupie the verses of that Marim, with the 2 other following it, more then this table.

Philo. You may do therein as you thinke best. But heere take this note also by the waye, that in all questions subiect to this rule, the greater aliquot parts that you take, the fewer shall you neede to make of the giuen price, and the fewer aliquot parts that you make of the giuen price, the fewer operations of diuision shall you neede to vse, whereby of consequence, you shall the sooner come to your conclusion. Note.

As for example. At 11.s.3.d. the yarde of Damaske, what is the price of Numero 294 yardes? Now if I for answere heer vnto should diuide the giuen price thus. 10 s. + 1 s. + 3.d. (as some peraduenture would do) you may perceiue forthwith by these 3. parts, that eyther there must be 3. seuerall diuisions made, or els for the second part I must take $\frac{1}{10}$ of the first quotient, and for the third part, I must take $\frac{1}{4}$ of the second. But I considering (both by the 6. Marim, and also by this table) that the giuen price needeth to bee parted but onely into 2. aliquot parts thus. 10 s. + 15 d. / $\frac{1}{2}$ £ $\frac{1}{10}$ £. Do make but onely 2. diuisions for the answering of the question, diuiding Numero 11. A question of Damaske.

first

first by 2. and secondly by 16. and so are the seuerall quotients, 147. l. and 18 l. 7 s. 6 d. both which added together, make 165 l. 7 s. 6 d as here appeareth. And so much is the price of the 294. yardes of Damask.

l	s	d
147	0	0
18	7	6
165	7	6

Eu. This note appeareth resonable, and I perceiue it well, remembering also by the 21. verse of your first booke, that that aliquot part is greatest, whose denominator is least.

Phi. It is well remembered, and now because I hope you haue all your aliquot parts perfectly in memory by this time, answer me to this question. Prunes being sold for 1 d. ½ the pound, what will Numero 1560. l. of suttle weight cost.

12. A question of Prunes.

Eu. To answer you, I finde by the 23 verse of the 7. marim that ½ d. the giuen price here is ½ s. wherfore I diuide Num. by 8. & the quotient is 195. which by the 7. verse of this chapter is shillings, and by reduction to grosser maketh 9 l. 15 s. and therfore so much will the 1560 l. of Prunes cost, but how may I be sure that this answer is true?

The triall.

Philo. Very easily by changing your aliquot partes, for if the answer be alwayes one, you may be assured the first answer is true: because truth neuer disagreeth with it selfe, wherfore now worke for a peny first, & then for the halfpeny: and so proue if the former answere returne, for if it do, it argueth the worke to be good, and the former answere to be true.

Eu. Then because I finde that a peny is ½ s. I diuide my Num. 1560. by 12. & the quotient is 130 s. & now for the ½ d. I perceiue by your former instruction, that sithe it is iust ½ of the former part, I need not to diuide Num. any more, but may take ½ of the quotiēt already found, which is 65. s. & adde it to the same quotient, & so is the totall answering number 195. s or 9 l. 15 s. as it was before, wherefore I affirme my former answer to be true, as by the whole worke heere appeareth.

		s	d	mi.
30	1560	130	0	0
1222		65	0	0
xx		195	0	0

Phi. You may be bold so to do, and it was both better & sooner done, to take ½ of the first quotient heere (as you did) then to make a new diuision by 24. as otherwise you must needs haue done: but now can you answer me to this question? At 3 s. 6 d. the yarde of Kersey, what will Nume. 256 ½ yards cost?

13. A question of Kerseye.

Euma. No verily, not by this rule, for I haue not seene Numero to consist of any mixt number before now.

Phi:

Philo. Then will I shew you presently how you shall answere both it and all other like questions, and it is thus: first looke how your Numero in this rule is esteemed or accompted, and so must you esteeme euery fraction therof after like rate, for when Numero is accompted for pounds, then is ½. of euery vnit therin to be accompted for 10. s and ⅓. for 6. s. 8. d. and ¼. for 5. s. and so forth of euery other fraction.

And againe, when Numero is accompted but for shillings, then is ½. of euery vnit therein, to be esteemed but for 6. pence ⅓. for 4 d. ¼. for 3. d. ⅛ for 2 d and so forth.

But if Numero be accompted for pence, then is ½ to be reckoned but for 12. mites, ⅓. for 8. mites, ¼ for 6. mites &c.

Euma This computation or estimation is both true and reasonable in mine opinion.

Phi. Then to proceed to the worke: first I set forth the aliquot parts of the giuen price thus. Which being all aliquot parts of a pound, assure me that Numero must be accompted for pounds, and the ½ for 10. s and so is my whole Nu. 256. l. 10. s. this done, I worke for the first part diuiding Nu. by 10. and the quotient is 25. l. and 6. l. remayneth, which reduced to shillings, make 120. s. whereto now I must add the 10. s. percell of Nu. which done, the totall is 130 s. which diuided also by 10. according to the 4 verse of this chapiter, quoteth 13. s. iust without any remaynder, so that the whole quotient for the first part is 25. l. 13. s. which now I haue found and know (seeing that each of the other parts is halfe his next fellow) I adde ½ thereof for the second part, and ½ of that second part for the third, and so haue I 44 l. 17 s. 9 d. for the answering nũber, as here appeareth by the addition.

2 s. + 1 s. + 6 d.
½. l. ⅓. l. ¼. l.

l	s	d
25	13	0
12	16	6
6	8	3
44	17	9

And so much will the 256 ½ yardes of Kersey cost. Wherefore now if by this example you vnderstand the effect of that which I haue said, then answere me to this question.

Spanish Taffata being solde at 3. s. 4. d. the yarde, what will Numero 256 ½ yardes cost? 14. 4 question of Taffata.

Euma. For answer, first heere I call Numero 256 l. 6 s. 8 d. as your doctrine importeth. And as for the giuen price, I see I neede make no parts of it, because it is a iust aliquot part of a pound, to wit ½ l. wherfore I diuide Numero by 6. and the quotient is 42 l. and there is a remainder of 4 l. left, which reduced to shillings make

make 80. s. whereto by your former instruction I must now add the 6. s remayning of Numero, and so the totall is 86. s. which eftsoones diuided by 6. the common diuisor quoteth 14. s. and yet there remaineth 2 s. which as I coniecture by the 4. verse, I think must be reduced to pence.

Philo. They must needes be so, which done you must add vnto them the 8. d. yet remaining of Numero, and then diuide the total by your first common diuisor, and so must continue reduction and diuision still, till either nothing remayne, or els that you can diuide the remainder no further, for so ỳ same 4. verse willeth & requireth.

Eu. Then to go forward, I reduce the same 2. s. to 24. d. whereto adding the 8. d. remayning of Numero, the totall is 32. pence which diuided also by 6. quoteth 5. pence, and yet there is a remayner of 2 pence left, which I can diuide no further.

Philo. No can, why so? doth not that verse byd you (if neede be) to reduce the remaynder to mites.

Eum. That is true, but because I haue not beene vsed to so small coynes, I assure you I neuer thought of them, wherefore now reducing the same 2. pence to 48. mites, I diuide their sum by 6. and so the quotient bringeth 8. mites iust without any remainder, wherefore finally it appeareth, that the 256 $\frac{1}{2}$. yardes of Taffata, will cost 42. l. 14. s. 5. pence. 8. mites.

Proofe or Triall. Philo. You haue answered truly, and for proofe thereof you may diuide your giuen price into 2. or 3. aliquot parts, and so working by this rule, if you haue done well, the selfe same answer shall returne euery way.

Eu. Very good, that will I do by my selfe at leasure.

15 A question of Coparose. Phi. Then answer me now to a little question which is this. At 3. d. 5. mites the pound of Coparose, what will Nume. 17 $\frac{1}{2}$ l. cost?

Eu. To answer this, assoone as I haue (for my more ease and better memory) set foorth the giuen price in the aliquot parts therof thus, I see the greatest aliquot part, is but $\frac{1}{4}$. of a shilling, wherefore I am to accompt Numero to be but 17. s. 6. pence, whereof I diuide the 17 s. first by 4. and it quoteth 4. s. and there remayneth 1. s. which I reduce to 12. pence, adding thereto the 6. pence parcell of Numero, so is it in all 18. d. which I also diude by 4 and it quoteth 4 pence, & there remayneth 2 pence, which reduced to mites risseth to 48. mites, and the same mites being eftsoones diuided by 4. (the common diuisor for this parte) quoteth

d	mites	mite.
3	4	1
$\frac{1}{4}$ s.	$\frac{1}{8}$ d.	$\frac{1}{12}$ d.

quoteth 12.iuſt without any remainer, and ſo the whole quotient for the firſt part is 4.ſ.4.d.12.mites, which I ſet aſide in ſome voide place thus:

ſ	d	mites.
4	4	12

Now for the ſecond part, I diuide Numero by 6. and the quotient is 2.ſ. and there remaineth 5.ſ.

Philo. Nay, you are greatly deceiued, for touching the ſecond and third parts of your price, as they are not now aliquot parts of a ſhilling, but of a peny: ſo by the third and eight verſes, you muſt now change your eſtimation of Numero accordingly, accompting it to be but pence, and not ſhillings, whereby the quotient ſhall be agreeable, as the ſaid 8. verſe affirmeth, and ſo muſt you do as often as the aliquot parts of the price change, from one coyne or valure to another.

Euma. I confeſſe my fault, I ſee I ſhould eſteeme Numero now to be but $17\frac{1}{2}$ d. that is 17.d.12.mites. Wherfore to begin where I left, I diuide Numero by 6, and the quotient is 2 d. and there remaineth 5.d. which I reduce to mites, and they yeeld 120. whereunto I adde the 12.mites, now parcell of Numero, and ſo the totall is 132.mites, which I alſo diuide by 6. and the quotient is iuſt 22. without any remainder, and ſo haue I for the quotient of the ſecond part onely 2.d.22.mites, which I ſubſcribe vnder the former quotient, ready for addition thus.

ſ	d	mites.
4	4	12
0	2	22

Finally, for the laſt part, I diuide $17\frac{1}{2}$ d. by 24. according to the gloſſe of the 78. verſe of your firſt booke, one- ne by drawing a lyne betweene them thus. $\frac{17\frac{1}{2}}{24}$

So is the quotient $17\frac{1}{2}$ mites, which I ſubſcribe to both the former quotients, and ſo adding them all together, the totall aunſwering number is 4 ſ. 8.d. 3.mites $\frac{1}{2}$. as heere appeareth by the addition, and ſo much will the $17\frac{1}{2}$ r. of Copperoſe coſt.

ſ	d	mites.
4	4	12
0	2	22
0	0	$17\frac{1}{2}$
4	8	$3\frac{1}{2}$

Philo. You haue ſaid trueth, and done well now, but you were about to haue gone awaye.

Euma. I aſſure you this pelting queſtion hath troubled mee more then I thinke a queſtion of farre greater valure would haue done.

Philo. Perhaps ſo, but I haue a queſtion in my minde which peraduenture would trouble you worſer, and therefore I will worke the anſwere my ſelfe before you, for your better inſtruction in all

other

other like, which if you marke well, may suffice aswell as a multitude of examples, and it is thus.

16. A question of Pepper.

A marchant bought Pepper Numero 2579 ¼ ℔. suttle at 2.ß.2.d 7.mites the pound weight, now the question is, what the whole Numero cost him?

For answer whereto, first I set out the parts of the giuen price thus, whereby I consider that Numero heere, is like to haue 3. seuerall kindes of estimations, because that of the aliquot parts, one is of a pound, another of a shilling, and the rest are of pence.

ß.	d.	mites	mites
2	+ 2	+ 4	+ 3.
½ ℔.	½ ß.	½ d.	½ d.

Wherefore first esteeming Numero, to be 2579.℔. 15.ß. I diuide it by 10. and the quotient is 257. ℔. and there remayneth 9.℔. which reduced to shillings yeeldeth 180 ß whereto, I add the 15. shillings remayning of my Numero, and so amounteth 195. ß. which also diuided by 10. quoteth 19.ß and there remayneth 5 ß. which reduced to 60. d. I eftsoones diuide by 10. and the quotient is 6. d. iust, wherefore I set aside all the coynes obteyned for this part thus.

℔	ß	d
257	19	6

Now comming to worke for the second part. I accompt Numero but for 2579.ß.9.d. diuiding it by 6. and it quoteth 429.ß. that is 21 ℔. 9 ß. and there resteth 5.ß. which I reduce to pence, adding thereto the 9.d remaining of Numero, so is their totall 69 d. which eftsoones diuided by 6. quoteth 11. d. and yet there remayneth 3.d which turned into 72. mites, I diuide also by 6. and so the quotient is 12. mites iust. These quotients I subscribe to the former thus.

℔	ß	d	mites
257	19	6	0
21	9	11	12

Thirdly, comming to the third parte, which is but the aliquot part of a peny, I accompt Numero now to be but 2579 d. 18. mites, which I diuide also by 6. and it quoteth 429.d. or 1.℔.15.ß.9.d. and there remaineth 5.d. which I reduce into mites, adding to them the 18. mites, parcell of Numero, so is the totall 138. mites, which diuided by 6. quoteth iust 23. mites; which quotients I also subscribe to the former, thus.

℔	ß	d	mites
257	19	6	0
21	9	11	12
1	15	9	23

Finally, for the last part, being of qualitie like the third, I diuide Nu. by 8. & the quotient bringeth 322 d. or 1.℔.6 ß.10.d leauing a remainder of 3 d. which I reduce into mites, adding to them also the 18. mites, parcell of my present Nu. so is the totall 90. mites, which I diuide also by 8. & the quotient is 11 ½ mites, which fraction cannot be reduced to any smaler money.

and

and therfore cannot be diuided any further, wherefore I subscribe these quotients also to the 3. former sums, adding them all together, and their totall is 282. l. 12. s. 1. d. 22 ¼ mites, as heere appeareth.

l	s	d	mites
257	19	6	0
21	9	11	12
1	15	9	23
1	6	10	11 ¼
282	12	1	22 ¼

And so much did Numero 2579 ¼ l. of Pepper cost.

Euma. Truly this is a busie question, but yet I haue so diligently obserued both it, and all your order of working thereof, that I hope I will not misse to performe the like in any other question that you shall propound.

Philo. I am glad to heere you say so. Wherefore now you shall further vnderstand, that Merchants apply this rule of theirs also, to 1. doozen, 1. grosse, 1. score, 1. quintall, and so to diuers other denominations. In doing whereof, although the first terme may seeme to exceede an vnit, yet is it but an vnit in effect, as thus.

At 6. s. 9. d. the doozen of Gloues, what will Numero 2670. doozen cost? or thus. At 14. d. the score, what will Numero 345. scores cost? or thus. At 20 d. the groace of Buttons, what will Numero 334. groace cost? or thus. At 24. s. 8 d. the quintall of woad, what will Numero 2465. quintals cost?

Euma. I perceiue that the rule may well extend it selfe to such like questions, because that as in them, the first terme signifieth not absolutelye 1. vnit, but 1. doozen, 1. score, or 1. groace, &c. So likewise Numero signifieth not barely so many vnits as it conteyneth, but so many doozens, scores, or groaces, as there are vnits in it. In which respect the first terme is but an vnit of that kinde, for otherwise I see and know the rule could not holde, as you sayd in the beginning, but I know not what a groace nor a quintal signifieth.

Philo. A Groace is 12. doozen of any thing that is vsually solde by the doozen, as of Buttons, Points, Gloues, Trenchers, Caps, Hats, and such like. And 12. of those groaces is called a great groace, that is 12. doozen doozens.

Now touching this word Quintall, it is a Spanish or Portugall worde (as I take it) but whence soeuer it is deriued it signifieth 100 l. weight, at 5. score to the 100. which is otherwise called suttle weight, or the suttle hundred.

Eu. I thanke you much, you haue satisfied me herein very well.

Philo. And euen so let this that hath beene sayd, satisfie you also for

for the marchants rule, which although some diuide into as many seuerall rules, as there are pence in a shilling, and shillings in a pound, or rather into no, yet I thinke he hath a very small capacitie, that cannot apply the same to any giuen price whatsoeuer, euen by the 4. first maxims onely of this chapter, and the examples and instructions which I haue taught you thereupon, wherefore here we will make an end therof, and now we will talke of gaines and losse in the 100. l of money.

Of Gaynes and losse in the 100. pound of money by the trade of Marchandise. Chapter. 2.

It is a common and vsuall practise among marchants, to search and know, how much they gayne or loose by euery 100. pound of money, which they disburse vpon any marchandise or commoditie, and so after the rate for a greater or lesser sum.

For your exercise wherein, some further varietie of questions and examples may be here propounded, then is mentioned in my second book, but other rules to dissolue them is there none, but the golden rule, in such order as I haue there taught you.

17. A question of the price of cloath.

As for example, if in 1. yarde of cloath retayled for 8. s. 6. d. the seller gayned after 27 ½. l. in the 100. l what did that 1. yarde cost the seller?

This question is best and most naturally answered by the golden rule direct, with an accompt proceeding, according to the order by me declared in the chapiter thereof, so that before any application of termes to the figure, the gaynes gotten by the 100. l. stocke, must be added to the same stock, so is the whole stocke and gaynes 127 ½. l. which done, I say thus: if 127 ½. l. stock and gaynes, come of 100. l. stock onely, of whom commeth 8. s. 6. d. the stock and gaines of 1. yarde of cloth? & so reducing the mixt numbers to simples, and obseruing the Golden rule, I finde the answering number to be 6 $\frac{14}{17}$ s. which fraction by abridgement commeth to $\frac{2}{7}$ s. that is 8. d. as here appeareth.

stock & gain.	stock
255 l	
x	100 l.
17	6. s. 8. d.
x	

Wherefore I conclude that a yarde of that cloath cost the Seller 6. s. 8. d at the first.

Euma. I remember this rule well, wherefore (as you say) except it be more for my exercise, then for any further learning, I hope you

you shall not neede to stand any longer about this.

Phi. Yet will I put a few questions more, to quicken you in the practise thereof, whereof this is one.

If 1. yard of cloath bought for 6 s. 8 d. be sold againe for 8 s. 6 d what gaines is that vppon the 100 l? 18. A question of gaines in the 100. pound.

Eu. As the other question required an accompt proceeding by addition, so I perceiue this requireth an accompt proceeding by subtraction, wherefore first abating the stock from the stock and gaines, I say thus, 8. s. 6. d — 6. s. 8. d leaueth 22. d. for neate gaines, wherevppon I resort to the golden rule saying thus: if $\frac{1}{3}$ l. stock procure $\frac{11}{2}$ s. gaines, what gaines shall 100. l. stock procure? and so working by the same rule in fractions, the answer is 550 s. as the fabrique here declareth.

stock	gaines.
$\frac{1}{3}$ l	$\frac{11}{2}$ s.
$\frac{100}{1}$ l	550 s.

Which by reduction to pounds is 27 l. 10. s. whereby the answere of the next question before, is iustified and confirmed to be true.

Philo. Well sayd: Now if a cloath cost me 6. s. 8. d. the yarde, at what price may I retayle it, to gaine after 27 $\frac{1}{2}$ l. in the 100. pounds disbursed? 19. A question how by retaile to gaine so much in the 100.

Euma. For as much as I perceiue plainly, that this question requireth an accompt proceeding by addition: first I say thus. 100 l. + 27 $\frac{1}{2}$. maketh 127 $\frac{1}{2}$ l. for stocke and gaines, which done, I come immediatly to the Golden rule with my termes, saying thus. If 100 l. stocke growe to 127 $\frac{1}{2}$ l. in stocke and gaines, whereunto shall 6. s. 8. d. or $\frac{1}{3}$ l. stocke growe? and the Golden rule in Fractions at the first chop, answereth $\frac{255}{800}$ l. as heere appeareth.

stock.	stock & gains
$\frac{300}{3}$ l.	$\frac{255}{2}$ l.
$\frac{1}{3}$ l.	$\frac{255}{800}$ l.

Which by abridging reduction commeth to $\frac{17}{40}$ l. that is 17. testerns, or 8 s. 6 d. and for so much must you sell the yarde to gaine after the rate of 27 $\frac{1}{2}$ l. in the 100.

Philo. Well said. Thus you see how euery of these questions approoueth the answere of other, but now tell me this. If 1. ell of Holland cloath, bought for 7. s. 8. d. be solde againe for 8. s. 6. d. what is there gained in laying out 20. pound vppon such commoditie? 20. A question of gaines in the score.

Euma. Because I see this question requireth subtraction proceeding: first I subtract the simple stock from the stock mixt with the gaynes, saying thus. 8. s 6 d. —— 7 s. 6. d. leaueth 10. d. for neate gaines, which done, I repaire to the golden rule of fractions,

saying, if 7. 1/2 s. stock bring 1/8 s. gaynes, what stock gaynes will 20 r. bring? and so reducing the termes and using the operation of the rule, I finde, the answer to be 2 4/13 r. as here appeareth.

23/2 s.	∠	1/8 s.
20 r.	—	2 4/13 r.

Which by reduction to coyne cometh to, 2. r. 3. s 5. d. 17/13. d, and so much is gotten cleere by the laying out of 20. pounds in that commoditie.

21. A question of losse by repentance.

Phi. Now put the case that A. had bought as much of this Holland cloath of B. as cost him 20. r. after 8. s. 6. d the ell, and that A. repenting of his bargaine offereth to lose 2. 4/13 r. of his principall to haue the rest in ready money, at what price may A. sell the ell to loose neither more nor lesse then his offer?

Addition of ground of arts fol. 468.

This question Maister Mellys propounded in his Arithmetick as a proofe or ratification of the answere to the last former question, and for answere thereto, he biddeth to adde the 2 4/13 pound lost, to the principall, so will it be 22 4/13 pound, which done (saith he) then say by the rule of 3. If 22 4/13 pound giue but 20 r. what shall come of 8 1/2 s.? and so working by the rule, he concludeth that you shall finde 7 s. 8. d. for the answere, which he saith is the iust price whereat A. ought to sell the ell, wherein he is greatly deceiued, which I speake not to disgrace him, whose paines hath deserued much praise and commendation, and whose louing affection towards that man of famous memory (Maister doctor Recorde) I cann ot but like well, and hartily reioyce at.

For as maister Mellys dooth, euen so do I acknowledge that Doctor Recordes bookes were the first fire whereat my simple candle tooke that light which it hath: wherfore I reuerence both the workes and the workeman, honoring the memoriall of his name, and professe vnfained loue to all such as loued him.

And therefore I protest, that my minde is not to write any thing to the derogation of any man, but onely to preferre trueth, and to reforme error, which vnto all mankinde, is as it were an accident inseperable, for the most circumspect may mistake at one time or other, and so dooth Maister Mellys herein: for certainly (in such cases) the losse ought not to be added to the stocke or principall, but it ought to be subtracted from the same.

Wherefore to answere this question exactly and truly, you must abate the 2 4/13. lost, from the 20. pound stocke, to make your second terme proportionall, and so resteth 17 9/13 pound, which being done, now may you come to the Golden rule, saying thus: If 20. pound

stocke

stocke decrease to 17 $\frac{19}{21}$ pound, whereto shall 8. shillings 6. pence, or $\frac{17}{40}$ pound decrease, and so reducing the mixt numbers to simples, and working by the Golden rule of fractions, the answer in least termes is $\frac{697}{1184}$ l. as heere the figure and fabrique declareth.

stock.	decrease.
$\frac{20}{1}$ l.	$\frac{419}{21}$ l.
$\frac{17}{40}$	$\frac{697}{1184}$ l.

Which by reduction to money is 7 $\frac{11}{21}$ s. or 7 s. 6. d. $\frac{21}{21}$ pence, and therefore (say I) for so much must A. sell euery ell of his cloth, to loose neither more nor lesse then 2. $\frac{4}{21}$ pound in his 20. pound disbursed, and not for 7. s. 8 d.

And for triall thereof, I aske this question of the Golden rule. If A. bestowed 20 l. in Holland at 8. s. 6. d the ell, how many ells had he for his 20 l? & it answereth me 47 $\frac{1}{17}$ ells, as here appeareth.

pound.	elles.
$\frac{17}{40}$	1
20 l.	47 $\frac{1}{17}$

22. A question what number of ells are bought.

Whereupon I inferre a second question, saying againe by the same rule. If 1. ell solde, yeelde $\frac{697}{1184}$ pound (that is 7. s. 6. d. $\frac{21}{21}$ d. as I affirmed before) then what shall 47 $\frac{1}{17}$ ells yeelde in sale? and the rule answereth 17 $\frac{19}{21}$ pound, as heere appeareth by the fabrique.

elles.	pound.
$\frac{1}{1}$	$\frac{697}{1184}$
$\frac{800}{17}$	17 $\frac{19}{21}$

23. A question of the price of cloath.

And so this answere verifieth my former answer, touching the price of the ell, for if the whole 47 $\frac{1}{17}$ elles of Holland (which did cost A. 20. pound) yeelde him but 17 $\frac{19}{21}$ pound, at that price, then hath he lost cleerely by the sale, 2 $\frac{4}{21}$ pound, as he was contented to do, for 17 $\frac{19}{21}$ + 2 $\frac{4}{21}$ pound, make iust 20. pound, whereby I conclude as before, that my former price of the ell is true.

A like error I finde also in maister Humfrey Bakers Arithmeticke, where he demaundeth this question. If 1. yarde of cloath of Tyssue be solde for 3. pound 15. shillings, wherein is lost after 10. in the 100. what did 12 $\frac{1}{2}$ yardes thereof coste? for solution whereof he willeth to adde $\frac{1}{10}$. of 3. pound 15. shillings the giuen price, vnto the same price, whereby the totall shall be 4. pound, 2. shillings. 6. d. Then (saith he) seeke by the Golden rule, whereto the 12 $\frac{1}{2}$ yardes will amount after 4. pound, 2. shillings, 6. pence the yarde, and you shall finde that they will come to 51. pound, 11. shillings, three pence, and indeede so dooth it, as heere appeareth by the figure.

yardes.	pound.
$\frac{1}{1}$	$\frac{21}{5}$ l.
$\frac{25}{2}$	5 l. 11 s. 3 d

And so much saith he, did the 12 $\frac{1}{2}$ yardes cost. But by his fauor it is not so, wherfore if I shall endeuor to reforme

24. A question of Tissue at the first peny. Wellspring of Sciences fol. 127.

and amend his error herein, I trust he wilbe no more offended with me, then I wilbe with him to redresse any like error committed by me (if any such be herein) for if any error or mistaking happen in computation, or in the Printing, it may be borne withall, but the martms, rules, precepts, & principles of the art, ought alwayes to be true, certayne, and infallible.

Wherefore with his friendly favor, I deny his rule, for I see no reason, why $\frac{1}{10}$. of the price, should be added to the whole price to make the first antecedent in the golden rule, except by such addition, he thinke to finde the first price or principall, which 1. parde did cost, whereof $\frac{1}{10}$. is lost, when there is 10. lost in the 100.

But $\frac{1}{10}$. of the 9. parts remayning after such losse, is not $\frac{1}{10}$ of the principall, for if you hauing an 100. l. in your purse, should casually loose $\frac{1}{10}$. thereof, what should you then haue remayning?

Euma. I should haue left 90. pounds.

Philo. You say truth, now if I would adde more thereto $\frac{1}{10}$. of 90 l. to restore your losse, what should you haue then?

Euma. Then should I haue 99. l. for $\frac{1}{10}$. of 90. l. is but 9 l.

Philo. You should haue so, but now you see that 99. pound lacketh money of 100. l. so that yet you lack 1. pound of your first sum.

Euma. I cannot deny that.

Philo. Truly euen so heere, when $\frac{1}{10}$ of the price which brought the losse is added to the totall price, it dooth not iustly make the first price which the parde cost, but wanteth some parcell thereof, for which cause (I say) the sequell of the worke cannot be exactly true: And therefore as I taught you before, I say againe, that in questions of losse, the losse is not to be added, but to be subtracted from the principall.

Wherefore sithe heere it is affirmed, that 10. pound is lost in the 100. pound, I subtract the same losse from the principall saying, 100. pound, —— 10. leaueth 90. pound. Then come I to the Golden rule, saying thus. If 90. pound be the remainder of 100. pound, of whome is 3. pound, 15 shil. the remainder?

and so reducing the mixt termes to simples, the opperation of the rule bringeth for answere, 4 $\frac{1}{2}$ pound, as heere the figure expresseth.

pound.	flocke.
$\frac{21}{7}$	$\frac{100}{7}$ l.
$\frac{15}{3}$	4 $\frac{1}{2}$.

Which

Which by reduction is 4 r. 3 s. 4. d. and therfore, so much (say I) did euery yarde of the Tyssue cost.

This done, I say by the same rule the second time, if 1. yard did cost $4\frac{1}{8}$ r. what did $12\frac{1}{2}$ yardes cost? and the rule executed, presently answereth $\frac{625}{16}$ pound as heere appeareth by the fabrique.

yardes.	pounds.
$\frac{1}{1}$	$\frac{25}{8}$
$\frac{25}{2}$	$\frac{625}{16}$

which reduced to coyne yeeldeth 52. pound, 1. s. 8 d. wherefore I auouche that so much did the $12\frac{1}{2}$. yards cost, and not 51 r. 11 s. 3. d. as maister Baker affirmeth.

And yet truly otherwise I do willingly confesse maister Bakers Arithemeticke, to be a worke worthy of greate comendations and of very ingenious, subtill, and rare inuention, especially for merchants and merchandise, wherefore to say as the Philosopher said, *Amicus mihi* Socrates, *& Amicus* Plato, *sed magis Amica veritas*. So likewise, I professe my selfe to be friend to maister Baker, and friend to maister Mellys, but most friend to the trueth. But now to returne to our purpose from whence vnawares I haue digressed.

Aristot. 6. cap. primi libri Ethicorum.

If a cloath cost me after $28\frac{1}{2}$ s. the yarde, for how much may I sell 10. yardes thereof, to gaine after 8 r. 6. s. 8 d. in the 100 r. stock disbursed?

15. A question of gaines in the 100.

Euma. Because the question is of gaines, I adde together the 100 r. stock, and the gaines thereby required to be gotten, which maketh $108\frac{1}{3}$ r. Then say I by the Golden rule, if 100 r. stocke, yeelde $108\frac{1}{3}$ r. for stock and gaines, what will $28\frac{1}{2}$ s stock yeeld? and so working by the rule, I finde $30\frac{7}{8}$ s. for the answer, as heere the fabrique sheweth.

stock.	stock & gain.
$\frac{100}{1}$ r	$\frac{325}{3}$ r.
$\frac{57}{2}$ s	$30\frac{7}{8}$ s.

Secondly (for I see this question is plurall) I say againe: If 1. yarde ought to yeeld $30\frac{7}{8}$ s. in stock and gaines, what ought 10. yardes to yeelde? and by following the operation of the rule, I finde $308\frac{1}{4}$ s. as heere the fabrique declareth, which by reduction to money is 15 r. 8 s. 9. d.

yardes.	stoc. & gain
$\frac{1}{1}$	$\frac{247}{8}$ s.
$\frac{10}{1}$	$308\frac{1}{4}$ s.

And therefore I conclude, that for so much must he sell his 10. yardes, to gaine after 8 r. 6 s 8 d. in the 100 r.

Philo. You haue said well. Now it is so, that a merchant selling certaine cloath for 15. r. 8. s. 9. d. gained neate by the sale 23. s. 9. pence: wherefore I would know what gaines that is in the 100. pound?

26. A question of gaines in the 100.

Euma. Here although the question be of gaynes and not of losse, yet I see the accompt proceeding the golden rule is to be made by subtraction, wherefore first I must abate the 23 s. 9 d. neare gaines from the 15 l. 8 s. 9 d. which containeth both the stock and gaines together, that so the remaynder being onely stocke, may appeare simplie, which done the stocke sheweth it selfe to be 14 l. 5 s. serving for the second terme in the golden rule, vnto whose figure now I applye the termes, saying thus. If 14 ½ pound stocke, get 23 ½ shillings gaines, what will 100 pound stocke get? And the rule aunswereth 166 $\frac{13}{17}$ s. as here appeareth.

stocke.	gaines.
5 l.	95 s.
100 l.	166 $\frac{13}{17}$ s.

Which reduced, maketh 8 l. 6 s. 8 d. confirming the aunswere of the last former question.

27. A question what price a yarde cost at the first peny.

Phi. Well yet for further approbation hereof, I aske this question, if in 10. yards of cloath retayled, for 15. l. 8. s. 9. d. there was gayned after the rate of 8 $\frac{1}{3}$ l. vppon the 100. what did a yard of that cloath cost the retayler?

Euma. I perceiue this question requireth plurallitie of proportion, wherefore for my first accesse vnto the golden rule I say thus, if 10. yardes yeelde 15. l. 8. s. 9. d. in principall and gaine, what doth 1. yard yeelde? & the rule aunswereth 30 $\frac{7}{12}$ s. as heere appeareth by the figure.

yardes.	stoc & gain
10	12 $\frac{5}{12}$ s.
1	30 $\frac{7}{12}$ s.

And for my second accesse, I adde the stock and gaines together, saying thus. If 108 $\frac{1}{3}$ l. stocke and gaines, came of 100. pound stock, of whome came 30 $\frac{7}{12}$ s. stocke and gaines. And by working with the rule, I finde 28 $\frac{1}{3}$ s. as heere appeareth.

stoc & gain.	stocke.
325 $\frac{1}{3}$ l.	100 l.
247 s.	28 $\frac{1}{3}$ s.

And so much did the yarde cost the retailer at the first peny.

38. A question how to gaine $\frac{1}{3}$ of the principall.

Philo. All this is very well and artificially aunswered. Now therefore, If 20 ½ yardes cost me 36 l. 10. s. at what price may I retaile the yarde to gaine $\frac{1}{3}$ of my principall?

Euma. For aunswere, first I adde the stocke and gaines required together, that is to say, I adde $\frac{1}{3}$ of 36 l. 10 s. which is 12. l. 3. s. 4 d. to the same 36 l. 10 s. & so the total is 48 l. 13 s. 4 d. which done, now I appose ỳ golden rule, saying thus. If I must sell 20 ½ yards for 48 $\frac{1}{3}$ l. for how much must I sell 1. yarde? and it aunswereth, that I must sell 1. yarde for 2 l. 8 $\frac{16}{141}$ s. as heere appeareth,

yardes.	pounds
81 $\frac{1}{4}$	146 $\frac{1}{3}$
1	2 l. 8 $\frac{16}{141}$ s.

Philo.

Philo. Item, if A. buye a peece of cloath for 8 s. 8 d. the ell, whereof he selleth 10 1/3 ells by great, for 5 l. 6 s. 8 d whether doth he win or loose, and how much vpon the 100. pound. 29 A question whether gaine or losse succeede.

Euma. Herein you aske me two questions, which cannot both haue solution at once, wherefore I perceiue the same belongeth also to plurall proportion: and for aunswere thereof, first I seeke by the golden rule in fractions, what the 10 1/3 ells solde did cost A. saying thus. If 1. ell did cost 8 1/3 s. what did 10 1/3 elles cost? and it aunswereth 91 s. or 4 l. 11 s. as heere appeareth.

elles. shillings.
1/1 — 8 1/3
10 1/3 — 91 1/3 Z 4 l. 11 s.

This being thus knowne to be the principall stocke, which the 10 1/3 ells did cost A. at the first. I abate the same from the 5 1/3 pound, receiued for the sale thereof (which conteineth both the stocke and gaines) and so resteth 1 l. 2. s. 4 d. for neate gaines. After this I vse the golden rule the second time, saying thus, if 4 l. 11 s. stocke bring 1 l. 2 s. 4 d. gaynes, what gaines doth 100 l. bring? and the rule aunswereth 24 l. 10 s. 310/173 s as heere the figure sheweth, which by further reduction commeth to 24. l. 10 s. 10 d. 10/51. d. and so much dooth A. gaine in the 100. l: after that rate.

91/1 s — 69/3 s.
100/1 l Z 24. 10 310/173 s.

Phi. Well then: If 12 1/3 pardes, elles, pounds, or peeces of any thing, cost the buyer 11. pound 5. shillings, who retaileth the same againe for 16. shillings the parde, ell, pound, or peece: whether dooth hee winne or loose, and how much in euery pounds woorth? 30. A question what is gained or lost in euery pounds woorth.

Euma. For aunswere, first I seeke by the Golden rule, what the whole 12 1/3. whether they be pardes, elles, &c. will yeelde at 16 s. as they are retayled, saying thus. If 1. yeelde 16. s. what will 12 1/3 yeelde? and I finde 200. shillings, as heere appeareth.

1 — 16 s.
12 1/3 — 200 s. Z

That is 10 pound. But seeing they cost 11. pound, 5. shillings, and yeelde but 10. pound, it is manifestly apparant, that the retayler loseth 1. pound, 5. shillings, by retailing at that price.

Now secondly, to knowe how much thereof is lost in euerye pound, I say thus. If 11 1/2 pound loose 1 1/2 pound, what dooth 1. pound loose. And the Golden rule presentlye aunswereth me, 1/11 pound, that is 1/5 pound, or 2. shillings, 2. pence, 2/3 pence, and so much dooth hee loose in euery pounds woorthe of that commoditie, as heere appeareth.

1/4 l — 1/4 l.
1 l. Z 1/11 l.

31. A question of gaines or losse by changing the price.

Philo. Well done. Now if a merchant in selling 100. pound weight of any commoditie for 4. pound, loose thereby after 10. in the 100. whether should he loose or gaine, if in case he had solde the 100. pound weight for 4. pound 10. shillings?

Euma. For answere hereunto, because the question is of losse: first I abate the 10. lost in the 100. from the same 100. so resteth 90.

Then taking the same remainder for second terme in the Golden rule, I say thus. If 4. l. returne but 90. for 100. what will 4 ½ l. returne? and the rule aunswereth 101 ¼ l. as heere appeareth.

pound.	pound.
4	90
4 ½	101 ¼.

which aunswering terme, seeing it exceedeth 100 by 1 ¼ pound, argueth plainly, that a: 4 l. 10 s. the Merchant had gained so much in the 100. to wit, 25. s.

Philo. Thus you see how vse and exercise, maketh that light and easie, which otherwise would seeme troublesome. And now heere is come to my minde one other rule touching gaines & losse vpon the 100. pound, which I my selfe for lacke of vse had almost forgotten, and it is thus.

29. To know what gaines in the 100. dooth growe,
When by retaile the price dooth rise or fall:
Adde the first gaines got by the 100. so,
vnto that 100. diuiding the totall,
By that same price, whereby such gaines was got,
whose quotient let, the latter price infolde,
So dooth it fall to the product by lot,
To tell you as much as needes to be tolde.
30. For looke how much the same is more or lesse
Then an 100. iust, so much is got or lost,
Whereof th'ouerplus, the gaines dooth expresse,
But the lack shewes, the losse of that it cost.

} 9

This rule or Maxim serueth, when a man knowing what he gaineth or looseth in the 100. pound, at one price, would know also what he should gaine or loose in the 100. at another price, as for example, I take the 51. question of the second booke, which was this in effect.

32. A question of gaines or losse in the 100. by change of price.

A Merchant retailing Salte at 8. d. a bushell, knew that thereby he gayned neare after 10. in the 100. but afterward he raised his price to 10. pence a bushell: wherefore it is demaunded

what

what he gayned in the 100 l. by that latter price.

For answere whereof I follow the prescript of this rule, adding 10 l. the first gaynes gotten by the 100. vnto the same 100. and so it is 110 l. which totall I diuide by that price which got this gaynes, that is by 8. pence, so is the quotient $\frac{110}{8}$ which quotient by this maxim, I must infold by 10. pence the latter price, and it produceth $\frac{1100}{8}$, which as this verse affirmeth, is as much as needeth to be told, for now looke how much this product $\frac{1100}{8}$ is ouerplus more then 100. So much is gotten in the 100. by 10. pence the latter price, but if the same be lesse then 100. then is there so much lost vpon the 100. by the latter price, as the same product wanteth of 100.

Wherefore I reduce $\frac{1100}{8}$ by distinct reduction, and it commeth to $137\frac{1}{2}$, which is more then 100. by $37\frac{1}{2}$. whereby I conclude that by the latter price, the merchant gayneth after $37\frac{1}{2}$ vpon the 100. how say you, vnderstand you this.

Euma. Yea I thinke so.

Philoma. Then admit that this merchant, hauing sold parcell of his salte at 8. pence a bushell, had afterwardes by ill market beene driuen to sell the rest but for 7. pence the bushell, whether had he then gayned or lost by that latter price, and how much vpon the 100. 33. A question of losse or gaynes by change of price.

Euma. You shall soone see by my worke, whether I vnderstand the effect of this maxim rightly or not, which to follow as neare as I can, first I adde 10. the gaynes gotten by the first price, to the 100. So is the totall 110 which totall I diuide by 8. the price that got that gaynes, and the quotient is $\frac{110}{8}$. and this quotient multiplied by 7. the latter price produceth $\frac{770}{8}$ which by distinct reduction is but $96\frac{1}{4}$, so that it lacketh $3\frac{1}{4}$ of 100. wherefore if I vnderstand the maxim rightly, the merchant by the latter price looseth after $3\frac{1}{4}$ in the 100. pound of his principall, or of that which the salt standeth him in.

Philomathes. He doth so indeede, and I perceiue you vnderstand the rule so well, that I neede not to stand any longer about this part thereof, wherefore I will go forward with the rest which is thus.

31. But if the first sale bring losse, mine aduise,
Is that from the 100. that losse you abate,
Diuiding the rest by that very price,
VVhich bred the losse by selling at that rate, } 10

VVhose

VVhose quotient then, the latter price infolding,
Shewes the thing sought for to all mens beholding

34. A question of gaynes or losse by change of price.

As for example, admit that this salt Merchant, had sold at the the first for 7 ß. the bushell, and so had lost in the sale by that price, $3\frac{1}{4}$ vpon the 100. and yet afterwardes the market amending, he had sold the rest for 8. pence, now the question is whether he winne or loose by the same price, and how much in the 100.

Here you see that 7 ß. the first price bringeth a losse of $3\frac{1}{4}$ in the 100. wherefore as this verse bidden me, I first abate the same losse from the 100. saying 100—$3\frac{1}{4}$ leaueth $96\frac{1}{4}$ that is, in simple termes $\frac{385}{4}$ which I diuide by 7. the price that brought or bred that losse, and the quotient is $\frac{385}{28}$ which infolded in 8. the latter price produceth $\frac{3081}{28}$. the very effect of that which I seeke, for by distinct reduction it commeth to 110. arguing manifestly that the 10. ouerplus aboue the 100. is now clerre gaynes gotten in the same 100. by 8. pence the latter price.

Euma. I thinke I vnderstand this verse present, also by this your example, for triall whereof I will with your fauour vse the 50. question which you propounded in the second booke which was this.

A marchant selling Wheate at 13 ß. the Alkeare, found that he lost after 9. in the 100. by that price, but afterwardes the market waxing worse and worse, he sold the rest for 12 ß. onely the Alkear, wherefore it was demaunded what he lost vpon the 100. by that latter price.

35. A question of gaines or losse by change of price.

Now to aunswere this question, by this rule present, because the first price or sale bringeth losse, I say thus 100 —9. leaueth 91. and 91. diuided by 13. pence (the price which brought the losse) quoteth 7. which multiplyed by 12. the latter price produceth 84. which product lacketh 16. of 100. and therefore so much dooth the Merchant loose vppon the 100. by his latter price.

Philo. He doth so, and thus you see how iustly the conclusions of the second booke, and this rule present agree together, for euen such answere as you found there, such you finde here.

Now let me see how by this rule you can answere the 31. question of this booke, and so we will end it.

The 31. question answered otherwise.

Euma. That question was thus, if a merchant in selling 100 ℔. weight of any commoditie for 4 ℔. loose thereby after 10. in the 100. whether should he winne or loose, if in case he had sold it for

4 ℔.

4 £. 10 S. and how much in the 100.

And for answere thereto, (because the the first price bringeth losse) I am to vse the order prescribed by this present verse, wherefore to doe accordingly, I abate the 10. lost from the 100. and so the remaynder is 90. pound, which I diuide by 4 £. the very price that brought that losse, and the quotient is $\frac{3}{4}$ which quotient infolded in 4 $\frac{1}{4}$ pound the latter price, produceth $\frac{45}{4}$ and that by distinct reduction turneth to 101 $\frac{1}{4}$ which beeing more then 100. by 1 $\frac{1}{4}$. pound, declareth that so much had beene clearely gayned in the 100. pound if the sayd 100. weight had beene sold for 4. pound 10. shillinges.

Philo. You haue sayd truth, and wrought very orderly, wherefore although many mo questions and examples might be diuised to like effect, yet sith inough is as good as a feast. Here we will end both this rule and chapiter, and will (as I may) go forward with other rules, which also concerne gaynes or losse in the 100 £. of money, but because they depend chiefly vpon time (which naturally requireth an other rule precedent, I will first acquaint you with that rule which (for a difference from the rest) I call by speciall name the Equation of time, the forme whereof insueth.

Of the Equation of sundrie times or dayes of payment, to one day or time. The 3. Chapiter.

23. To bring diuers dayes for payment assign'd,
To one onely time proportionally,
First make each payment appeare in his kinde,
As part of the whole, and then by and by,
Reducing the times to firnames alike,
Fold euery payment in his proper time,
VVhose products added, yeelds that which you seeke,
So iust that you cannot accuse it of crime. } 11

As for example, a marchant is to pay 1000. pound at 3. seuerall times or termes, to wit 500 £ at 5. weekes, 300 £ at 3. monethes and 200 £. at a yeere, the question is at what time the totall debt ought to be payd at once, keeping still the first proportion of time appointed to the 3. seuerall paymentes. 36. A question at what time diuers paiments shal be payd together.

For aunswere hereunto, this verse willeth me first to make each payment appeare in his kinde as part of the whole, that is to say,

I must first set foorth euery payment in such fraction or part of the totall as it contayneth or is.

Which (by the 55. and 56. maxims of my first booke) is soone done by diuision, for if I diuide the totall 1000 l. by 500 l. which is the first payment, the quotient is 2. and so by the sayd 55. Maxim 500 l. is $\frac{1}{2}$ of the totall.

Secondly diuiding the same totall by 300. which is the second payment the quotient is $\frac{10}{3}$ Wherefore (by the sayd 56 maxim) the same 300 l. is $\frac{3}{10}$ of the totall.

Lastly diuiding the same totall by 200 l. the third payment, there issueth 5. in the quotient, which by the 55. maxim aforesaid, sheweth the same payment to be $\frac{1}{5}$ of the totall debt.

So these 3. seuerall paymentes appearing now in their kinde as part or partes of the total thus, $\frac{1}{2}$ $\frac{3}{10}$ $\frac{1}{5}$, the next thing to be done is to reduce all the times to like sirnames, but the sirnames of the times here mentioned, is weekes, moneths, and yeeres, wherfore I reduce them all to weekes which is the least sirname expresst in the question: wherein I take the 5. weekes as they are, but for the 3 moneths I take 12. weekes, & in stead of 1. yeere I take 52. weeks which done (thē as this rule prescribeth further bidde-th me) I fold euery payment in his proper time, that is $\frac{1}{2}$ in $\frac{1}{2}$ weekes, $\frac{3}{10}$ in $\frac{12}{1}$ weekes and $\frac{1}{5}$ in $\frac{52}{1}$ weeks, and their seuerall products are $\frac{1}{2} + \frac{12}{5} + \frac{52}{5}$ weeks, all which products added together make 16 $\frac{1}{2}$ weekes, being the very iust time for which I seeke, for at the end of 16 $\frac{1}{2}$ weekes, the whole 1000. pounds ought to be payd at once, according to the first proportion of time limited for the 3. former seuerall payments.

Euma. Your playne example hath so lightned this Maxim or rule, that I thinke I vnderstand both the meaning of it, and also the explication and practise of it.

37. A question to like purpose with the former.

Philo. Then aunswere me to this, a Marchant hath 800 l. to pay, whereof he must pay $\frac{1}{2}$ in readie money, $\frac{1}{3}$ at 2. moneths, $\frac{1}{5}$ at 4. moneths, and the rest at a yeere, the question is, if he would pay all at once, what time ought to be giuen him.

Euma. For answere hereof I follow this rule, which bideth me first make euery payment appeare in his kinde as part of the whole, but all the paymentes herein mentioned appeare in that forme alreadie, sauing the last, wherefore to find what the last payment is (as reason willeth me) I subtract $\frac{1}{2} + \frac{1}{4} + \frac{1}{5}$ from the totall, and so resteth $\frac{1}{20}$ for the last payment.

Thus

Thus all the partes appearing in such forme as the rule requireth, now must I by and by reduce the times to like surnames, but they are so alreadie, sauing the first and the last, whereof I can easily reduce the last (which is 1. peere) to 12. monethes, but the first payment hath no time at all as farre as I can see, so that now without your helpe, I wot not what to do with it.

Philo. As for the readie money that is payd in hand, it must neuer enter into the question at all, for you know that the time which ought to be giuen to the debter, is giuen onely in respect of such paymentes as require time of forbearance, which heere are the 3. last payments onely, for as you your selfe do confesse, there is no time assigned to the first payment, but it is to be payd presently, extempore, without any time of forbearance, wherefore there is no reason to seeke any day or time of payment, for that portion which is knowne alreadie to be payable presently.

Euma. Very good, I must needes acknowledge it to be true which you say, wherefore now to proceede, I omit the first payment quite, multiplying each of the other paymentes, by his owne time, that is to say, $\frac{1}{4}$ by $\frac{3}{12}$ monethes, $\frac{1}{3}$ by $\frac{4}{12}$ monethes, and $\frac{1}{12}$ by $\frac{12}{12}$ monethes, so are their seuerall products $\frac{3}{48} + \frac{4}{36} + \frac{12}{144}$ monethes, all which added together make $\frac{19}{48}$ monethes, that is $3\frac{1}{2}$ monethes, which by this rule is the iust time that ought to be giuen to the debter for payment of the whole 800 l. at once.

Philo. It is so, and this is one of the rules especially that cannot be wrought by any thing taught you in my 2. former bookes, wherefore note it well, and for your better exercise therein, take a question or two more, whereof the first is this.

38. A question at what time diuers paiments are payable together.

A. oweth to B. 600 l. whereof he is bound to pay 100 l. at 2. monethes, 150 l. at 4. monethes, 200 l. at 8 monethes, and the rest at 12. monethes, now A. being loth to stand so long in bond, and yet willing to haue the full vse of the money as long as he might, would pay it all at once, whereunto B. agreeing, the question is, how long time he ought to forbeare the totall sum to receiue it altogether?

Euma. For answer, I do as the rule present biddeth me, which is first to make euery payment appeare in his kinde, as part of the whole, for performance whereof, I diuide 600. the totall debt by 100. l (the first payment) and the quetient is 6 wherefore by the 55. maxime aforesayd, the same payment is $\frac{1}{6}$. of the totall.

&c.

Secondly the same totall being diuided by 150 r. (the second payment) quoteth 4 wherefore by the same maxim that payment is $\frac{1}{4}$ of the totall.

Thirdly I diuide the totall by 200. pound (which is the third payment) and the quotient yeeldeth 3. so arguing it to be $\frac{1}{3}$ of the totall.

Lastly the same totall beeing diuided by 150. (which is all the rest of the debt remayning for the fourth payment) there issueth 4. in the quotient, which by the sayd 55. maxim, sheweth the same payment also to be $\frac{1}{4}$ of the totall.

This being done, I should by and by reduce the times to like surnames, but here needeth no such reduction, because they are all of one surname alreadie (viz. moneths) wherefore to proceede, I multiplie euery of these fractions or parts of the total seuerally, by the proper time thereto assigned, that is $\frac{1}{2}$ by $\frac{1}{1}$ moneths, $\frac{1}{4}$ by $\frac{4}{1}$ months, $\frac{1}{3}$ by $\frac{8}{1}$ monethes, and $\frac{1}{4}$ by $\frac{12}{1}$ monethes, and their seuerall products are $\frac{2}{2} + \frac{1}{1} + \frac{8}{3} + \frac{12}{4}$ moneths, all which added together make 7. intire moneths, wherefore I conclude, that at 7. moneths end A. ought to pay P. the whole 600. pound.

Philo. Your conclusion is true, wherefore answere me to this one question more, and then we will go forward with the next rule, my question is this.

31. Another question to like effect

A Merchant is to pay 400. pound in forme following, to wit, 90. pound in hand presently, 160. pound at the end of 20. dayes, and the rest at 4. monethes (reckoning 30. dayes to a moneth) wherefore I demaunde at what time both the latter payments ought to be payd at once.

Euma. For answere to this, I omit the first payment vtterly as you taught me before, because it is payde extempore, or without time, seeking after the former maner, to finde what part or partes the other payments are of the totall.

So by the 56. maxim aforesayd, I find 160. pound the second payment to be $\frac{2}{5}$ thereof, and by subtracting the 2. first paymentes from the totall, I haue the remaynder left for the last payment, to be 150. pound which by the sayd 56. maxim is $\frac{3}{5}$ of the totall, and so the paymentes being thus proportionated to the totall, now by the present rule it followeth, that I must next reduce the times to like surnames, which times here are named to be 20. dayes, & 4. monethes, wherefore accompting 30. dayes to a moneth, I reduce the same 4. monethes to 120. dayes, which done, I multiplie the

the second paymeut, viz. $\frac{3}{7}$ by $\frac{30}{7}$ dayes, and $\frac{4}{7}$ the last payment, by $\frac{120}{7}$ dayes, and the seuerall products are 8. and 45. which added together make 53. dayes, at the end of which time both the sayde paymentes (by this rule) ought to bee payde together.

Philo. They ought so. That is at the end of 1. moneth, 3. weeks and 2 dayes. And thus much for this rule of the equation of tyme may suffice, wherfore nowe we will go forth with other rules, touching gaynes and losse in the 100. r. in order following.

Of gaynes or losse vppon the hundred pound in respect of one onely day or time of payment. The 4. Chapter.

33 Questions of gayne or losse in the hundred,
By one day or time, do sometimes require,
Plurall proportion, whereat some haue wondred,
That no other rule can thereto aspire,
Sith that sometimes the golden rule compound,
Direct or conuerst doth answere them round. } 12

As for examele, If a Retayler, selling 1. yard 1. ell, 1. pounde weight or 1. peece of any thing, for 2. s. 6. d. to be payde at 3. moneths, did gayne thereby after the rate of 50. r. vpon the 100. in a yeare or 12. moneths, I demaund what that yard, ell, pound, weyght, or peece, did cost the Retayler at the first. 30. A question of the first price by reason of the gaines gotten.

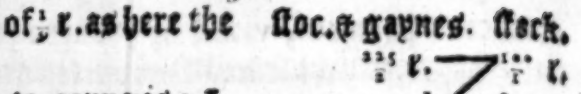
This question I assure you cannot be answered neither by the golden rule compound directly, nor conuersedly, nor otherwise then by the Golden rule plurallye, that is at twice, wherein also the time must be first begun withall, as I shewed you in the 45. question of my second booke.

So that first I must say thus, if 12. monethes gayne 50. r. what shall 3. monethes gayne, and the rule aunswereth $12\frac{1}{2}$ r. as heere appeareth.

monethes	pounds
12	50
3	$12\frac{1}{2}$

Which done, I must adde this $12\frac{1}{2}$ pound gained in the 3. moneths, to the 100. r. stocke which gayned it, to make the first terme of my second proportion, so haue I $112\frac{1}{2}$ r. for the first antecedent in that proportion wherin to proceede, now I say thus, if $112\frac{1}{2}$ r. stock & gains together come of 100 r. stock or principal, of whom came 2 s 6 d which containeth both ỹ stock & gains together of the thing retailed, & the golden

golden rule aunswereth of $\frac{1}{5}$ l. as here the figure sheweth.



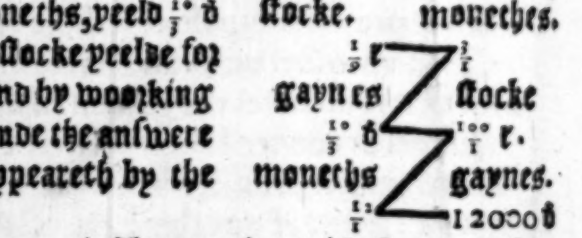
Which by reduction to coyne is 2 s. 2 d. $\frac{2}{5}$ d. and so much did the same 1. yarde, ell, pound, or peece cost him that retayled it.

And yet will I moue 4. questions more, which grow as it were twigs all out of one branch, which I doubt not but you can readily aunswere by some one rule of compound proportion: the first whereof is thus.

21. A question of gaynes in the 100. by the lak.

If 1. yarde, ell, pound weight, or peece of any thing cost $\frac{1}{5}$ l. which afterwardes is sold agayne for 2 s. 6 d. payale at 3. moneths, what gaynes is that vpon the 100. in 12 monethes.

Euma. Indeede I can answere this by the compound rule direct, after that I haue take the difference of the first price from the latter (for so the neate gaynes may appeare) which done, I finde the gaines to be 3 $\frac{1}{5}$ d. wherefore now I come to the same rule, saying thus. If $\frac{1}{5}$ l. stock in 3. moneths, yeeld $\frac{1}{5}$ d gaynes, what shall 100 l. stocke yeelde for gaynes in 12. moneths, and by woorking as the rule requireth, I finde the answere to be 12000 d. as here appeareth by the fabrique.



Which by reduction is 50 l. iustly agreeing with the conclusion of the former question.

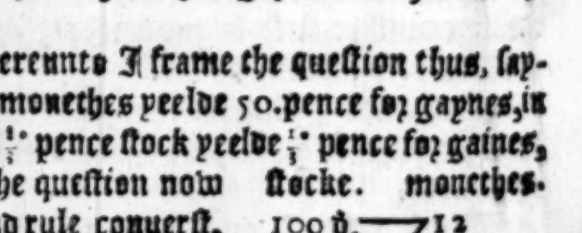
Philo. You say truth, then secondly answere me to this.

32. A question What time & price will gayne $\frac{1}{2}$. in $\frac{1}{2}$ on the 100.

If 1. yarde, ell, pound, or peece of any thing, cost the buier $\frac{1}{5}$ l. readie money, which he retaileth againe for 2 s. 6 d. how long time may he giue for payment thereof, and yet gayne after 50 l. in the 100. for 12. monethes.

Euma. For answere hereunto I frame the question thus, saying if 100 d. stocke in 12. monethes yeelde 50. pence for gaynes, in how many monethes will $\frac{1}{5}$ pence stock yeelde $\frac{1}{5}$ pence for gaines, and I see presently that the question now is subiect to the compound rule conuerst, by whose operation I finde 3. moneths, as here the fabrique declareth.

33. A question how to gayne $\frac{1}{2}$. in $\frac{1}{2}$. by the 100. in 12. moneths.



Wherefore I conclude, that so long time the retayler may giue to the buyer.

Philo. Well then thirdly, if 1. yard, ell, pound weight, or peece of any thing, cost the buyer 2 s. 2 d. $\frac{2}{5}$ d. that is $\frac{1}{5}$ l. for how much more

more must he retayle the same, giuing 3 moneths day of payment to gayne after 50. in the 100. for 12. months.

Euma. This question I put to the compound rule in this order saying thus: if 100. l. stock in 12. moneths bring 50. l. for gaynes, what shall $\frac{1}{3}$. l. stock gayne in 3. moneths? and so working by the compound rule direct (to whome it appeareth subiect) I finde the aunswering terme to be $\frac{1}{72}$. l. as this figure sheweth, which by reduction to coyne is 3 $\frac{1}{3}$ d. agreeable with the former conclusions, wherefore the saide 1. yarde, ell, pound weight, or peece, must be retailed for so much more then it cost, to gaine after the rate required, that is, for 2 s. 6 d.

stock.	moneths.
100 l.	12.
gains.	stocke.
50 l	$\frac{1}{3}$ l.
more	gaines.
3	$\frac{1}{72}$ l.

Philo. Well yet fourthly I aske this one question more. If 100 l. stock in 12. moneths, bring 50 l. for gaines, what stocke in 3. moneths will bring $\frac{1}{72}$ pounds for gaines?

34. A question what stock will bring the gaines appointed.

Euma For aunswere hereunto, assoone as I haue annexed the termes to the figure of the compound rule, in their due places, I espie it to be subiect to the compound rule conuerse: wherefore following the order of worke belonging to that rule. I finde $\frac{1}{3}$ l. for the aunswering number, as here appeareth, which is the stocke that in 3. moneths will gaine $\frac{1}{72}$ pound.

stock.	moneths.
100 l	12
gains	month.
50 l.	3
gains	
$\frac{1}{72}$ l	$\frac{1}{3}$ l.

And thus euery of these aunswers confirme other, as iust as can be wished, whereby the certaintie of the worke is made vndoubted: but yet as the verse present saith, I cannot choose but wonder, why the first question of this chapter cannot be answered after the like forme, except perhaps the cause be, that it is not within the compasse of the 221. nor 228. maxim of your second booke.

Phi. You haue said truth, that is the very cause thereof indeed, for if it were within any of the verses of those 2. maxims, then might it be answered either by the compound rule directly or conuersedly.

Of gaines or losse vpon the hundred pound of money, in respect of diuers dayes or times of paiment. The 5. Chap.

34. When any Merchant in buying dooth craue,
To haue diuers dayes of payment assign'd,
Wherein the seller determines to haue,
Such gaines in the hundred as liketh his minde.

Then

Then must those dayes first be brought all to one,
Coequall in time, for paiment of all,
So the compound rule, directly alone,
shews what gaines vnto the stock forborne doth fall. 13

As for examples. A Merchant buyeth Pepper for 2. s. 1. d. the pound, ready money, which he may sell againe presently for reasonable gaynes, but the buyer craueth diuers dayes of payment namely, 6. moneths and 6. moneths, that is to say, he requireth to haue 6. moneths day for the payment of ½. of the money, and 6. moneths more after that, for the payment of the other halfe, that is 12. moneths for the last payment.

35. A question of Pepper solde for time.

Now the seller determineth with him selfe that if he giue time for the payment, then he will gayne after 20 l. in the 100. for a yeere, or else he will keepe it. Wherefore the question is, at what price this merchant may sell the pound of his Pepper, giuing the dayes required, and yet gaine after the rate by himselfe desired?

For aunswer whereof, as the first thing that this verse requireth, is, that both the dayes of payment be brought to one, that is to say, to finde at what time both payments are proportionablye due to be paide at once, by the equation of time, taught before in the 32. verse. So by the same verse, I finde them both to be due at 9. moneths.

For seeing the paiments heere are proportionated already (for they are expressed in parts or fractions) thus, ½ at 6. moneths and ½ at 12 moneths. Therefore I neede do no more but folde each part in his proper time, and so the products are 6/2 and 12/2, which added together, make 9. entire moneths, the very time when both payments are due together.

Which equalitie of time being thus knowne: now saith this present maxim, the compound rule direct shall shew, what gaines ought to fall or growe vnto the stocke forborne, by the supposition of the question: whereupon I come thereto, saying thus. If 100. pound stocke in 12. moneths, bring 20. pounds gaines, what gaines ought 2. shillings 1. peny stock (which is the stock disbursed for 1. pound of Pepper, and here forborne by supposition of the question) to bring in 9. moneths? and so working as the rule requireth, the aunswering terme sheweth, that he ought to gee 3/40 pound in euery pound weight, to make his gaynes after the rate of 20. pound in the 100. for a yeere, as heere this following

fabrique

fabrique testifieth.

That is by reduction $3\frac{1}{4}$ d. whereby it followeth that he must sell his Pepper for so much in a pound aboue that it cost him, which is in all, for 2 s. $4\frac{1}{4}$ d. the pound weight.

stocke.	moneths.
100 l.	12
gains.	stocke.
20 l.	$\frac{25}{144}$ l.
month.	gains.
5	$\frac{1}{87}$ l

Againe a Merchant bought Cloues at 6 s. 8 d. the pound readie money, which he selleth againe, to be payde $\frac{1}{4}$ in hand, $\frac{1}{4}$ at three moneths, $\frac{1}{4}$ at 6. moneths, and the other $\frac{1}{4}$ at 12. moneths end. The question therefore is, how much he must rayse the price in the sale of euery pound weight of his Cloues, to gaine iust after 20. in the 100. for a yeeres forbearance?

36. A question what price brings the desired gaines.

Euma. This question with your fauour, I will assaye to answere, for performance whereof, as I see that heere are diuers dayes of paiment assigned, so first I seeke by the 32. verse, at what time the 3. latter payments ought to be payde at once. For touching the first payment, being ready money payde presently, as I remember well, that you tolde me before, it must neuer enter into the question at all. So heere also I consider, that seeing this question concerneth the gaines due, aswell by the time of forbearance of the stocke, as by the quantitie thereof: therfore reason requireth, that the gaines of the forbearance rise vppon those three paiments onely which are forborne.

Philo. Your consideration is good, and according to reason and truth.

Euma. Then omitting the first payment heere also: I multiplie each of the other 3. payments seuerally by his owne tyme (as the sayd 32. verse willeth,) and the totall of their products is $\frac{21}{4}$. moneths. So shewing me the same to be the verye time when all ought to be paide at once.

Wherfore this equalitie of time being found and knowne, now I come to the compound rule direct, questioning with it thus. If 100 l. stocke in 12. moneths, bring 20 l. gaines, what gaines will $\frac{1}{3}$ l. stock bring in $\frac{21}{4}$ moneths? and the operation of the rule answereth me $\frac{7}{144}$ l. as here the figure and fabrique expresseth.

stocke.	moneths.
100 l.	12
gaines	stocke.
20 l.	$\frac{1}{3}$ l.
month	gaines
$\frac{21}{4}$	$\frac{7}{144}$ l.

That is 7. d. and so much must he raise the price in euery pound weight, aboue that it cost him, to gaine after 20. in the 100. for a yeere, giuing the dayes aforesayde:

wherefore he must sell them at 7. s. 3. d. the pound.

37. A question of gaines in the 100. by 2. daies of paimēt.

Philo. You haue answered truly, and wrought the question very well, wherefore I would see how you can answere mee to this. A Merchant bought Maces for 8 s. the pound ready money, which he solde againe for 10 s the pound, to be paid $\frac{1}{2}$ at 3. moneths, and the other $\frac{1}{2}$ at 5. moneths. The question therefore is, how much the seller gaineth vppon the 100. in a yeere, or 12. moneths, after this rate?

Euma. I will doe my good will: wherefore to begin. First I seeke at what time both these paiments ought to be paide together, which by the 32. verse I finde to be iust at 4. moneths: so this equalitie of time being once knowne, I come forthwith to the compound rule direct (as before) saying thus. If 8 pound stock in 4. moneths, get 2. shillings gaines, what gaines dooth 100. pound stock bring in 12. moneths? and the opperation of the rule answereth 75. pound, as heere appeareth. And so much doth the seller gaine vpon the 100. in a yeere or 12. moneths by the sale of his Maces.

stocke.	moneths.
8 s.	4
gains	stock.
2 s.	100 l.
mon.	gaines.
12	75 l.

Philo. Well said: Now will I put 2. or 3. questions more, which I will also answer my selfe: the first whereof is thus.

38. A question of gaynes by sale of cloathes.

A Merchant selling 50 clothes at 19 $\frac{1}{2}$ pound the peece, to be paide $\frac{1}{2}$ at 4. moneths, $\frac{1}{3}$ at 5. moneths, and $\frac{1}{6}$ at 7. moneths, gained by the sale of them after 8. pound in the hundred for 12. moneths: Wherefore I would know, how much he gained in all by the sale of the same 50. cloathes?

And for answer hereof: first I seeke at what time all the payments would be payable at once, which by the 32. verse, I finde to be at $\frac{29}{6}$ moneths. This being knowne, I come immediatly to the compound rule direct, with the rest of the question, saying thus. If 100. pound forborne 12. moneths, yeelde 8. pound gaines, what will 475 pound (the price of all the cloathes) being forborne $\frac{29}{6}$. moneths, yeeld for gaines?

And the rule answereth 15 $\frac{11}{12}$ l. as heere appeareth by the fabrique.

And so much is the neate gaynes which the Merchant hath gotten by sale of his cloathes after that rate.

forborne.	moneths.
100 l.	12
gaines.	forborne.
8 l.	475 l.
month.	gaines
$\frac{29}{6}$	15 $\frac{11}{12}$ l.

Euma. But heere I perceiue you sought the totall summe due for the cloathes, to make the first demaun-

demaunding terme of the question.

Philo. Reason it selfe teacheth so, and it is a thing soone done as you know, by multipling the 50. cloathes in $9\frac{1}{2}$. l. their owne price: wherefore I presume I neede not giue you speciall rules for euery such trifle.

As for example, another question is this. A merchant buying 30. cloathes for 6. pound the peece readie money, solde 10. of them againe for 7. pound the peece, to be paide at 3 moneths: and the other 20. he solde for 8. pound a peece, to be paide at 4. moneths: wherefore it is demaunded what he gained in the 100. for 12. moneths by these sales?

39. A question of gaines in the 100. by sale of clothes at 2. prices.

Heere because there are aswell diuers prises, as also diuers dayes of payment. There is somewhat more worke to be done, then the present rule mencioneth, which must be supplyed by the reason and discretion of the workeman.

For first the totall price which the cloathes cost, is to be sought and abated from that summe which they yelde by their sales, that so the neat gaines (being seperated from the stock and gaines confused together) may appeare by it selfe.

So that heerein, there is as it were an accompt proceeding to be vsed, which being performed, next both dayes of paiment must be brought to one equall in time (by the 32. verse.) For doing whereof, I seeke what parts the 2. payments, 70. pound, and 160. pound are of the totall 230. pound. And I finde the first to be $\frac{7}{23}$. and the latter $\frac{16}{23}$. Wherefore multiplying each of these partes by the time due to it, that is $\frac{7}{23}$, by $\frac{3}{1}$ moneths, and $\frac{16}{23}$ by $\frac{4}{1}$ moneths, the seuerall products are $\frac{21}{23}$ moneths, and $\frac{64}{23}$ moneths, whose totall is $\frac{85}{23}$ moneths, the iust time equall to both the former times limitted.

Wherefore now to finish the aunswer by the compound rule direct, I say thus. If 180. pound stocke in $\frac{85}{23}$ moneths, gaine 50 l. what doth 100. pound stock gaine in 12 moneths? and so working by the rule, I finde the aunswere $90\frac{10}{11}$ pound, as heere the figure sheweth.

stocke.	moneths.
180 l.	$\frac{85}{23}$
gains.	stocke.
50 l.	100 l.
mouth	gaines.
12	$90\frac{10}{11}$ l.

And so much doth the seller gaine vpon the 100. in 12. moneths after that rate.

40. A question what price will gayne 10. in 100. in a yeere.

Againe, another question is thus. A Merchant buying Cinamon for 9. shillings the pound ready money, would know how he

may sell the 112. l. weight thereof, to be paid ¼ at 2. moneths, and the rest at 5. moneths, and yet gaine after 10. vppon the 100. for 12. moneths?

Heere sithe the giuen price is but of 1. pound, and the question is of 112. pound, therefore the first thing to be sought, is what the 112. pound did cost the merchant, which eyther by the single golden rule, or by the Merchants rule is foorthwith found, to be 50 l. 8 s. or rather $50\frac{2}{7}$ l.

This being done. Secondly both paiments are to be brought to one in proportion of time, which by the 32. verse, is found to be due at $\frac{11}{4}$ moneths. Now these things thus knowne, the compound rule direct is lastly to do his office, to whome I say thus. If 100 l. stocke in 12. moneths bring 10 l. gaines, what gaines shall $\frac{2}{7}$ l. stocke bring in $\frac{11}{4}$ moneths? and the rule duly executed answereth $1\frac{31}{280}$ l. as here the fabrique sheweth.

stocke.	moneths.
100 l.	12
gaines	stocke.
10 l.	$\frac{2}{7}$ l.
mont.	gaines.
$\frac{11}{4}$	$1\frac{31}{280}$ l.

Which by reduction is 23 s. $1\frac{5}{7}$ d. and so much must the Marchant get in the sale of 112. l. of his Cinamon, to gaine after 10. in the 100. pound of money for a yeere, giuing the dayes aforesaid.

Wherefore, finally adding this gaines to the 50. pound, 8. shillings, which it did coste the merchant, there resulteth 51. pound 11. shillings, $1\frac{5}{7}$ pence, the very price of the 112. pound, as he must sell it, and thus much may suffice for losse and gaine vpon the 100. pound of money. Wherefore now shall follow certaine briefe rules, touching the 100. pound weight.

Of buying and selling by the hundred weight, after 112. pound to the hundred, which is the common weight of England. The 6. Chap.

35. To finde out the price of any one pound,
By naming the price of an 100, pound weight,
When 5. score and 12. in the 100. is found,
Let 15. folde the pounds of money streight
Contayned in the price, which done without deceit,
That product diuided by 7. doth showe,
To how many pence one pound weight doth grow.

As for example. At 8. pound, the 112. pound weight of any mer-

merchandise, what dooth 1. pound thereof cost?

41. A question of the price of 1. pound by so much the 100. pound.

Now heere as you see, the price of the 100. weight is named to be 8. pound, by which giuen price I would learne the price of 1. pound weight thereof: for which cause (as this verse willeth me) I folde 8. the pounds of money conteined in the giuen price, by 15, and it produceth 120. Now saith the verse, this product diuided by 7. sheweth how many pence is the price of 1. pound weight, wherfore diuiding 120. by 7. there issueth $17\frac{1}{7}$ in the quotient, which sheweth that so many pence, euery pound of that 100. weight dooth cost.

42. A question of like effect to the former.

Euma. This is very easie to doe. As for example, I will proone to finde what 1. pound is woorth, when the 112. pound is solde for 12. pound. Wherefore I multiply the same 12. pound of money, by 15. and the product is 180. which diuided by 7. quoteth $25\frac{5}{7}$ d. that is 2. s. $1\frac{5}{7}$ d. and in so much (by this rule) dooth euery pound of that hundred stand the buyer.

43. Another question to like purpose.

Philo. It dooth so: But what say you to this. At 7. l. 16 s. 8 d. the 112. pound weight, what dooth 1. pound cost?

Euma. I pray you aunswer this question your selfe for my better instruction therein.

Philo. Contented: but before I proceede therein, I must tell you that the 16 s. 8 d. parcell of the giuen price, must be changed in aliquot parts of a pound: so that for 10 s. parcell thereof, there must be taken $\frac{1}{2}$ pound, and for the 6 s. 8 d. residue, must be taken $\frac{1}{3}$ pound, which being done, then must those parts be seuerally multiplyed by 15. (the common multiplyer in this maxim) and their seuerall products diuided by 7. the common diuisor, and then finally their seuerall quotients all added together, maketh the answering number.

Euma. This worke is not much vnlike the worke of the Merchants rule, wherefore I thinke I can aunswere the question my selfe now. And for proofe thereof, I set foorth the giuen price of the hundred, in poundes, and Aliquot partes of a pound, thus. 7. l. + $\frac{1}{2}$ l. + $\frac{1}{3}$ l.

Which done, first I multiply 7. by 15. the ordinarye multiplier in this rule, and the product is 105. which diuided by 7. the common diuisor quoteth 15. d.

Secondly, I multiplie $\frac{1}{2}$ l. by 15. and the product is $\frac{15}{2}$. which diuided by 7. quoteth $1\frac{1}{14}$ d.

Thirdly I multiply $\frac{1}{2}$ r. by 15. and the product is $\frac{15}{2}$, which diuided also by 7. quoteth $\frac{5}{7}$ pence. Then finally I adde together all these quotients, to wit, 15 d. + 1 $\frac{1}{12}$ d + $\frac{5}{7}$ d. and the totall is 16 $\frac{11}{12}$ d. as heere appeareth by the worke of the addition annexed.

And so much by your saying is euery pound weight of that 112. pound woorthe.

1 + 10

16 d. | $\frac{r}{r4}$ + $\frac{5}{7}$

14

Philo. It is so, you vnderstand my saying well, and haue wrought the question as well, which course is to be kept in all other like questions, for the numbers multiplyed in this rule, must euer either be iust pounds, or els reduced to such aliquot fractions of a pound, as you did heere.

And the quotients are neuer to be accompted or taken for other but for pence, if they be integers, and for parts of a peny, if they be fractions.

36. But if by the price of one pound at most,
you seeke for the price of the 100. and twelue,
Then folding the pence, which one pound dooth cost,
But soly in seauen, least it ouerwhelue.
Let the same product by 15. be sundred,
So haue you in pounds, the price of that hundred. } 15.

Loe sir, heere is a contrarie rule to that last before, for nowe, If by the price of 1. pound weight, you seeke to know the price of the 112. pound: then by this verse, you must multiply the pence which 1. pound dooth cost, by 7. onely, who now is become the common multiplyer heere, and then must you diuide the product thereof by 15. which now is become the common Diuisor, and so haue you in the Quotient, not the pence, nor the shillings, but the pounds especiallye, that such an hundred weight will cost.

So that in this rule the money to be multiplyed, must alwayes be pence, or reduced to pence: and the number issuing in the quotient, is alwayes pounds and fractions of pounds. As for example.

44. A question what the 100. pound weight costeth by the price of 1. pound.

At 9. pence the pound weight of any thing, what will the 112. pound cost? For aunswere, I multiplye 9. pence by 7. and the product is 63. which I diuide by 15. and there issueth 4 $\frac{1}{5}$ pound, that is to say, 4. pound, 4. shillings. And so much will the 112 l. cost at that price.

So

So likewise at 2 s. the pound weight, what will the 112 l. cost: here I must first reduce this 2 s. the given price wholy into pence before I proceede to multiplication, so is it 24 d. which multiplied by 7. produceth 168. and that being diuided by 15. quoteth 11 $\frac{1}{5}$, that is 11 l. 4 s. and so much will the 112. pound cost at that price. 45. Another question to like intent

Euma. This is very easie to doe, for I perceiue it is but as it were the conuerse of the rule next before it, wherefore I presume I can do it well inough, but I pray you how should I answere the question by this rule, if it were put thus, at 3. pence $\frac{1}{2}$ 4. Wites the pound weight, what will the 112 l. cost. 46. A question of the price of the 112 pound, by the price of 1. pound.

Philo. In that case and all other like, you must reduce all your coynes or moneyes lesse then a peny into the aliquot fractions of a peny, and so set them forth with the pence thus, 3 d + $\frac{1}{2}$ d + $\frac{1}{4}$ d. and so multiplying euery part of the giuen price by 7. the common multiplier, you must diuide euery product seuerally by 15. the common diuisor.

And finally adding all the quotients together, you haue the pounds in money (if they be Integers) that the 112 l weight will cost, but if there be any fractions, they alwayes signifie fractions, or parts of a pound in money.

Eu. I did gesse euen so, by that you told me touching the last rule before, wherefore now to answer mine owne question my selfe, I set forth the giuen price of the pound weight in pence, and partes of a peny thus, 3 d + $\frac{1}{2}$ d + $\frac{1}{4}$ d. which done, first I multiplie the 3. pence by 7. and it produceth 21. which diuided by 15. quoteth 1 $\frac{6}{5}$ pound, that is 1 pound 8. shillings, which I set aside by it selfe as a parcell of the answering number as I did in the merchants rule thus.

l.	s.	d.
1	8	0

Then do I multiplie the $\frac{1}{2}$ d. by 7. and the product is $\frac{7}{2}$ which diuided by 15. quoteth $\frac{7}{30}$ l. which both by the 12. and 17. verses I know to be 4. shillings 8. pence, wherefore I subscribe that vnder the former quotient, readie for addition thus.

l.	s.	d.
1	8	0
0	4	8

Lastly I multiplie the $\frac{1}{4}$ d. by 7. and the product is $\frac{7}{4}$ which diuided by 15. quoteth $\frac{7}{60}$ pound, that is 18 $\frac{2}{5}$ d. which I subscribe vnder the former quotients, and so adde them all together thus as you see.

l.	s.	d.
1	8	0
0	4	8
0	1	6 $\frac{2}{5}$
1	14	2 $\frac{2}{5}$

Whereby it appeareth that at 3 $\frac{1}{4}$ pence and 4. Wites the pound, the 112 pound weight, will cost 1l.

1. pound 14. shillings 2 $\frac{2}{7}$ pence.

Philo. It will cost euen so as you say, and seeing you are so perfect in this rule alreadie, I esteeme it but superfluous to put any mo questions touching the same, wherefore we will proceede to other rules.

Of buying and selling by the 100. after the rate of 120. to the 100. The 7. Chapiter.

37. If by the price giuen, of 1. you would know.
The price of an hundred contayning 6. score,
Double the price of that one giuen so,
Reducing that double to pence euermore,
VVhich done, diuide that sum of pence by foure. } 16
So doth the quotient giue answere in post,
VVhat number of pounds that hundred will cost.

47 A question how by the price of 1. to know the price of 120.

This rule is very playne, as for example, at 14 d. the ell of canuasse, what will 120. elles cost.

For answere hereof (as this rule biddeth me) I double 14. pence the giuen price of the one ell, and it is 2. shillings 4. pence, which reduced to pence make 28. pence, wherefore I diuide the same 21. by 4. and the quotient is 7. that is to say 7. pounds, and so much will that hundred of canuasse cost.

Euma. As you say, I perceiue the rule is very playne and easie inough, as long as the giuen price is all entire pence, but what reason there is to make so many kindes of hundreds I cannot see, for by the 4. maxim of your first booke, I learned that 10. times 10. was 100. and that 10. hundreds make 1000. and sith that time you lately reckoned 112. pound weight for an 100. and now you account 120. for an hundred, so that by this reason, I may alwell reckon 140. or 150. for a hundred, and 30. for a score, and 14. or 15. but for a dozen, and so should neyther the name of hundred, score, or dozen, containe any number certayne.

Philoma. As I told you in the Chapiter of Reduction, that in Hauerdepoise weight onely 112. pound is allowed to the 100. So you must vnderstand, that of some wares onely, 6. scores, or 120. is allowed to the 100. as namely of Canuasse 120. elles is commonly allowed to the 100. and likewise of Herrings, Lings, Codds, Stockfishes, Nayles, and many other things, 120. is accompted

compted but for 100. and therefore such hundreds are commonly called the great hundred, or a hundred of great tale.

And although it seeme straunge vnto you, that any mo should be accompted to a score then 20. or to a dozen then 12. yet you shall vnderstand that in diuers places, and of diuers things, there are 21. accompted to a score and 13. to the dozen, which ouerplus, no doubt began at the first by agreement or offer of the seller, as an aduantage for the retayler to gayne by, to the intent that if the buier had sold as good cheape as he bought, yet should he gayne euen by the first sellers good will or liberalitie, 1. in the score, and 1. in the dozen.

Which thing afterwarde by processe of time, grew to such a custome, that now the buyer may and doth chalenge it as of dutie, in so much if that you buye a score of Wheate, Barly, or other corne here at Cychester, you shall euen by the common custome of the place and countrie haue 21. bushels for 20. and 21. quarters for 20. and so if you buye a dozen of bread, you shall haue 13. penniworth for 12 pence, and in some places you shall haue 13. poundes of candles for 12.

Euma. Then I perceiue this ouerplus aboue the iust numbers groweth rather by custome, then by law or order of Arithmeticke, but as this present rule is easie of it selfe, when the giuen price of the thing is euen pence, so I coniecture that if the price of one ell, or any other entire Unit, were pence ioyned with some lesser coynes, that then all the lesser coynes ought to bee expresst in aliquot fractions of a penie, and so the aunswere to bee concluded after the example which you taught me in the first Chapiter.

Philo. You coniecture rightly, and as the truth is, and therefore if the question were thus, at 2 $\frac{1}{3}$ d. 3. mites the stockfish, what will the 120. amount vnto. 48. A question of stockfishes.

Fyrst I would set foorth the giuen price of the 1. fish thus, 2. pence + $\frac{1}{3}$ pence + $\frac{1}{7}$ pence, and then would I double it all thus, 5. pence + $\frac{1}{7}$ pence, which doone, I diuide 5. pence by 4. and the quotient is 1 $\frac{1}{4}$ pounde, that is 1. pound 5. shillings, or 25. shillings.

Secondly I diuide $\frac{1}{7}$ d. by 4. and the quotient is $\frac{1}{28}$ p. which by the 14. & 20. verses of the 6. Maxim, is 1 s. 3 d. finally, both these quotients added together make 26 s. 3 d. and vnto so much will the 120. Stockfishes amount at that price.

Euma.

Euma. All this I conceiue very readily perceiuing withall, that as the money here multiplied must alwayes be pence, or aliquot partes of a peny, so the quotient alwayes yeeldeth pounds or partes of a pound, which differeth not much from the last rule before, in qualitie and order of woorking.

Philo. You say very truly.

38. And when by the price of any 6. score,
you couet to serue the price but of one.
Infolding the pounds of that price by foure,
Diuide the product by two all alone,
So doth the quotient shew you very prone,
VVhat number of pence that vnit will cost.
VVithout ouerplus, or any thing lost. } 17

Euma. This rule is much like in effect to the 35. verse, sauing that there the question was but vpon the 112. and here it is vppon the 120. and agayne there the common multiplier was 15. and the diuisor 7. but here the common multiplier is 4. and the ordinarie diuisor 2. and as for all other qualities and effects they are agreeable in my minde.

49. A question of Ling.

Philoma. They are so, as for example, when Lings are at 8. pound the 120. what is that a peece, heere as this verse requireth, I multiplie 8. which are the poundes contayned in the giuen price, by 4. whereby I produce 32. which diuided onely by 2. quoteth 16. and so many pence doth euery Ling of that 120. cost.

And therefore if (beside such pounds) there be any shillings or pence contayned in the giuen price, the same must be set foorth in aliquot parts or fractions of a pound, as was there taught, and the woorke pursued and accomplished as was there practised, as for example, I aske this question.

50. A question of Codfish.

When Codfish is sold for 3 l. 6 s. 8 d. the 120. what is that a peece.

Euma. This question with your fauour I will vndertake to answere, for which cause, after that I haue set foorth the giuen price in poundes and partes of a pound thus, 3 pound + $\frac{1}{3}$ pound, I multiplie first the 3. pound by 4. as this present verse willeth me, and the product is 12. which diuided by 2. quoteth 6. pence, then for the second woorke, I multiplie the $\frac{1}{3}$ pound by 4. also, and it produceth $\frac{4}{3}$ pound, which beeing likewise diuided by 2. the common diuisor quoteth $\frac{2}{3}$ pence or 16. Mices, wherefore adding both these

quotients

quotients together, I conclude that the codfishes do cost ½ 6 s. 16. mites a peece.

Philo. You haue done well, whereby I perceiue you are perfect inough in these rules, and therefore I will inferre no mo questions touching the same, for it is no difficultie to apply their vse to any like question, wherefore nowe we will say somewhat of the rules of factorshippe, vsed betweene merchants and their factors, in order following.

Of the rule of factorship, and of the estimation of the factors person. The 8. Chapter.

39. VVhen any merchant a factor doth make.
To do his affayres for parte of the gaynes,
VVhich factor doth nothing els vndertake,
But onely to venture his person and paynes, } 18
Then looke how his gaynes is in proportion,
Vnto the gaynes which the merchant doth take,
Such is the rate of the factors person,
Vnto the whole stocke the merchant doth stake.

51. A question of the estimatiō of factors person.

As for example, a merchant maketh a young man his factor to deale for him beyonde the seas, committing to his gouernance 200 r. to be imployd in the trade of merchandise, vpon such condition that he shall haue ½ of the gaynes thereof growing for his labor, now it is demaunded at what rate, or at how much, the factors person is esteemed or valued.

For answere whereof, sith it is manifest, that if the factor haue ½ of the gaynes onely for his person and paynes taking, then the merchant can haue but the other ½, wherefore by this verse or rule, as the factors ½ of the gaynes, is to the merchants ½ of the gaynes, so is the valure of the factors person to the merchants whole stocke, staked downe or disbursed.

Euma. So I perceiue hereby, that the person of the factor is esteemed woorth 200 r. for as ½ to ½ is the proportion of equalitie, so must the estimation of the factors person be equall to the merchants stocke of 200. pound.

51. A question of a factors estimatiō.

Philo. You haue sayd very truly, and because you vnderstand the rule so quickly, answere me to this, a Merchant hath deliuered to his factor 900 r. to be employed in the trade of Merchandise, vpon condition that he shall haue ½ of the gaynes for his labour, if

any

any thing be gayned, and that he shall also beare $\frac{2}{3}$ of the losse, if ought be lost, now the question is at how much, or at what rate the factors person is esteemed.

Euma. First it is certayne, that whether it be losse or gayne, whereof the factor is partaker, his share thereof is $\frac{1}{3}$ whereby it followeth that the merchants share shalbe the other $\frac{2}{3}$ residue.

Wherefore me thinkes by this maxim I am as it were erbozted to vse the golden rule direct, saying thus, as $\frac{2}{3}$ to $\frac{1}{3}$ or rather thus, as 2. to 1. so is 900. to whom: and the rule immediately answereth 450. pound, as here appeareth by the figure. Whereby I conclude that so much is the estimation of the factors person.

2 Z 1
900 r — 450 r

Philo. It is so, and you haue found out the same way (in effect) by your selfe, to answere these and such like questions, which I was about to haue taught you, if your owne forwardnesse had not preuented me.

And yet you may answere it by the golden rule after this fashion also, saying thus, if $\frac{2}{3}$ come of 900 r. of whom commeth $\frac{1}{3}$ or rather thus omitting the equall denominators, if 2. come of 900 r. of whom commeth 1. and so both wayes the answer is 450 r. as here appeareth.

2 Z 900 r
1 — 450 r.

53. A question of the estimation of a factors person.

But what say you to this, a merchant deliuered to his factor 400 r. to be employed in merchandise, on condition that he shal haue $\frac{2}{3}$ of the gaynes for his trauell, I demaund at how much the person of this factor is rated or esteemed.

Euma. For answere I see foorthwith, that if the factor haue $\frac{2}{3}$ then the merchant must needs haue $\frac{1}{3}$, wherefore by this rule, as the factors 2. is to the merchants 3. so is the estimation of the factors person, to the merchants stocke of 400 pound.

And alternately as the merchants 3. is to his stocke of 400 r. so is the factors 2. to the estimation of his person, wherefore I conclude by the golden rule direct, that if 3. come of 400 r. stocke, then 2. commeth of 266 $\frac{2}{3}$ r. as here appeareth by the figure. And therefore so much is the person of the factor esteemed to be worth.

3 Z 400 r.
2 — 266 $\frac{2}{3}$

Philo. You say truth, it is euen so.

40 But when a factor besides his person,
Putteth some stocke in aduenture also,
As the merchants gaynes is then in proportion,
To his proper stocke which he doth forgoe,

19

So

So the factors gaynes in like rate doth growe,
Vnto the whole stocke, he ought to defray,
VVherefore looke howe much he lesse doth bestow,
So much is his person esteemed alway,

As if a Merchant commit 400 r. to the discretion of his factor, to be employed in traffique vpon such condition that the factor putting in 80. r. more thereto shall haue $\frac{1}{5}$ of the gaynes for his trauell.

54. Another question touching the estimatiō of the factor.

Nowe by this maxim, as $\frac{1}{5}$ which is the merchants gaynes is in proportion to 400 r. his proper stock which he foregoeth or disburseth, so is the other $\frac{4}{5}$ which is the factors gaynes, vnto the whole stocke which he ought to defray or disburse, which stocke therefore by the golden rule should also be 400 r. as here the figure sheweth.

$\frac{1}{5}$ — 400 r.
$\frac{4}{5}$ — 400 r.

But now the factor as you see disburseth not so much by 320 r. (for he is sayd to put in but 80 r.) wherefore subtracting the same 80 r. from 400 r. there resteth 320 r. which by the intent of this verse present is the true estimation of the factors person.

Euma. This I suppose I vnderstand well inough.

Philo. Then aunswere me to this, a merchant committeth the employment of 600 r. to his factor, & the factor aduentureth 250. pound more therewith of his owne, and his person, for which cause it is agreed betweene them, that the factor shal haue $\frac{2}{5}$ of the gains wherefore I demaund at how much his person is esteemed in this case.

55. A question of the estimatiō of factors person.

Euma. Here it is apparent, that sith the factor is to haue $\frac{2}{5}$ of the gaynes, therefore the merchant must needs haue the other $\frac{3}{5}$ thereof, whereupon I frame the question to the Golden rule, by the purport of this verse, saying thus, if $\frac{3}{5}$ the merchantes gaynes come of 600. pound his proper stock, of whom commeth $\frac{2}{5}$ the factors gains but because these fractions haue both like basses, I omit them or cancell them in the operation, vsing onely their Numerators as here you see. And so woorking by the rule I finde 400. pounde, as appeareth.

$\frac{3}{5}$ — 600 r.
$\frac{2}{5}$ — 400 r.

Wherefore by this present maxim, so much ought this factors whole stocke to haue bene, but it is but 250 r. which is lesse then 400. pound by 150. pound, as appeareth by subtracting 250. from 400. wherefore I conclude that his person is esteemed at the residue, to wit, at 150 r.

Philo. You haue done well, it is euen so.

41. And

41 And when a factor is satisfied sole,
With gaynes of some part of the stock for his pains,
Then looke what part, such part is of the whole,
Such part of th'encrease, shalbe the factors gaynes,
So that diuers wayes his share may appeare,
But yet the golden rule shews it most cleare.

56. A question what part of the gaynes is due to the factor.

As for example, a merchant committing 800 l. to his factors gouernment, promiseth him that he shall haue the gaynes onely of 160 l. thereof, for the aduenture of his person and for his paynes, whereunto the factor agreeeth, whereupon it is demaunded what portion of the gaynes is due to the factor.

Now here you see, the factor layeth out nothing himselfe, neyther is it expressly sayd, that he shall haue $\frac{1}{5}$ $\frac{1}{3}$ $\frac{1}{4}$ or other part of the gaynes, but he is satisfied only with the gains of 160 l. as of a part of the whole stock, wherefore by this present verse, looke what part this 160 l. is of the whole 800 l. stocke, such part of the encrease or gaynes ought the factor to haue, so (that as this verse sayth) his share of the gaynes may be found diuers wayes, for by the 55. maxim of my first booke, 160 l. is $\frac{1}{5}$ of 800 l. wherefore I may conclude thereby that the factor shall haue $\frac{1}{5}$ of the gaynes.

But the golden rule sheweth it most playnely and clearely, to whom I may say thus, if 800. pound the entire stocke require the entire gaynes, what doth 160. pound thereof require, and the rule answereth $\frac{1}{5}$ as here appeareth.

pound	gaines
800	1
160	$\frac{1}{5}$

Wherefore I conclude as before, that so much of the whole gaynes belongeth to the factor.

Euma. This is very easie to do, for by 1. vnit or integer the entire gaynes is truly represented.

Philo. It is so, but if their agreement had beene such, that the factor should haue layd in 160. pound to the merchants stocke, and then should haue had such part of the gaynes as that 160. pound is of the whole stocke, then I say the factor should haue had but $\frac{1}{6}$ of the whole gaynes, for when the whole stocke, by the addition therto of the factors 160. pound, is become 960. pound, then is the factors 160. but $\frac{1}{6}$ of that totall. Wherefore by the golden rule as before, if 960 l. take 1. entirely for gains, what shall 160 l. take, & the rule answereth $\frac{1}{6}$ as here you see.

pound	gaines
960	1
160	$\frac{1}{6}$

And so here we will leaue off factorage, and go in hand with bartering or trucquing.

Of

Of the exchanging of wares for wares, or partly for wares and partly for ready money, commonly called bartering or trucquing.

The 9. Chapter.

42. Of Barters or trucques, there are diuers kindes,
VVhereof the first, is when the Trucquers take,
But ware for ware, by agreement of mindes,
No partie grating, greater gaines to make,
The qualitie of which exchange of wares,
The compound rule ascending sole declares.

Hereby is declared vnto you, that there are diuers kindes of trucques, whereof the first (which is simplest and plainest) is when the trucquers change one ware for another only, without grating or crauing any further gaynes one of the other but onely equalitie of valure, as the same wares are presently worth to be solde for redy money, that is as you would say, one peny worth for another, in which case the compound rule ascending with out other helpe, doth quickly dispatch the matter.

As for example, A. and B. are agreed to trucque or barter, A. hath 11. Karsies worth 5 r. 5 s. a peece ready money, and B hath Quichenella, worth 12 s. the pound ready money: the question is how many pounds of Quichenella counterualeth the 12. Karsies to make the trucque equall.

57 A question of Trucqueing Kersies for Quichenella.

Here seeing there is nothing els required but ware for ware equall in valure, therefore by this verse, the compound rule ascending onely shall declare that equalitie, saying thus, if 1. Karsie be worth 5 ¼ r. and ½ r. worth 1. pound of Quichenella, how many pound of Quichenella is 12 Karsies worth: which termes reduced to simples, applied to the figure, and vsed as the compound rule ascending requireth, aunswereth that the 12. Kersies are worth 105 r. of Quichenella, as heere the fabrique witnesseth. Wherefore so many pound weight of Quichenella,

Kersies.	money
1	5¼ r.
money	quiche.
½ r	1 r.
Kersies	quiche.
12	105 r.

ought A. to receiue for his 12. Kersies, to make the trucque equall.

Eu. This rule is not hard to do, considering what I haue learned already.

58. A question for proofe of the next before.

Philo. And the proofe thereof is as easie, euen by the selfe same rule, as for example, A. and B. will trucque ware for ware,

now A hath 105 ℔. weight of Quichenella worth 12 s. a pound, and B. hath Kerseys worth 5. pound 5. shillings a peece: the question therefore is, how many Kersies A. shall haue for his 105 ℔ of Quichenella.

Euma. This I doubt not to aunswere my selfe euen very presently, and to that end I say thus, if 1 ℔. of Quichenella be worth ¼ ℔. and ¼ ℔. worth 1. Kersie, how many Kersies are 105 ℔. of Quichenella worth, & so working by the compound rule ascending, to whom the question is apparantly subiect, I finde 12. Kersies for the aunswering number, as heere appeareth by the fabrique.

quiche.	money.
1 ℔	¼ ℔.
mony	Kersie
¼ ℔.	1
qui.	Kersie
105 ℔	12

Whereby, the answer of the former question is iustified & confirmed.

Philo. It is so, wherefore now let vs goe to the next kinde of trucque, which is thus.

43. The second trucque is when 2. marchants minde,
To trucque their wares, but yet with such deuice,
That one of them thinks some vantage to finde,
By prising his wares aboue their true price,
VVhich craft espied, the tother doth preuent,
By the golden rule, applied for that entent. 22

59. A question of trucquing Pepper for Cinamon.

As for example, A. hath Pepper worth 2 s. 4 d. the pound ready money, but in trucque he rateth it at 3 s. the pound, and B. hath Cinamon, worth 5 s. 6 d. the pound ready money: now at what price ought B. to deliuer his Cinamon to ballance the trucque, and be no looser thereby?

The losse which B. here is like to incurre, is easily preuented, if he also prise his Cinamon aboue the true price proportionally, as he may soone do, by the golden rule direct, saying thus: if 7. groes worth of Pepper be worth 3 s. in trucque, what shall 5 ½ s. of Cinamon be worth in trucque?

Or thus, if Pepper of 2 ⅓ s. yeelde 3. shilling in trucque, what shall Cinamon of 5 ½ s. yeelde in trucque: and the operation of the rule aunswereth 7 ½₇ s. as here the figure sheweth.

⅓ s	¼ s.
11/2 s	7 ½₇ s.

And for so much, must B. prise and deliuer the pound of his Cinamon to make the Trucque equall, and saue him selfe harmelesse.

60. A question of trucquing for proof of the next former.

Euma. This appeareth good pollice, & is no hard matter to vs.

Philo. You say truth, and for triall of this rule, you may now sup-

ſuppoſe þ A. hath Cinamon of 5 s. 6 pence the pound ready money which he will trucque with B. at 7 1/12 s. for Pepper, of 2 1/3 s. ready money, whereuppon it is demaunded, what price B. ſhall make of the pound of his Pepper to ballance the trucque?

Euma. Then for aunſwer I come to the golden rule ſaying, (as you ſayd before) if Cinamon of 5 1/3 s. ready money peelde 7 1/12 s. in trucque, what ſhall Pepper of 2 1/3 s. ready money peelde in trucque? and the rule anſwereth 3 s. as heere the ſubtique declareth.

11/3 s. Z 22/12 s.
7/3 s. — 3 s.

Whereby the anſwer of the laſt former queſtion is fully approoued and verified.

Phi. It is ſo, and as this preſent order ſerueth for the equifieng or equalling of ſuch trucques or barters as this is, ſo doth it alſo ſerue to finde out the valure of the true and iuſt price of ſuch wares as are ſurpriſed or enhaunſed by the trucque, as for example by the laſt former queſtion.

61. A queſtion to finde the true valure of wares ſurpriſed.

A. and B. trucque togeathers, A. hath Cinamon woorth 5 1/3 s the pound ready money, but in trucque he priſeth it at 7 1/12. s. the pound, and B. hath Pepper which he (being politique inough to countermatch A. in trucquing) deliuereth to A. in equalitie of trucque, at 3 s. the pound: now the queſtion is, what a pound of B. his Pepper is woorth ready money?

For anſwer whereof I vſe the golden rule onely, ſaying thus: if Cinamon of 7 1/12 s the pound in trucque, be woorth but 5 1/3 s. ready money, what is Pepper of 3 s. the pound in trucque, woorth in ready money? and the rule anſwereth 2 s. 4 d as here appeareth.

5 1/4 s Z 5 1/3 s
1/3 — 2 1/3

And thus the true prices aſwell as the ſurprices of the wares exchaunged are found all by one rule onely, by chaunging the ſituation of the termes in the golden rule, the triall whereof I doubt not you your ſelfe can make, aſwell as you did of the other queſtion before.

62. Another queſtion to know the true price of the wares.

Euma. I truſt ſo, and for that purpoſe I will ſuppoſe that A. and B. will trucque, and that now A. hath Pepper woorth 2 1/3 s. the pound ready money, which in trucque he rateth at 3 s. and that B. hauing Cinamon deliuereth the ſame to A. in equalitie of trucque at 7 1/12. s. the pound, whereuppon I am demaunded, what the pound of B. his Cinamon is woorth ready money?

For anſwer whereof, I ſay to the golden rule thus, if Pepper of 3 s. the pound in trucque, be woorth but 2 1/3 s. ready money what

is Cinamon of 7 1/14 s. the pound in trucque, worth in ready money, and the rule aunswereth me 5 s. 6 d. as here appeareth.

trucque.	money
1/14 s.	7/8 s.
99/14	5 s. 6 s.

Iustly agreeable to the former questions.

Phi. You haue done well, and euen as you haue done this, so by vsing the golden rule, twice, thrice, or oftner (as occasion serueth) you may dissolue diuers other questions of trucques, which seeme to be of another kinde, but are also within the compas of this rule, as for example.

63. A question who getteth or looseth by the trucq.

A. and B. desire to trucque togethers, A. hath Allom worth 25. shillings the quintall ready money, which he will trucque at 30 s. the quintall with B. to take Pepper for it at 3 s. 4 d. the pound, which is worth but 3 s. the pound ready money: now B being vnskilfull to make the trucque equall, giueth him Pepper for 100. quintalls of his Allom at the prices aforesaid, the question is which of them hath the aduantage of other, and what the looser looseth?

For answer whereunto, first I say by the golden rule (after the former fashion) if Allom of 25 s. the quintall ready money, yeeld 30 s in trucque, what shall Pepper of 3 s. the pound ready money yeelde in trucque, (or the answer is 3 s. 7 1/7 d. as here appereth.

money.	trucque.
25 s	30 s.
3 s	3 s. 7 1/7 d.

Now sith that B. selleth his Pepper but for 3 s. 4 d. in trucque, which by due proportion of price ought to yeeld 3 s. 7 1/7 d. who seeth not presently, that he looseth 3 1/7 d. in euery 3 s. 7 1/7 d. worth of Pepper deliuered? whereby it is manifest that A. hath a great deale the aduantage of him.

Note here how the answering terme of euery proportiō proceeding, is made the demaunding terme in the next proportion succeeding.

Secondly, to finde how many pound of Pepper B. deliuered for the 100. quintals of Allom, I must first see whereto the 100. quintalls of Allom amount at 30 s. the quintall, saying by the golden rule againe thus, if 1. quintall cost 30 s. what doth 100. quintals cost, and the rule answereth 3000 s. as here appeareth.

quintalls.	shillings
1	30.
100	3000

Thirdly I aske the same rule this question, saying: if 3 s. 4 d. pay for 1 l. of Pepper, for how many pound will 3000 s. pay? and it answereth me for 900. pound as here the figure expresseth. wherefore, so many pound weight of Pepper shall A. haue for his 100. quintalls of Allom.

shill.	l. weight
10/3	1
3000	900.

Fourthly I repayre once againe to the same rule, saying thus.

If

if in so much Pepper as is woorth 3 s. 7 1/7 d. there be lost 3 1/7 d. what is there lost in so much Pepper as is worth 2700. s. (which is the whole valure of all the Pepper at the iust price of 3. shillings a pound?

Or thus, if 3 s. 7 1/7 d. loose 3 1/7 d. what shall 2700 s. loose? and the aunswer is 200 s. as heere appeareth by the figure.

stock.	losse.
216/7 d	16/7 d.
2700 s	200 s.

Which reduced, maketh iust 10 r. and so much hath B. lost in this trucque by his ignorance in Arithmaticke.

Euma. I vnderstand this application of the rule well, wherein I perceiue also, that he that will barter wisely, hath no lesse neede to be guided by reason then by rules of arte, or els he may be deceiued sometimes.

Philo. That is without question, for though the rules be neuer so exactly framed and composed by most perfect art, and probable demonstration, yet if they be not aptly applied as reason requireth, they profit the workeman no more, then a Sawe doth serue a Carpenter to make an Aulgar hole, or an Aulgar to slit a board, but because you say you vnderstand the application of the rule in this behalfe well inough, aunswere me to this question.

64. A question who gaineth by the Trucque.

A. hauing 100. peeces of Silke vendible for 3. r. the peece ready money, putteth them away in barter to B. for 4. r. the peece, taking wooll in exchaunge at 7 r. 10 s. the 100 r. weight, which indeede is worth no more but 6. r. the 100. ready money, now I demaund what quantitie of wooll payeth for the 100. peeces of Silke, and which of the trucquers dooth gayne or loose by the trucque.

For aunswer, first I vse the golden rule as you did before, saying thus, if Silke of 3 r. ready money, yeelde 4. r. in trucque, what shall wooll of 6. pound ready money yeeld in trucque? and it aunswereth 8 r. as here appeareth by the figure.

money	trucq.
3 r	4 r.
6	8.

Whereby I perceiue presently that A. hath the aduantage of B because A. taketh that wooll but for 7 1/7 r. which prised proportionally to his Silkes is worth 8 r. so that he gaineth 10 s cleare in euery 100. weight of woll.

Phi. Not in euery 100. weight, but in euery 8 r. worth of wool.

Euma. Why, is it not all one?

Phi. No in deede, as you shall see anon, but in the meane season go forward.

Note here also, how the answering terme of euery proportiõ proceeding, is become the demaunding term in the next proportion succeeding.

Eu. Then secondly vsing the rule agayne I say thus. If 1. peece of Silke. yeeld 4. r. what will 100. peeces yeelde? and it answereth 400. r. as heere appeareth by the fabrique.

peeces	pounds.
1	4.
100	400.

Thirdly to finde what quantitie of woll payeth for the silkes, I vse the rule yet againe, saying. If 7 ½ pound, paye for 1. hundred weight of woll, for how much will 400 r. paye? and it answereth me for 53 ⅓ hundred weight, as heere appeareth. Whereby it is certaine, that so much Woll payeth for the silkes.

pounds.	C. weight
15/2	1
400	53 ⅓

Lastly, occupying the same rule once moze, I say thus. If 1. hundred of woll be woorth 6. pound ready money, what is 53 ⅓ hundred woorth? and I finde 320 r. as heere appeareth. Which is moze by 20. pounds, then the silkes were woorth in ready money, whereby it is euident that B. looseth 20 r. by this trucque.

C. weight.	pounds.
1/3	6/1
160/3	320

Philo. He dooth so: and the like should you haue found, if you had applyed the former losse found, to the golden rule, saying thus. If 8. pounds woorth of woll loose 10. shillings, what dooth the whole 320. pounds woorth loose? and the rule answereth 20 r. as heere the figure sheweth.

woll.	money.
8 r.	½ r.
320	20

But if you (supposing that B. looseth 10 s. in euery hundred weight of woll) should say thus. If 1. C. lose ½ r. what shall 53 ⅓ C. loose? the rule would answer you 26 ⅔ r. that is to say, 26 r. 13 s. 4 d. as here appereth: which is a very false answer.

wooll.	money.
1. C.	⅓ r.
160/3	26 ⅔.

Eu. Very good, I thinke I see the diuersitie now betweene 1. hundred weight of wooll, and 8. pounds woorth of wooll, for 100. weight of wooll according to the true valure or iust price thereof, is woorth but 6. r. But 8. r. after that rate will buy 1 ⅓ C. weight and so 8. pounds woorth of wooll is 1 ⅓ C. weight.

Phi. You say truth, and for that cause when you reckoned that A. gayned 10 s. in euery 100. weight, you were deceiued, for hee gayneth but 10. s. in euery 1 ⅓ C. and so consequently in the whole 53 ⅓ C. he gaineth iust 20 r.

And so likewise in the 63. question, where B. looseth 3 ½ d. I say not, that he looseth so much in euery r. weight of his Pepper, but that he looseth so much in euery 3 s. 7 ½ d woorth of his Pepper. for otherwise I should haue concluded erroniously, but now seeing you vnderstand the application of the rule so well, we will go to the next, which is thus.

44. The 3. trucque is when, a marchant doth craue,
Not onely to raiſe his price in the trucque,
But alſo determines beſides that to haue,
Halfe ready money to bring him good luck,
Or els ⅓. a third, or a fift,
Or other like portion obteyned by his drift,
45. VVhich trucque to ballance and ſet counterpayſed,
Abate the part ſo in money requeſted,
Firſt from his iuſt price, and then from the raiſed,
So the 2. remnants placed and digeſted,
As vppermoſt termes in the rule of 3.
Then take the others iuſt price for the thurd,
And worke by the rule whereby you ſhall ſee,
The trucque made as euen as arte can affoorde. } 23

As for example A. and B. are about to trucque, A. hath Claret wine of 14 r. the tunne ready money, but in trucque he priſeth it at 18 r. and yet he will haue ⅓. of the ſame ſurprice in ready money, or els he will not trucque, and B. hath furres, to wit, Gray Jenets ſeaſoned worth 2 s. 8 d. the ſkin ready money, wherefore the queſtion is, at what price B. ſhall deliuer his Jenets, to make the trucque equall and ſaue him ſelfe from loſſe? 65. A queſtion of trucquing Wines for Furres.

For aunſwer whereof, firſt ſeeing A. hath priced his wine at 18 r. the tunne in trucque, whereof he will needes alſo haue ⅓. in ready money, which is 6 r. Therefore (as the latter verſe of this preſent maxim willeth me) I abate the ſame 6 r. requeſted in money, firſt from 14 r. his iuſt price, and the remaynder is 8 r. and next from 18 r. the raiſed price and the remayner thereof is 12 r which 2. remaynders, taken orderly as they come by this maxim, I make the 2. firſt or vppermoſt tearmes in the Golden rule, Taking 8. groates, the iuſt price of the others Jennets for the third, ſaying thus.

If 8 r. ready money, riſeth to 12 r. in trucque, whereto ſhall 8. groates ready money riſe in trucque: and the rule anſwereth to 12. groates as here appeareth. And for ſo much ought B. to deliuer euery ſkinne of his furres: to auoide loſſe, and make the exchange equall.

money.	trucque
8 r.	12 r.
groats	groates
8	12

Euma. By this your playne example I hope I vnderſtand the whole maxim very well. 66. A queſtion of Chamlets trucqued for Canvile.

Phi. Then anſwer me to this, A. and B. will trucque, A. hath

Chamblets of 45. s. the peece ready money, but in Trucque, hee will not vtter them vnder 48 s. the peece, and yet he will haue $\frac{1}{4}$ of his surprice or raised price in ready money, and B. hath Canuas of 16. d. the ell ready money, wherefore the question is, at what price B. may deliuer his Canuas to equifie the trucque and be no looser thereby?

Euma. For aunswer hereof, I follow the precept of this present maxim, abating 12. s. (which is $\frac{1}{4}$ of the surprice of A. his Chamblets, or the portion of ready money by him required) first from 45 s. his true price, and next from 48. s. his raised price, and so the 2. remaynders are 33. s. and 36. s. which 2. remaynders I place in the figure of the golden rule orderly as I found them, by this maxim, that is, the remaynder of the true price for the first terme, and the other for the second: which done I take 16 d. the iust price of the others Canuas for the third, saying thus, if 33 s. ready money grow to 36. s. in trucque, whereto shall 16. d. ready money grow in trucque? and so working by the rule it answereth me, that it shall growe to 17 $\frac{1}{11}$ d. as here appeareth.

33 s. ∠ 36 s.
16 d. ∠ 17 $\frac{1}{11}$ d.

Wherefore I conclude, that B. must deliuer his Canuas at that price the ell to be no looser.

Eu. Your conclusion is true, and if you list to proue it, that may you do by the verse insuing.

Proofe.

46. For proofe whereof, place the remnant or rest,
Of th'ouer price first, and next of the true,
And let the others surprice be addrest,
To make vp the third, as one of the crue,
VVhich done, if you vse the golden rule duly,
The answere shall shew you the thing you seeke truly. 24

67. A question how to finde the true valure of wares surpriced in trucquing

As for example, whereby to cleare the meaning of this verse, suppose that A. and B. are willing to trucque together, and that A. hath Chamblets worth 45 s. the peece ready money, but in trucque he will haue 48. s. a peece, and yet he will haue $\frac{1}{4}$ of that surprice in ready money also. But B. hauing Canuasse, and being wylye inough to matche A. in the Trucque, so that he will be no looser thereby, deliuered his Canuasse at 17 $\frac{1}{11}$ pence the Ell, in equalitye of Trucque. The question therefore is, what the iust price of an Ell of B. his Canuasse is worthe in readye money?

For aunswere heereof, after that I haue abated 12 s. (which is $\frac{1}{4}$ of the surprice or summe of ready money demaunded.) Fyrst from

from 45. the true price of the chamblets, and next from 48 s. their ouer price or rayſed price, I finde their remayners to be 33 s. and 36 s. (as before) but now as this verſe preſent bideth me, I place 36 s. which is the remnant of the ouer price firſt, and 33 s. the remaynder of the true price next or ſecond, and whereas before I tooke the other truques true price for the third terme, now I take 17 $\frac{1}{11}$ d. his furprice inſtead thereof, for the third terme in the golden rule, ſaying thus.

If 36 s. in truque be woorth but 33 s. readie money, what is 17 $\frac{1}{11}$ d. in truque woorth in readie money, and truq. money.
by vſing the operation of the rule I am anſwe- 36 s. 33 s.
red 16 pence as here appeareth. $\frac{121}{11}$ d. 16 d.

And therefore 16 d. is the true and iuſt price of the ell of B. his Canuaſſe in readie money, which (as this verſe ſayth) is the very thing I ſought for, and ſo you ſee that this aunſwere approoueth the anſwere of the next former queſtion to be true.

Euma. This proofe muſt needes be true, the concluſion is ſo agreeable to the premiſes.

Philo Now will I ſhew you a fourth kinde of truque or barter, which is yet more intricate then any of the former, and it is thus.

47. VVhen one truequer ſeeking tother to preuent,
Surpriceth his wares thinking in his hart,
To win by the truque, and yet not content,
VVill haue readie money, ſome parcell or part,
If then the tother (his craft vnderſtoode,
VVill gayne in the 100. as he thinketh good.

48 Then muſt be ſought firſt, how much the true price,
Of his wares will coſt, which gaynes by the hundred,
After the rate ſo ſet by his deuice,
VVhich by the golden rule truly found and numbred
Rebate the money by tother deſired,
Both from his true price and ſurprice alſo,
Setting what reſteth in order required,
For firſt 2. termes in golden rule to goe,
Making the third, the laſt terme found before,
So woorking the rule your lacke doth reſtore.

48. A queſtion at what price caps may be truequed for laies to gayne 10. in leD.

As for example A. hath Sayes woorth 20 s. 10 d. the peece readie money, which he would truque with B. for caps woorth 35 s. the peece readie money, but becauſe A. thinketh ſo to preuent or rather to

to circumuent B. that he will gayne by the trucque aboue the equalitie of wares, therefore he will not sell his Sayes in barter vnder 25. shillings the peece, and yet he will haue ½ of his surprice also in readie money, but B. vnderstanding his subtiltie yeeldeth to the trucque vpon condition that he may gayne onely after 10. in the 100 l. by his wares trucqued.

Wherefore now the question is, at what price B. shall deliuer a dozen of his cappes in this trucque, to gayne after that rate.

For answere whereunto the second verse of this maxim willeth me first to seeke by the golden rule, how much the true price of his wares will yeelde, which gayneth by the 100. after such rate as he hath set or diuised to haue.

Now here it is B. that will gayne in the 100. and the rate which he hath set to gayne is 10. wherefore I seeke by the golden rule, what 35. shillings (which is the true price of his caps) doth yeeld after that rate of 10. in the 100. saying thus, if 100 l. grow to 110. pound, whereto will 35. shillings grow, and the operation of the rule aunswereth 38. shillings 6. pence, as heere the fabrique sheweth.

100 l. — 110 l.
35 s. — 38 s. ½

This aunswere beeing thus found, the rule present willeth me further to rebate the readie money desired by the other (that is to say, by A.) both from 20. shillings 10. pence his true price, and also from 25. his raysed surprice. But the money which he desireth, is ½ of his surprice: Now his surprice beeing 25. shillings, ½ thereof is 6. shillings 3. pence, which I abate both from (20. shillings 10. pence) his true price, and also from (25. shillings) the surprice it selfe, so the two remaynders resting are 14. s. 7. pence, and 18. shillings 9. pence, which 2. remainders I set orderly for the 2. first termes in the golden rule, and for the third, I take 38 ½ shillings the last terme shilling. shilling found before by the former golden rule, as here you see them.

175/13 — 75/4
77/5 — 49 s. 6 d.

Which done, working as the rule requireth, the aunswering terme restoreth the lacke for which I seeke, but I seeke for the price of a dozen of cappes in this barter, wherefore the aunswering terme here being 49 s. 6 d. assureth me that B. must deliuer a dozen of his caps at that price in trucque to gayne after 10. in the 100.

For proofe whereof, let A. haue 96. peeces of Sayes, which at 20 s. 10 d. the peece, come iust to 100 l. but when he will sell them

them at 25. shillings the peece, he mindeth to make 120. pound of them, whereof sith he will also haue $\frac{1}{4}$ that is 30. l. in ready money. Therefore it followeth that he can haue cappes but onely for the 90. l. remaining.

And yet by equitie of the trucque, he must haue no moe caps for that 90. pound, but so many as are woorthe 70. l. at 38. s. 6. d. the dozen (which with the gaynes after 10. in the 100. is their true valor or price.

Then suppose that B. hath $57\frac{1}{7}$ dozen of caps, which at 35. s. the dozen amount allso iust to 100. l. whereof (sith he payde 30 l. to A. in readie money) he must deliuer but so many onely, as with his gaynes after 10 in the 100. amount to 70 l. that is to say, so many as are woorth $63\frac{7}{11}$ l. and yet A. is to receiue the same caps for 90. pound.

Now therefore by the golden rule I say thus, if $63\frac{7}{11}$ pound be deliuered to pay or satisfie 90. pound, what shall 35. shillings the true price or valure of 1. dozen of caps satisfie, and it answereth me 49 s. 6 d agreeable with the former conclusion, as here appeareth in the figure.

pound.	pound.
$\frac{7}{11}$	20
$\frac{7}{4}$ l.	2 l. 9 $\frac{1}{2}$ s.

Euma. This trucque is more benificiall to B. then to A. in my minde, for although A. thought to get the aduantage of B. by surprising his Sayes and getting $\frac{1}{4}$ of his surprice in readie money, yet it seemeth to me that for his 100. pounds woorth of Sayes, he hath but only 30. pound in money, and as many cappes as are woorth in readie money $63\frac{7}{11}$ pound, all which amounteth but to $93\frac{7}{11}$ which being lesse then 100 l. by $6\frac{4}{11}$ l. argueth that he hath lost so much by the trucque.

Philo. He hath so, and so much hath B. gayned after the rate of 10. in the 100. in the sale of his $63\frac{7}{11}$ poundes woorth of cappes, deliuered in trucque, for by the golden rule, if 100 pound gayne 10. pound, then $63\frac{7}{11}$ l shal gaine $6\frac{4}{11}$ l. as here appeareth.

pound.	pound.
100	10
$\frac{7}{11}$ l.	$6\frac{4}{11}$

And thus you see whilest A. couetously endeauored to deceiue B. he is deceiued himselfe, according to an olde prouerbe which sayth thus.

Deceit deceiueth, and shalbe deceiued,
For who so loueth to practise deceit,
Though his deceite be not foorthwith perceiued,
Yet shall he at length, be caught with like bate.

And

And thus much may well suffice for ordinarie trucques or barters, for although some writers appoint mo kinds of trucques, and mo rules concerning the same, yet partly for that such rules are very vnruly and tedious to reduce into meeter, and partlye because such manner of trucques seldome or neuer come in vse, but specially because their chiefe rules and foundations conclude falsely, I vtterly omit them.

Of the names, rates, and valuations of diuers and sundrie Coynes and monyes, aswell forraigne as English, with the maner how to reduce them to an equalitie one with another. The 10. Chap.

To instruct you in the rates and values of forraigne Coynes and moneyes, is a matter very hard for me to do, not onely because I my selfe being a small traueiler am (as I will not sticke to confesse) vtterly ignorant of them by my proper knowledge, but also because their values in diuers places are very mutable and vncertaine, wherefore herein I can say no more then I finde in other authors, be it true or vntrue.

Ground of artes fol. 157. And therefore touching French money (with which I will begin) I finde in M. Doctor Records booke, that the merchants in France ordinarily make their accompts by Deniers, Soulx, and Franks, wherein 12. Deniers make 1. Soulx, & 20. Soulx (which we commonly call Souses) make 1. Franke or Liure, which is the pound French or Turnois.

So that these 3. kindes of monyes agree much like to the rate of our English money, for as 12 d. make 1 s so 12. deniers make 1 soulx, and as 20 s. English make 1 £ so 20. soulx French make 1. franke or liure, French or Turnois.

But herein they differ, that 1. peny sterling is as much as 9. deniers French, 1 s English is worth 9. soulx French, and so consequently 1 £. sterling is worth 9. frankes or 9 £. French.

Ground of artes, fol. 549. And yet in the selfe same Author afterwardes, I finde that our pound sterling maketh but 8 ⅔ £. French, our shilling but 8 ⅔ soulx French, and our peny but 8 ⅔ d. French, so that beeing in doubt which of these rates I might beleeue, I vsed some conference touching the same, with such merchants and traueilers as I was acquainted, who haue credibly enformed me, that neither of both those rates are true at this day, but affirme for certaintie, that our pound sterling now maketh 10. frankes or liuers French, our shilling 10. soulx French, and our sterling peny 10. deniers French, and that

the French crowne at this day is vsualy woorth 60. Soulx or 3. franks in France, and so is commonly currant there.

And yet (as M. Baker sayth) the French crowne is commonly currant among merchants in France for 51. soulx turnoys, although in exchange they will giue but 50. soulx for a crowne, & the like saith M. Mellis also, and both he and M. Dionis Gray affirme, that this Character ▽ representeth the French crowne by exchange, which they say is euer constantly 50. soulx turnoys or French money. we'spring of Sciences 168. Mellis 526. Gray fol. 82.

Euma. Then it seemeth to me hereby that a French crowne should be woorth lesse of our sterling siluer money in France, then it is woorth with vs here in England, for such crownes as are allowed to be currant here, are currant for 6 s. or 18. groats a peece, but 51 soulx after that reckoning are not woorth aboue 5 $\frac{1}{10}$ s. sterling.

Philo. You say well, agayne I finde that at Lions in France, the merchants keepe their reckonings and books of accompts by crownes de Marke which are esteemed at 45. soulx the peece, which they diuide into 20. soulx, and 1. soulx into 12. deniers; and there they also vse Markes of golde, whereof 1. Marke maketh 65. ▽. de Marke which serueth for exchange, and it is diuided into ounces, and euery ounce into 24. deniers, and the denier into 24. graynes. Ground of artes. fol. 553.

Now touching Flemish money, I finde that at Andwerpe they vse to make their accompts by deniers de grosse, that is to say, as I finde it interpreted by pence Flemish, whereof 12. make 1 s. Flemish, and 20 s. make 1 r. Flemish, which they call a r. de grosse.

And further I finde that 7 s. Flemish, make but 6 s. English, and that 7 s. Flemish, is woorth but 6 s. English, and consequently that 7 r. Flemish, are equall in value to 6 r. sterling or English. Ground of artes. fol. 530.

Eu. So then that after that rate 1 r. English maketh 1 $\frac{1}{6}$ r. Flemish, and 1 s. English maketh 1 $\frac{1}{6}$ s. Flemish, and 1. peny English maketh 1 $\frac{1}{6}$ d. Flemish.

Philo. You say truth, and now concerning Spanish money, you shall vnderstand, that the merchants there make their accompts generally by Maruedies, Royals, and Ducats, whereof 34 Maruedies make 1. Royall commonly called by vs, a Spanish Royall, or Royall of plate, which although it be woorth somewhat aboue 6 s. sterling in other countries, yet is accepted with vs for no more at this day, and 11. of those Royals make 1. Ducat.

Euma. So then I perceiue that 5 $\frac{2}{3}$ Maruedies make the valure of our peny sterling, and that their ducats is woorth 5 s. 6 d. English. because euery ducat as you say is woorth 11. Royals, and euery Roall woorth 6 d. sterling.

Philo.

Philo. Merchants commonly esteeme a ducat to be worth about 5 s. 10 d. sterling by exchange, because euery Royal as I told you, is somewhat better worth then our 6. pence.

Agayne in Portugall and all the Isles of Surrey and Madera, they make theyr accompts chiefly by Res, vingtins, Royalls and ducats. The Res it selfe is a base coyne, whereof 20. make 1. vingtin, which is a siluer coyne, taking his name of the number 20. as you would say Vigintires, and the same Vingtin is worth our Trip or English 3 d. or worth halfe a Spanish Royall, so that 40. Res or 2. Vingtins make 1. Royall, and 10. Royals there make 1. ducat.

Euma. Then it seemeth that their Vingtins, are equiualente to our Trip or halfe Testerne, their Royals to our testernes, and their ducat to our crowne of 5 s. but their Res is not agreeable to any of our coynes, for 6 ⅔ Res make but our peny.

Philo. They are so as you say, sauing that their Royals are the very coyne of Hilspayne, and therefore are esteemed somewhat better with them then our 6 d. and so are they also in France, Flaunders, and generally euery where, for indeede they are almost ⅓ peny weight heauier then our testerns, I meane the weight of our coyned halfepeny.

Ground of Artes. 159.

Now touching Danske money, they haue their soulr, whereof 20. make 1. pound or Liure, which is worth 2. shillinge sterling, so that their Liure, and the Liure French or Turnois seeme to be equiualent, they haue also their Grasshe, whereof 30. make 1. Gilderne which is worth 4. shillings sterling, & they haue also Dollars olde and new, their common olde Dollar is 35. grasshe, but of their new Dollars some are worth 24. grasshe, some 26. and some 30.

And because that in all Realmes Christian there are diuers and sundry coynes, both of gold and siluer currant, varying and differing greatly in their valuatiõs, I haue thought it not impertinent here to annexe a table, which comprehendeth the most parte of the golden coynes, with their iust weights and valures, wherein I haue not put the valures of the forraigne coynes, as they are currant in their own naturall countreys where they were first coyned, but as they may be accepted in payment in this realme (if law did so allow) without losse to the receiuer, for so the proportion of their valures is reduced to an equalitie with our sterling money, but first I begin with our owne country golde.

A Table of the names and values of the moſt vſuall golde Coynes in *Chriſtendome, with their ſeuerall weights of pence and graines, and what* they are woorth of currant Engliſh money.

The names and titles of the golde coynes.	The weight pence	grain	The value. shillin	pence
Roſe noble, or Royal.	4	23	15	0
Halfe Roſe Noble.	2	11	7	6
Olde Noble.	4	9	13	4
Halfe olde Noble.	2	4	6	8
Angell.	3	8	10	0
Halfe Angel.	1	16	5	0
Salute.	2	5	6	4
¼ of a Salute.	1	11	4	2
George Noble.	3	0	9	0
Halfe George Noble	1	12	4	6
Olde Crowne *K. H.*	2	9	6	4
Baſe Crowne *K. H.*	2	0	5	0
Patte Crowne.	1	0	2	6
Soueraigne of *K.H*	4	0	10	0
Other ſou. of al ſorts.	3	14	10	0
Double Soueraigne.	6	16	20	0
Great Soueraigne.	10	0	30	0
Halfe Soueraigne.	1	19	5	0
Unicorne of Scotlãd	2	10	6	0
Scottiſh Crowne.	2	5	6	0
French Noble.	4	16	13	4
All French Crownes	2	5	6	0
Flaunders Ryder.	2	6	6	6
Gelders Ryder.	2	2	3	6
Keyſers Royall.	3	10	10	0
Philips Royall.	3	10	10	0
Philips Crowne.	2	5	5	0
Colleyn Gilden.	2	2	4	8
Old Andrews Gilde.	2	2	4	10

The names & titles of the golde coynes.	weight ℔	grat.	value ß.	₰
New Andrew gilde	2	3	5	0
Flaunders Noble.	4	10	12	0
½ Flaunders Noble.	2	6	5	0
Flemiſh angel beſt.	3	6	9	0
Carolous gylden.	1	21	3	6
Flaunders Royall.	2	6	5	0
Flaunders Crowne.	2	5	6	0
Philips Gylden.	2	3	4	2
Halfe Phil. Gylden.	1	1	2	1
Golden Lyon.	2	16	7	8
¼ of a Golden Lyon.	0	21	2	5
½ of a Golden Lyon.	1	19	4	11
Dauids gylden.	2	2	4	0
Horne Gylden.	1	12	4	11
Saxon Gilden	2	2	4	8
Cruſado +.	2	6	6	0
Cruſado +.	2	6	6	2
Pillereis.	4	20	13	4
Halfe Pylleries.	2	10	6	8
Portigue. 1. ounce.	2	16	68	0
Golden Caſtilion.	2	23	8	10
Ducat of Arragon.	2	6	6	6
Ducat of Spraine.	2	6	6	6
Double Ducat.	4	11	13	0
Piſtolet ſingle.	2	4	5	10
Piſtolet Double.	4	8	11	8
Ducat of Hungary	2	7	6	4
Ducat of Florence	2	5	6	4
D. Ducat of Rome.	4	13	12	8

And if you lust to see the very forme and pictures of these coynes, you shall find the most parte of them in certeyne writing Tables, set foorth by one Francke Addams booke binder of London.

But here I must tell you by the waye, that by the peny weyghts here mencioned in this table, is not ment a peny of our siluer mony (which at this daye weigheth but 10.½ graynes) but a penye of goldsmiths weight, which weigheth 24 graynes of coyns of dyze Barley. And of such pence 20. make 1. ounce, and 12. such ounces make the pound Troy.

Ground of artes, fol. 156.

Euma. I am very glad that you tell me so, for I vnderstoode nothing els before but a very peny weight of our sterling money.

Philo. Now for asmuch as my purpose is to shewe you, how you shall reduce all maner of moneyes to an equalitie in valure touching exchanges, I will heere first insert such notes touching the monyes of diuers other citties and places, as M. Mellis hath painfully collected in his addition to the Ground of arts, which are these insuing.

Ground of artes fol. 538. Naples.

At Geane (sayth he) they vse the Soulx, one Ducat maketh 3 pound.

At Naples they vse Ducats, Tarris and Graynes, the Ducat maketh 5. Tarris, and 1. Tarrie 20. Graynes, but they take 6. Ducats (which maketh 30. Tarris) for the ounce.

A Ducat maketh 10. Carlins, and 1. Carlin 10. Graynes, so that 2. Carlins make a Tarry, and 100 graynes make a Ducat.

Rome.

At Rome they vse the ducats of the Chamber, whereof 1. maketh 12. guyles, and euery guyle 10. soulx.

Venice.

At Uenice they vse ducats currants at 124. soulx the peece, or 24. deniers, and euery denier is wooxth 32 picolis.

Palerma and Messina.

At Palerma and Messina, they wite after the ounce, Tarry, and graynes, and 1. ounce is equall to 6. ducats or 30. tarris, euery tarrie is 20. graynes, and euery grayne 6. picolis, and also 1. ducat is wooxth 24. carlins.

Millane.

At Millane they vse r. s. and d. of ducats imperiall, and the ▽ of exchange is wooxth 4. pound.

Lucques. Florence and Aucona.

At Lucques, Florence, and Aucona, they vse the ▽ of golde in golde, the French crowne is wooxth 7. pound, but at Bolonia it is 3. pound 10. shillings.

Barselona.

At Barselone they vse Soulx, the Ducat of exchange is wooxth 22. Soulx.

Valentia and Saragossa.

At Ualentia and Saragossa, they vse the Liure, Soulx, and

Denier

denier the french ▽. of exchange is worth 20. soulx, and 1. soulx is 12. deniers.

At the fayres of Castile they vse marueydies, and the ducat is worth 374. marueydies as I tolde you before, and sometimes 375. Castile.

At Lisbourne, they vse the Res, one Ducat of exchaunge is worth 400. Res. Lisbona.

At Nureinberge, Frankforde and Augusta in Germanie are vsed Kretzers, whereof 60. make a florine. Nuremberg Frankford &c.

At Andwerp (as I told you before) they vse shillings and deniers de groes, to wit our English peny. Andwerp.

As for London, I thinke I shall not need to tell you that there they vse pounds, shillings, and pence sterling, and that they exchange by the peny sterling. London.

And forasmuch as the greatest diuersitie of exchaunges in any one place, is at Lions in Fraunce, therefore I will first begin to shew you the orders of that place, where you shall vnderstand they haue ordinarily 4. fayres in a yeere, at which they commonly make their exchange, which are from 3. moneths to 3. moneths, and that in 3. seuerall sorts for the Cities and Townes following. Ground of Arts. fol. 537.

First they deliuer at Lions 1. Marke of gold to haue or receiue at Naples, almost 41 ½. ducats: at Venice 70. ducats Currants, at Roome 63. ducats of the chamber, at Lucques and Florence 65. ▽. of golde, and at Millaine 82. ▽. The exchange at Lions to other places.

And contrariwise at the said Cities, they giue so much of their moneys to haue a Marke at Lyons.

Secondly at Lyons they giue 1. ▽. de Marke of 45. soulx Turnois, to haue therefore at Geane almost 68. soulx: at Palerma and Messina, almost 24. carlyns: at Barselone 22. soulx: at Valentia or Sarragossa 20. soulx, at the fayres of Castile 350. marueidies, at Lisbone 360. Res, at Andwerpe 57. Deniers de grosse, and at London 70. pence, or 5. shillings 10. pence sterling.

And contrariwise, in the saide cities and places, they giue almost as much of their moneys to haue one Crowne de Marke at Lyons.

Thirdly at Lyons they giue a crowne of the Sunne to haue almost 93 Kretzers, at Francfeforde, Augusta, Nuremberge, and other cities of Almayne.

Also at Lyons onely, they paye ½ of their exchange in golde,

and $\frac{1}{3}$. in money, or els all in money for giuing $1\frac{1}{3}$. vppon the 100.

Euma. It seemeth by this which you haue sayd, that Lions is a very famous and notable mart Towne.

The exchange of Naples to other places.

Philo. There is no doubt of that, but to proceede with the exchaunges of other places, at Naples they deliuer almost 112. ducats, to receiue at Rome 100. ducats of the chamber of the olde valure.

Through Lucques and Florence, they deliuer 100. ducats Carlins to receiue there almost 86 ▽. of gold.

Through Palerma and Millina one ducat of 5. taris to receiue there almost 164. graynes.

Throughe Millan 1. ducat, to receiue there almost 90. soulz.

Through Geane 1. ducat, to receiue there almost 65. soulz, the whole sum to be paide within 10. dayes after sight of the billes of exchaunge.

Also at Naples they deliuered one ducat to receiue in Andwerp almost 67. deniers de grotes, within two moneths, and at London almost 5 s. starling in 3. moneths, at Barsalon almost 20. soulz within two moneths, at Valencia, almost 18. soulz within two moneths, at Lisbourne 333. Res within three moneths, and at the faire of Castile almost 340. Marbeydies at the same fayre.

The exchange of Venice to other places.

At Venice, they deliuer 100. ducats currant to receiue in Almayne almost 140. Florence, at 60. Kretzers the peece, at Lucques and Florence almost 108. ▽. of gold in 10 dayes.

Likewise at Venice they deliuer 1. ducat currant to receiue at Palerma or Millina, almost 21. Carlins, at Millayne almost 93. soulz, at Geane almost 62. soulz, the whole at 10. dayes end.

Also touching exchange, it is necessarie to vnderstand, what the Pair (or as the Italians call it the Pari) signifieth, which is nothing els, but to make the exchaunge of one place or Citie, like, or equall in valure to the money of other Cities or places, whereby the gaynes or losse vppon the exchaunge is found or knowne, which word as I suppose, is deriued from the Latin word Par or Paritas, which signifieth likenesse or euenesse, as for example, suppose I haue receiued aduise from Andwerpe that the ▽. is there woorth 7 s. Flemish, I demaund what the same ▽. is woorth at London when the Pari or exchaunge at Andwerp goeth for 23 s.

69. A question of the valure of the French crowne.

For

For answer hereto, I resort to the golden rule direct, saying thus, If 23 s. Flemish, be worthe but 20. s. sterling at London, what is 7 s. flemish worthe there, and the rule answereth me $6\frac{5}{11}$ s. as heere appeareth.

flemish.	sterling.
23 s.	20 s.
7	$6\frac{5}{11}$

That is 6 s. 1 d. $\frac{1}{11}$ d and so much is the ▽. then worth at London, by paritie or equitie of the exchange.

Now these foundations being laide, and taken (or at the least admitted) for true. I trust God willing to shew you the reduction of these forreigne Coynes or moneys to ours, or of any one of them to other, without any tables or other rules then you haue already learned.

For if it be true, that 6. pound English make 7. pound flemish, and that 1. pound English maketh 10. pound French or Turnoys, although I know nothing what proportion is betweene Frenche money and flemish in equalitie, yet may I quickly finde it by one of the compound rules discending or ascending, which rules you haue learned already.

And for an example. If I would know how many pound flemish 400. Frankes or Liuers Tournoys make, I neede to say but thus. If 7. pound flemish = 6 pound sterling, and 1. pound starling = 10. pound Turnoys, how many pound flemish = 400. frankes turnoys? and so annexing the termes to the figure, the question forthwith sheweth it selfe (by the 284. verse) of the 226. Maxim) to be subiect to the compound rule discending, whose operation being performed according to that Maxim, bringeth $46\frac{2}{3}$ pound Flemish for the answering terme, as heere appeareth.

70. A question of reducing Flemish money into French.

flemish.	sterling.
7. l.	6. l.
sterlin	turnoys
1. l.	10. l.
turnois	flemish
400. l.	$46\frac{2}{3}$ l.

And so many pounds flemish is equall in valure to 400. pound French or Turnoys.

Agayne, if I know the equalitie betweene French money and English, and betweene french and flemish money yet may I soone finde it by one of the compound rules aforesayd, as for example. I would know how much flemish money is equall in valure to 100. pound starling: for the finding whereof, I come forthwith to the figure with my numbers, saying thus, if 1 l. starling make 10. l. french, and 400 l. french make $46\frac{2}{3}$ l flemish, how many pound flemish will one hundred pound starling make? and seeing the

71. A question of reducing Flemish money to sterling.

queſtion by the 290. verſe, appeareth ſubiect to the compound rule aſcending, I vſe the operation of that rule accordingly, whereby I finde the anſwere 116 $\frac{2}{3}$ l. flemiſh as appeareth by the fabrique: wherefore I conclude that ſo many pound Flemiſh, is equivalent to 100. pound ſterling.

ſterling.	French
1. l.	10. l.
french	flemiſh
400 l.	14 $\frac{4}{7}$ l.
ſterlin.	flemiſh
100. l.	116 $\frac{2}{3}$ l.

72. A queſtiō of reducing Spaniſh money to ſterling.

Euma. Truly, now I perceiue very plainlye, that by the ſaid compound rules diſcending and aſcending, any other like moneys may be reduced to an equalitie, for I may aſwell ſay. If 70. ß ſterling be woꝛth 1. ▽. de marke, and 1. ▽. de marke woꝛth 350. Marueidies, how many marueydies are woꝛth 100. l. ſterling?

Philo. Yea, you may ſay ſo very well, but then in application of your termes to the figure, you muſt eyther turne the 100. pound all into pence, oꝛ els you muſt turne the 70. pence into partes of a pound, that ſo they may matche together, & be genderlike, as the rule requireth, and as you ſee me to haue ſet them heere.

pound ſterl.	▽.
$\frac{7}{24}$	1.
▽.	maruel.
1.	350
ſterl.	
100 l.	

Euma. That I conſider to be reaſonable: which done, the queſtion declareth it ſelfe ſubiect to the compound rule aſcending, by perfoꝛming whoſe operation, I finde the laſt terme to be 120000 Marueydies: and therefoꝛe I conclude that ſo many Marueydies are equiualent to 100. pound ſterling.

73. A queſtion of reducing money of Palerma to ſtarling

Philo. They are ſo: now let me ſee how you can finde what ſterling money one of the Carlyns of Palerma is woꝛth?

Euma. I remember you ſaid not long agon, that at Lyons they delyuer a Crowne de Marke to receiue therefoꝛe at Palerma 24. Carlins, wherfoꝛe ſeeing that 24. Carlins are woꝛth 1. ▽ de mark, and that 1 ▽. de marke, is ſayd to be woꝛth 70. ß. ſterling, I ſay thus by the compound rule diſcending. If 70 ß. = 1. ▽ de mark, & 1. ▽ de marke = 24 Carlins, how many pence, oꝛ how much ſterling money is equall to 1. Carlyn?

ſterling.	de marke.
70 ß.	1. ▽.
de marke	carlins.
1. ▽.	24
Carlins	ſterlinꝝ
1	2 $\frac{11}{12}$. ß.

And the operation of the rule anſwereth 2 $\frac{11}{12}$ pence, as heere appeareth by the fabrique: and therefoꝛe I conclude, that one of the Carlins of Palerma is woꝛth ſo much ſterling.

Phil. Thus you ſee, how by one of theſe two rules, the values of all foꝛreine coynes and moneys, may be reduced both in equali-

equalitie to ours, and also one forreyn money to another, for if the question had beene, how many Carlins is worth 240. ß. sterling, then the former supposition of the question, continuing as it doth, the question had beene subiect to the compound rule ascending, and the answere would haue been 82 3/7 Carlins, as here appeareth.

74. A question of reducing Carlins to sterling.

sterling	demarke.
70 ß	1 ▽.
demarke	Carlins
1 ▽.	24.
sterling	Carlins
240 ß	82 3/7.

Whereby it is manyfest that 82. 3/7. Carlins are equiualent to our 20 s. or pound sterling.

Euma. All this I perceiue well, and further I imagyn that by the selfe same rules, the weights and measures of forrayne countries, may be reduced to an equalitie with ours, and ours with theirs, and so each of them with other.

Philo. Your imagination is true, as I will shew you by examples anon, but first I will propound some questions incident to the exchaunges of money, whereof as yet I haue sayd nothing in effect.

Of the exchaunges of money, from one Realme or Citie to another. The 11. Chapter.

Forasmuch as I haue told you before, that the equalitie of the exchange of money dependeth chiefly vpon the 2. compound rules discending and ascending, in the practise whereof I finde you very perfect and ready, and for asmuch also as there are no newe rules to be learned touching the same, therefore I say, (without distrusting your habilitie to answer any question in this behalfe) I will proceede to the same questions immediately, wherin first I begyn thus.

A marchant deliuered in exchange 100. l. sterling in London to receiue 1000. franks Turnoys therfore in Roan at sight, which when he had there receiued, he reuelliered the same there to another marchant by exchange, to receiue the valure thereof in Flemish money at Andwerp at sight, now I demaund how many l. Flemish he ought to receiue for those 1000. Frankes Turnoys.

75. A question of reducing Flemish money to Turnoys.

Eu. For answere hereunto, sith it is affirmed before that 7 l. Flemish, make 6 l. sterling, and that 1 l. sterling maketh 10. frankes or 10 l. Turnoys. Therefore I forme the question to the compound rule, saying thus. If 7 l flemish make 6, l. sterling, and

1 l. sterling make 10 l. turnoys how many pound flemish will 1000 l. turnoys make? and so applieng the termes orderly to the figure, the question appeareth presently to be subiect to the compound rule discending, by performing whose operation, I finde the answering number to be 116 2/3 pound, as heere appeareth. And so many pounds flemish ought he to receiue at Andwerpe.

Flemish.	sterling
7 l.	6 l.
sterling.	turnoys.
1 l.	10. l.
turnois.	flemish.
1000 l.	116 2/3

76. A question of reducing Spanish money to English.

Philo. You haue done well: againe. A merchant deliuered in London 100 l. starling in exchange, to receiue for the same in Bayon or any other Citie of Hispaine 1. Duccat of 374. Maruedies for euery 5 s. 10 d. of the laid 100. pound. I demaund how many Maruedies payeth the billes of exchange?

Euma. For answer to this, I forme the question thus, saying. If 374. Maruedies = 1. Duccat, and 1. Duccat = 70 d sterling, how many Maruedies = 24000 d. sterling? for sithe the rule requireth, that each couple of the bynas termes match together in sirname, and seeing also that the 5 s. 10. d sterling, cannot aptlye be put in any other sirname then pence. Therefore I reduced the 100 l. sterling to like sirname, and so applying them to the figure, I see straight wayes that the question is subiect to the compound rule discending, by the performance whereof, I finde the answering terme to be 128228 4/7. and so many Maruedies shall he receiue for his 100 pound sterling, as here the present fabrique sheweth.

Maruedeis.	Ducats.
374	1.
Ducats.	sterling.
1	70 d.
sterling.	Maruedes.
24000 d	128228 4/7.

Philo. You haue answered truly, and wrought orderly, and it was not onely well done of you, to reduce all the sterling money to pence, as you did, but also the very best way to auoyde fractions.

77. A question of reducing sterling money to Spanish.

Agayne a marchant deliuered in Bayon, 5000. spanish Royals, to receiue for the same in London 5 s. 10 d. sterling for euery ducat of 11. Royalls, I demaund how much sterling money payeth the bills of exchange.

Euma. To answer this I reduce, the sterling money to pence agayne as before saying thus, if 11 Royalls = 1 ducat and 1 ducat = 70. d. sterling, how many pence sterling = 5000. Royalls? and by applycation of the termes to the figure I perceiue immediatly that the question is subiect to the compound rule ascending, where-

fore

fore working after the order belonging to that rule, I finde the answering terme to be 3 1818 3/11 d. as here the fabrique therof declareth: which by reduction commeth to 132 l. 11 s. 6 3/11 d. sterl. so much sterling mony paieth þ bils of exchange at Londō.

Royal. ducats.
11 — x
ducat. sterling.
x — 70 d.
Roial sterling.
5000 — 3 1818 3/11 d

Phi. It doth so, and (as I sayd before) although such reduction to the least denomination be best for the auoiding of fractions, yet if you would you might haue vsed these termes in the figure which I haue vsed, and so working by the rule, you should haue found the aunswere 132 10/11 l. sterling, which fractiō reduced maketh iust 11 s. 6 3/11 d. as before.

Royalls ducats
11 — x
ducats sterling
x — 7/24 l.
Royalls sterling
5000 — 132 10/11 l.

Euma. I perceiue it well, but I vsed the former order purposely as that which I liked better at that time.

Philo. You did well: but what say you to this? a marchant deliuered in London 100. pound sterling to receiue in Paris 1 french crowne or 50. soulx turnois for euery 5 s. 3 d. of the sayd 100. l. now how much money turnoys satisfieth the billes of exchange.

78 A question of reducing French money to sterling.

Euma. For answer hereof, although I might reduce all the sterling money to pence agayne as I did before, yet now for a change I will put the 5 s. 3 d. into parts of a pound sterling, so will it be 21/20 l. which done, I say thus.

If 50. soulx = 1. ▽. and 1. ▽. = 21/20 l. sterling, how many soulx = 100 l. sterling? and so working by the compound rule discending, to whome the question sheweth it self subiect, I finde the answer to be 19047 13/21 soulx as appeareth by the fabrique: wherfore I conclude that so much French or turnois money satisfieth the billes of exchange.

soulx ▽.
50 — x
▽ sterling
x — 21/20 l.
ster. soulx
100 l — 19047 12/21.

Phi. It doth so, and if the question had beene how many french crownes the marchant should haue receiued, you might haue framed the question thus, saying, if 1 ▽ = 50. soulx and 50. soulx = 21/20 l. sterling, how many ▽ = 100 l. sterling, and so working by the former rule (vnto which the question continueth still apparantly subiect) you should haue found the answering number to be 380 20/21 ▽. as heere appeareth.

79. A question of reduing French crownes to sterling money.

Crownes. Soulx.
1 — 50
soulx. sterling.
50 — 21/20 l.
sterling Crownes.
100 l — 380 20/21

And so many french Crownes should he haue receiued for his 100. pounds sterling.

Eu. Very good this I conceiue easily.

Phi. Then answer me to this, a marchant deliuereth at Lisbourn in Portugall 2500. Royalls of plate, to receiue for euery ducats of 10. Royalls the peece 4. Dollars at Hamborough, I demaund how many Dollars payeth the billes of exchaunge.

80. A question of reducing Ducats & Dolors together.

Eu. For answer I frame the question thus, saying: if 10. Royals = 1. ducat, & 3. ducats = 4 Dollars, how many Dollars = 2500 Royalls, the termes of which question being orderly placed in the figure, immediately argue the same to be answerable by the compound rule ascending, whose operation performed as that rule requireth, bringeth 333 $\frac{1}{3}$ Dollors for the answering terme, whereupon I affirme that so many dollars discharge the billes of exchange, as here the fabrique expresseth.

Royals	ducats
10	1
ducat	dollo.
3	4
royal	dollo.
2500	333 $\frac{1}{3}$

Phi. Well now suppose that I take vp money at Andwerp after 198. 6 d. the pound Flemish, to pay for the same at London 20 s sterling, and when the day of payment is come, my factor there, for want of readie money, is forced to take vp as much money of some other merchant in London, as will serue to discharge my billes of exchange, which he returneth to me agayne to pay at Andwerpe by other bils of exchange, at 19. shillings 9. pence Flemish, for euery pound sterling by him so taken vp.

81. A question what is wonne or losse by the exchange.

Now I demaund whether by this exchange I win or loose, and how much vpon the 100 pound.

Euma. I pray you worke the solution of this question your selfe for my better instruction in the like.

Philo. Contented, and for performance thereof after that I haue reduced the mixt numbers, I frame it in this order saying, if 1. pound sterling yeelde $\frac{19}{20}$ shillings Flemish, and $\frac{19}{4}$ shillings Flemish, yeelde 1. pound returned, how many pound returned will 100 l. sterling yeelde, which termes annexed to the figure orderly, shew forthwith that solution of the question pertayneth to the compound rule ascending by performing wherof I finde the answering terme to be 98 $\frac{18}{19}$ l. as here the fabrique witnesseth:

sterling.	Flemish.
1 l.	$\frac{19}{20}$ s.
Flem.	retur.
$\frac{19}{4}$ s.	1 l.
sterl.	returned
100 l.	98 $\frac{18}{19}$ l.

Whereby it is easie to be gathered, that if

if 100 r. sterling thus returned yeeld me but 98 $\frac{11}{73}$ r. then I must needs loose by this exchange so much in the 100. as 98 $\frac{11}{73}$ r. lacketh of 100. which by subtraction appeareth to be 1 $\frac{62}{73}$ pound.

Euma. It must necessarily be so by common reason.

Philo. Yet this question may be aunswered also, by the single golden rule direct, aplying onely the Flemish money thereto, and omitting the sterling money, because the valure thereof is constant and not changeable, as the Flemish money is, whereof seeing that for 19 $\frac{1}{2}$ s. receiued, I must repay 19 $\frac{1}{4}$ s. agayne, it is apparant that I loose 3 d. in euery 19 s. 9 d. payd. The same question otherwise answered.

Wherefore I may by the sayd rule best and soonest say thus: if 19 $\frac{1}{4}$ s. loose 3 d. what doth 100 r. loose, and the rule answereth 1 $\frac{62}{73}$ r. as here appeareth. and now do you answere me to a like question which is this.

72 s.	1	s.
A		A
100 r.	1 $\frac{62}{73}$	r.

A merchant of London taketh vp there 20 s. sterling to pay therefore at Andwerpe 21 s. 8 d. Flemish, and when the day of payment is come, his factor for lacke of readie money, taketh vp money agayne at Andwerpe, wherewith to satisfie the billes of exchange due, which he returneth agayne to his maister by other bils of exchange to pay at London 20 s. sterling, for euery 22 s. Flemish by him so taken vp. 82. A question of loose or gaynes by the exchange.

Now the question is, how much this merchant doth win or leese by this exchange vpon the 100. pound.

Eu. For as much as his factor first receiues at Andwerpe 22 s. Flemish, for that which afterwarde he payd but 21 s. 8 d. Flemish it is manifest that he saueth or winneth 4 d. cleare in the payment of euery 21 s. 8 d. Flemish (for as the sterling money, I cannot see but the one 20 s. payeth the other, & for that cause I perceiue ỳ same may be wholy omitted) and therefore vsing but the single golden rule as you did, I say thus: if 65. grots gaine 1. grote, what shall 100. r. gaine? and so vsing the rule, the quotient yeeldeth 1 $\frac{7}{13}$ r. as here appeareth: wherefore I conclude that by this exchaunge he gaineth after so much in ỳ 100 r. which by reductiõ is 30 s. 9 $\frac{1}{13}$ d.

65 grs.	1	gro.
100 r.	1 $\frac{7}{13}$	r.

stock gaynes.

Phi. You haue done well, and in truth haue gone the neerest way to worke, but yet let me aske you this question. 83. A question of exchange whereby to gaine 8 in 100.

A marchant in London deliuered 200 r. sterling in exchange for Paris at 5 s. 10 d. the ▽. of 50. soulz turneys, the question is at what price the same ▽. ought to be returned againe to London, there

there to gayne after 8 r.in the 100 sterling by the exchange?

Euma. For answere hereunto I say thus, if 1 ▽ make $\frac{7}{12}$ r.ster. and 100 r.sterling, make 108 r.returned, what money returned, shall 1.▽ make or yeelde, now tye termes are no sooner duly setled in the figure, but the question by and by appeareth triable by the compound rule ascending, whose operation performed byngeth 75.$\frac{5}{7}$ d. for the aunswering terme as here the figure sheweth.

▽	1	sterling.
sterlin	$\frac{7}{12}$	pound.
returned	100 r	returned
▽	1	108 r.
returned	75$\frac{5}{7}$	d.

And therfore I conclude that for so much must the ▽ be returned in exchange to London to gayne thereby after 8.in the 100.pound.

Philo. You haue done well, wherefore I will put but one question more touching exchanges, which I will also aunswere my selfe, and so we will end, and it is thus.

84. A question how to gayne 10.in 100 by the exchange.

I deliuer 200 pound ster.in London to receiue at some one towne in Spayne for euery 5 s.10 d.thereof, 1.ducat of 11. Royals, or 374. Maruedies, which by aduice I wil my factor there, to returne for London agayne by exchange, but yet so that I may gayne thereby after 10.in the 100.

Wherefore the question is, at how many maruedies he ought to returne the ducats to satisfie my request.

Now to answere this and all other such such like questions, first I must seeke by the single golden rule direct, how many maruedies are contayned in the whole 200.pound, saying thus, if 5 s.10 d. sterling, yeelde 374.meruedies, what will pound ster. marue. 200.pound yeelde, and the rule answereth 256457$\frac{1}{7}$ maruedies as here appeareth.

	$\frac{7}{12}$	374.
	200	256457$\frac{1}{7}$

This being knowen I adde the gaynes required, to the stocke employed, and it maketh 220.pound for stocke and gaynes (for you know that seeing I require to gayne 10.in the 100. therefore my 200.pound stocke must needes yeelde me 220.r.or els my request is not performed) which being done I resort once agayne to the golden rule saying thus, if 220 r.stock, ought to come of 256457.$\frac{1}{7}$ maruedies stocke, what ought there to come of 5 s.10 d.or $\frac{7}{12}$ r. stock, and the rule answereth 340 maruedies as here appeareth by the fabrique.

sto.& gay.	220 r	179$\frac{5}{7}$
marue.	$\frac{7}{12}$ r	340

And for so many Maruedies ought my factor to retume the ducat to the end I may gayne 10.pound in the 100.by the exchange, and thus much for exchanges.

Of

Of the Reduction of diuers and ſundry Weights and meaſures to ours, and ours to theirs, and of reducing each of them to other. The 12. Chapter.

That the compound rules deſcending and aſcending all maner of weights and meaſures, may be reduced to an equalitie, you your ſelfe could partly eſpie and perceiue, as appeareth by your owne woordes heretofore, but for playner declaration thereof, I wil propound ſome queſtions ſeruing to that purpoſe, whereof take this for the firſt.

If four 112. pound weight of London or rather of England, make 98 pound at Roane, and that 100. pound of Roan weight maketh 112. pound at Seuill, (as I am enformed it doth) then how many pound of London or Engliſh weight maketh 100. pound of Seuill weight. 85. A queſtion how to reduce London weight to Seuill weight.

Euma. This I can anſwere very readily, for hauing applied the termes to the figure, their ſituation ſheweth preſently that the aunſwere belongeth to the compound rule deſcending, by whoſe operation I finde the aunſwering terme to be 102 $\frac{2}{49}$ as here the fabrique declareth.

London	Roane
112	98 l.
Roan	Seuill
100 l.	112 l.
Seu	Lond.
100 l.	102 $\frac{2}{49}$ l.

Whereby I conclude that 102 $\frac{2}{49}$ l. of London weight maketh 100 l of Seuill weight.

Philo. You ſay true, and ſeeing you are ſo quicke, now tell me alſo how many pounds of Seuill weight maketh 112. pound of London weight. 86. A queſtion how to reduce Seuill weight to London weight.

Euma. For anſwere thereto, I perceiue the termes of the former ſuppoſition continue ſtill as they did, but the demaunding terme onely is chaunged, wherefore after that the termes are duly placed in the figure, I may ſee foorthwith that the queſtion now is become ſubiect to the compound rule aſcending whoſe operation perfomed (as the 231 maxim of the ſecond booke erhorteth (deliuereth 109. $\frac{15}{17}$ pound for the aunſwering terme, as heere the fabrique ſheweth.

London	Roane.
112 l.	98 l.
Roan	Seuill
100 l.	112 l.
Lond	Seuill
112 l.	109 $\frac{15}{17}$ l.

And therefore I ſay that ſo much Seuill weight = 112. pound of London weight. 87. A queſtion how to reduce Engliſh weight to Rochell weight.

Philo. It is ſo, now ſuppoſe that 26. pound of London weight maketh 25 l. Flemmiſh or of Andwerpe weight, and that 25. l. Flemiſh.

Flemish = 29 l. of Rochell weight. Then I demaund how much London weight = 100. l. of Rochell weight?

Euma. These numbers truly placed in the figure, shew foorthwith that the question is vnder the compound rule descending, by whose operation (abridged as the aforesaid 231. Marim requireth) I finde the answering terme to be 8 $\frac{12}{13}$ l. & so many pound of London weight will make or be equall to 100 l. of Rochell weight, as heere the fabrique testifieth.

London.	flemish.
26 l.	25 l.
Flemi.	Roche.
25 l.	29 l.
Rochel	Londō
100 l.	89 $\frac{12}{13}$ l.

88. A question how to reduce Rochell weight to English weight.

Philo. Well said: Now declare likewise how many pound of Rochell weight will make 112. l. English.

Euma. To do that, I do but change the demaunding terme onely of the former question and fabrique, from 100. pound of Rochell weight, to 112. pound English, by doing whereof, the question foorthwith chaungeth also, becomming now subiect to the compound rule ascending: and therfore persecuting the worke, as that rule requireth (and abridging the labour therein also by the sayd 231. Marim, as I did in the former question) I finde the answer to be 124 $\frac{12}{13}$ pound, as heere the figure sheweth.

English.	flemish.
26 l.	25 l.
flemish.	Roch.
25 l.	29 l.
English	Roch.
112 l.	124 $\frac{12}{13}$ l.

Wherefore I conclude, that so many pound of Rochell weight = 112. pound of English weight.

Philo. Very well. Thus as you haue learned to reduce the weights of London, Roane, Siuill, Andwerpe, and Rochell, each to other: so may you (by these two rules) doe with the weights or measures of any other Cities or countries (whose proportion together is knowne) without any further rules or Tables, and yet for your more ease therein, after the end of this Booke, you shall finde two Tables, one of the proportion of the weights, and the other of the proportion of the measures of diuers forreigne cities to ours: which two Tables containe the effect of 42. seuerall tables, commonly printed with Maister Humfrey Bakers Arithmeticke, which I speake not in disgrace of him, whom I reuerence, nor of his worke, which I greatly commend, but onely to shew the matter with some more breuitie. And in the meane season, I will infer certaine questions and familiar examples, touching the reduction of some forreine measures to an equalitie, & so will commit you to God, leauing the further vse and application which thereby may be collected

collected to your industrious consideration at more leasure.

And to begin, you shall understand that 10. elles Flemish, make 6. elles English, and 10 elles English make 18. braces of Uenice, wherefore I would know how many elles Flemish, make 100. braces of Uenice.

89. A question to reduce flem. ells to Venice braces.

Euma. That will I tell you shortly Godwilling, wherefore I applye the numbers orderly to the figure, which done the question sheweth it selfe by & by to belong to the compound rule descending, by the vse of whose operation I finde the answere to be 92 16/17. ells Flemish as here appeareth.

Flemish	English
elles 10	6. elles
Engl.	venice.
els 10	18. bra
Uenice	Flem.
br. 100	92 16/17 el

And therefore I conclude that so many ells Flemish will make (or are equall to) 100 elles Flemish.

Philo. You say true, now tell likewise how many braces of Uenice, will make 100. elles Flemish.

90. A question to reduce venicio braces to flemish ells.

Euma. To do that I change the demaunding terme onely of the former figure, whereby the question changeth also, ipso facto, and becometh subiect to the compound rule ascending, by the execution of whose operation I finde the answere to be 108. wherefore I conclude that so many Uenician braces = 100. elles Flemish as here appeareth by the fabrique.

Flemish.	English.
elles 10	6. elles
Engli.	Uenice
elles 10	18. bra.
Flemish	Uenice
els. 100	108 br.

Philo. All this is well done, now let me see, if you can finde how many braces of Uenice will make 100. ells English?

91 A question of reducing brace of Venice to ells English.

Euma. That shall you also soone see me do, and to performe it I say thus. If 6 ells English make 10. ells flemish, and 100. ells flemish, make 108 braces of Uenice, how many braces of Uenice make 100. ells English? and so applieng the numbers to the figure I see immediatly that the question is to be aunswered by the compound rule ascending, which executed accordingly bringeth 180. for the aunswer as heere appeareth: wherefore I conclude that 180. Uenician braces = 100. ells English.

English.	flemish.
els 6.	10 els
flemish.	venice.
els 100.	108 br.
English.	venice.
ells 100.	180 br.

Phi. You haue done well inough, but yet in this case, forasmuch as you knew before that 10. ells English make 18. braces of Uenice, you neede not to haue vsed but onely the single golden rule, saying thus. If 10. ells English make 18. braces, how many will make 100. ells English? and the rule an-

answereth you 180. braces euen as before, as this figure sheweth.

elles eng. braces.
10 — 18
100 — 180

92. A question to reduce palmes of Geane to els English.

But now if 5. yardes english = 4. elles English, and 15. elles of Palmes of Geane = 13. yardes English (as indeede they are) how many Palmes = 100 elles English.

Euma. This question I see is propounded disorderly, wherefore first I reduce it to order thus, saying if 4. elles English = 5. yards and 13. yardes = 50. Palmes, how many palmes = 100 elles English, and so it sheweth it selfe forthwith answerable by the compound rule ascending, the practise of whose operation yeeldeth 480. $\frac{10}{11}$ palmes for aunswere, wherefore I conclude that so many palmes of Geane = 100. elles English as here appeareth.

elles. yardes.
4 — 5
yards palmes
13 — 50
elles palmes
100 — 480 $\frac{10}{11}$

93. A question reducing English els to Palmes of Geane.

Philo. Well sayd, now declare likewise how many elles English are equall to 100. palmes of Geane.

Euma. That I will quickly do now, euen by the single golden rule onely, for seeing I know that 480. $\frac{10}{11}$ palmes, are equall to 100 elles English, I say thus, if 480. $\frac{10}{11}$ palmes = 100. elles English, how many elles English = 100. palmes, and the rule answereth me 20 $\frac{5}{11}$ elles English as here appeareth.

palmes. elles.
$\frac{525.0}{11}$ — 100
100 — 20 $\frac{5}{11}$

Wherefore I conclude that so many elles English make (or are equall to) 100. palmes of Geane.

94. A question reducing Lucques mesure to Andwerp measure.

Philo. Well yet let me aske you this question, if 4. elles at Andwerpe = 5. elles at Franckfoord, and 25. elles at Franckfoord = 24. braces at Lucques (as indeede they are) how many braces of Lucques measure = 100. elles of Andwerpe measure.

95. A question reducing Flemish elles to Braces of Lucques.

And agayne how many elles of Andwerpe measure = to 100. braces at Lucques.

Eu. The first of your questions is subiect to the compound rule ascending, by whose operation I finde that 120. braces of Lucques measure are equall to 100. elles of Andwerpe measure as the fabrique here expresseth.

Andwerp Franckfoord
elles 4. — 5. elles.
Franck. Lucques
elles 25 — 24. braces.
Andwe. Lucques
els 100 — 120. braces.

And the second is vnder the compound discending, whose operation orderly accomplished sheweth that 100. braces at Lucques = 83 $\frac{1}{3}$ elles of Andwerpe measure as here

here the fabrique maketh mention.

Phi. You haue answered well, and euen after the like maner may you reduce any other measures to an equalitie together, whether they be yardes, or elles of England, Alues of France, Uares of Spayne, or Portugal, Palmes of Geane Braces of Italy, or Elles of Germanie, whereof to set any mo particuler examples may seeme rather superfluous then necessarie, especially to you that are so well practised in the application of these rules to that purpose alreadie.

Andwerpe.	Franckford.
elles 4.	5 elles.
Franckf.	Lucques
elles 25.	24. braces
Lucques.	Andwerpe.
braces 100.	83 $\frac{1}{7}$ elles.

For after the same maner you are to reduce all other measures as Dutch miles, whether they be great or small, French leagues, Italian miles, or miles of any other countries, also Lasts, Muyes, Vanages, Alkers, quarters, bushels, gallons or other measures whatsouer, wet or drie.

Wherefore trusting that this which I haue sayd is sufficient to make you a skilfull Merchant, I will here Claudere riuos, quia iam sat prata biberunt, and so will bid you farewell till I meete you agayne, what time godwilling I will teach you the extraction of rootes and arte of Arithmeticke figurall, commonly called Algebra.

And in the meane season doe not you cease to practise, that which you haue alreadie learned, till you feele your selfe fully & throughly ripe in euery point thereof.

Euma. That shall I not onely do withall mine endeauour, but also pray dayly to God, to send you such good health and leasure to accomplish your promise, as may discharge you thereof, and satisfie my expectation.

Finis laus Deo.

A plentifull table in order of the alphabet whereby the reader may readily finde any queſtion or other principall matter contayned in theſe 3. bookes, wherein the firſt number ſignifieth the leafe, the letter b. the ſecond ſide of the leafe, and v. the verſe.

ABatement what it ſignifieth. fol. 19
Abridgement of multipli. 29. 81. b
Abridgement of diuiſion. 41. b
Abridging Reduction. 75 b. v. 161
Abſolute numbers defined. 54
Abſtract numbers which. 52
Abſurditie in operation. 12. b. v. 26
Abſurde queſtions. 157. b
Accompt proceeding why. 129. b
Ace what it is. 52
Addition of Integers. 14. v. 30
Addition of fractions. 75. v. 160
Additiõ of 2. mixt nũbers. 76. b. v. 162
Additiõ of ſubtils with groſſe. 79 v. 166
Additiõ of proportions. 89. 151. v. 257
Aditiõ of terms proportiõal. 152. v. 259
Age in queſtiõ. 103. 104. 210. b. 202
Aliquot part what. 44. b
Aliquot part of a l. ſ. d. 231. b. 232.
Allome in queſtion. 258. b
Alloy what it is. 200
Alloy in queſtion. 200. b. 101.
Alloy needeful or not. 206. b. 207. 109
Almonds in queſtion. 227
Alkere what 143
Alternation what. 194. b
Amplification of termes. 110. b
Analogiſmus what. 182.
Angell in value. 67. b
Anſwering terme. 121. b. 169. 212. b
Antecedent what. 48. 88. b
Apples ſtolne in queſtion. 218
Arithmeticke defined. 1. v. 1
Article numbers which. 7. v. 15

B.

Badge or ſigne of addition. 17. v. 35.
Badge of ſubtraction. 18. v. 44
Barrell by aſſiſe. 66. b
Barters or trucques beginning. 255
Baſſe or denominator common. 69. b
Baſſe denominator & ſirname one. 53
Bate for rebate. 17. v. 39
Beaſtes carrying burdens. 21
Bequeſts diuided. 118. 155. 164. 190
Bias croſſe vſed. 29. 32. 43. 81. 212
Building of a ſhip in common. 154. b
Buying of a ſhip in common. 164. b
Blade ſheathed in numbers. 39
Boule or ceſterne filled when. 133. 147
Bouchers partners in paſture. 160
Bottle of hay in queſtion. 136. 185
Bounds of a number. 8. b. v. 18
Breaking of Integers. 77. b. v. 164. 84
Bricke wall in queſtion. 142
Broken numbers or fractions. 7
Buckerham and veluet in queſtion. 126
Buſhell by aſſiſe. 66

C.

Camericke in queſtion. 148. b
Caps trucqued for ſayes. 259
Carriage in queſtion. 169. 179
Carpenters woorke in queſtion. 148
Center receptacle of the anſwere. 212
Ceſterne emptied. 147. b
Ceſterne filled. 133. 147
Chalder what it is. 66. b
Chamblet trucqued for Canuaſſe. 258
Cinamon in queſtion. 183. 255. b
Citie beſieged. 183. b
Cloth in queſtion. 228. 238. b
Cloues in queſtion. 132. b. 204
Codfiſh in queſtion. 252. b
Commenſals expenſes. 148. b. 184. b
Common baſſe. 70
Cõmon diuidend found. 64. v. 150. 151
Common meaſure what. 4

7.v.15.
tie. 47.b.v.200.
10.b
223.b.v.316
6.12.113
queſtion. 131
ers. 52.v.107
76.v.163
allig.19.b v.305
64.v.150.151
eginning. 32
83.v.171
obers. 85.v. 175
s to no vſe. 151
.85.v.172.173
onſtrated. 83.b
46.b
211.b
d, 133
tion, 204
66.b
ic, 5.b
67
tion, 139.b
3.147.b.211
67.b.238.
67
67
67
149.184
89.b.
97.v.203
97.b.v.204
hy 180.b
ed, 180.v.293
55.v.109
ich, 55.v.112
rning, 124.b
eometri 108

Excesse

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ABatement w
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Abridging Redu
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Alloy in queſtio
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Alternation wh
Amplification o
Analogiſmus v
Angell in value
Anſwering tern
Antecedent wh
Apples ſtolne in
Arithmeticke d
Article numbe

Badge or ſigne
Badge of ſubtr
Barrell by aſſiſ

Common Pasture hired. 160
Commons weekely in question. 121
Compoũded nũbers which. 54.v.111
Compound proportion, 166
Coumpt preceeding. 130
Conclusion contrarie to the posi. 166.
Consequent what. 48.88.b
Continuall proportiõ what. 94.v.198
Continued proportion. 187.v.294
Contract numbers which. 52.53.b
Conuerse of a proposition what. 50
Conuersed proportion what. 49.96.b
Copper mixt with Siluer. 201
Coperose in question, 236.b.
Corne mixed. 194.197
Coynes valure in question. 176.b.178
Crosse byas vsed. 28.30.32.43 82,
Crosse playne added. 17.16.v.35
Cubit by assise. 67.
Conduict water in question. 133.147
Currance in question. 227.b.
Cypher set last is a disorder. 321.6

D.

Damaske in question, 233
Daye artificiall, 221
Day naturall. 67
Dayes in a weeke, 67
Dayes in a yeere, 67
Debt forgotten, 107
Debt in question. 79.106
Deduction or Abatement one, 18.b.
Degrees or places. 4.b.
Demaunding terme, which 169.
Demonstration Geometricall, 137.b.
Diaper in question. 181.b
Difference, excesse, and odds. 89.v.181
Difference betweene a part &c. 44.b.
Difference of errors first propor. 215.
Diff. of positiõs 2. proportionall. 216.
Difference of prises. 194.
Digit numbers which. 7.v.15.
Direct proportionalitie. 47.b.v.200.
Disguised numbers. 10.b
Disproportiõ what. 223.b.v.316
Distance like requisite. 6.12.113
Distance of miles in question. 131
Distinction of numbers. 52.v.107
Distinct Reduction. 76.v.163
Diuer. of answers by allig. 19.b v.305
Diuidend least. 64.v.150.151
Diuision of Integers beginning. 32
Diuision of fractions. 83.v.171
Diuision of mixt numbers. 85.v.175
Diuision of proportiõs to no vse. 151
Diuision abridged. 41.85.v.172.173
Diuision of fract. demonstrated. 83.b
Diuisor greatest, 46.b
Double position what, 211.b
Double rule mentioned, 133
Dredge pouder in question, 204
Drye measures what. 66.b

E

Egge compared to vnitie, 5.b
Elle by assise, 67
Elle of Holland in question, 139.b
Emptie vessell filled, 133.147.b.211
English coynes 67.b.238.
English measures, 67
English mile what, 67
English weights, 67
Entrenching of a Campe, 149.184
Equalitie of proportion, 89.b.
Equalitie ordinate, 97.v.203
Equalitie perturbate, 97.b.v.204
Equal termes omitted, why 180.b
Equal matchtermsomitted, 180.v.293
Euen numbers defined. 55.v.109
Euenly euen numbers which, 55.v.112
Examples the light of lerning, 124.b
Excellencie of propor. Geometri 108

Exceſſe difference and ods, 86.v.181
Exceſſe of childrens age. 103
Exception in diuiſion. 35.b.v. 68
Exceptions in the Marchants rule 227
Exchange in queſt. 196.265.166.267
Exchaning reduction. 87.v.177
Expences of bread in queſtion. 185
Expences for commons. 121.b
Expences of Gunpouder. 184
Expences of prouender. 136
Expences of vittayls. 183
Expences of wyne. 148.184.b
Extrems of a proportionality. 31.123
Extrems by place onely. 217.188
Extrems by quantitie onely 31.193.b
Extrems both by place & quantity. 31

F

Factors perſon in queſtion. 253.
Fadom by aſſiſe. 67
Falſe concluſion. 159
Falſe poſition ſingle. 207
Fals poſition doble. 211
Fals ſeruant ouertaken. 134.b
Figures numerall ten. fo.4.v.8
Figure of nought. 4.v.8
Figure of the golden rule. 121
Figure of the goldē rule cōpound. 121
Figure of a long ſword for diuiſion. 39
Figure of the rule of fals poſitiōs. 213
Finding of the matchterms. 96.12
Finding of the meane Arith. 96.v.205
Finding of the meane geo. 104.v.223
Finding of the 3. propor. 104.v.224
Finding of 3. terms propor. 105.v.226
Finding of any meane term in progreſſion Arithmeticall. 88.b.v.214
Finding of any mene terme in progreon Geometricall. 110.b.233
Finding of extrems in progreſſion Arithmeticall. 101.b.v.215
Finding of the laſt term in progreſſion Geometricall. 111.b.v.233
Finding of numbers continu pro. 112
Finding of the gre.com.mea. 64.v.148
Finding of the leſt com.diui. 64.v.150
Finding of the princi. diſburſed 156.b
Fineneſſe of the Maſe in queſtion. 205
Finger breadth by aſsiſe. 67
Firken by aſsiſe. 66
Fiſhe monſtrous in queſtion. 219
Fold, infold and multiply idem. 52
Fole and her dam. 220
Foote by aſsiſe. 67
Footemen iourneying. 104.131.145
Fountayne fayre in queſtion. 221
Forme or ſhape of the figures. 4
Fraction what it is. 7.v.17
Fraction of fractions what. 10.v.23
Fraction of fraction demonſtrated. 11
Fraction added or ſubtracted. 69.v.155
Fraction multiplied. 80.v.168
Fraction diuided. 83.v.171
Fraction returning to integers pro. 84
Fractiō proportionall which. 50 v.104
Fraction reduced, looke in reduction.
Fraction in queſtion. 137.138.b
French meaſures in queſtion 186.259
French money in queſtion. 186
Furlong what. 67
Furres trucqued for wyne. 258
Fuſtians in queſtion 182.b

G

Gaynes by cloth in queſtion. 238.b
Gayns by cloues in queſtion. 132.b
Gayne by intereſt. 135.141.b
Gaynes by kerſeis in queſtion. 132
Gaynes by Salt in queſtion 142.b.242
Gaynes by ſuger in queſtion. 133.b
Gayns by tinne diuided. 154
Gains vpon the 100.li. in que. 12.131

Gaine vpon the 100. by time 246 180
Gallon by aſsiſe. 66.b
Genderlike termes which 97.v.202
Generation of termes proportio. 112
Geometricall proportion continually in three termes onely. 108.
Geometrical proportion diſcont. 119
Geometricall progreſſion. 110
Golden rule ſingle directly. 121 v. 244
Golden rule in fractions. 129 v. 249
Golden rule di. with accompt pro. 130
Golden rule conuerſed or recipro 135
Golden rule couerſed in fractions 136
Golden rule direct demonſtrated. 113
Golden rule conuerſed demonſt. 137
Golden rule plurally direct. 138.b
Golden rule plurally, partly direct & partly conuerſed. 148
Goldē rule compounded by addi 151
Golden rule compounded by ſub. 152
Golden rule compounded by Mul. 166
Goldē rule compounded directly. 167
Golden rule com. dir. in fra. 170. v. 279
Golden rule com. cōuerſed. 172. v. 280
Golden rule com. in fract. 275. v. 282
Golden rule com. diſcen. 176. b. v. 283
Golden rule compound diſcen in fractions folio. 176 v. 286
Golden rule comp. aſcen. 177. v. 289
Golden rule comp. in fra. 178. v. 292
Gradnall numbers which. 112
Grayne weight. 66
Greateſt common meaſure. 63. v. 148
Greateſt number vnknowne. 2. v. 4. 5.
Greateſt fracti. of on ſing. p. 10. v. 22
Greater inequality. 90. b. v. 186
Groſſe reduced to ſubtills. 68. v. 153
Groſſe and ſubt. added pre. 79. v. 166
Groſſe and ſubtills ſub pr. 79. v. 167

H

Halfe a tunn what it is. 67
Handbredth in meaſure. 67
Hauerdeupoiſe weight what. 67
Hawke ſold in queſtion. 130
Hed and taile of the figures which. 121
Hilt of the ſword what. 39
Hogſhead of wine. by aſſiſe. 66
Holland cloth in queſtion. 139. 204 226.
Horſe bought, at what price. 167. b
Horſmens wages in queſtions. 184. b
Houres in the day naturall. 67.

I

Imperfect numbers, abounding &. 55
Improper reduction 78. v. 164. 165
Inch by aſsiſe. 66
Inckle in queſtion. 227
Integer what it ſignifieth. fol. 1
Intereſt in queſt. 135. 141. b. 170
Intractable numbers reclaymed. 86
Inuerſed proportion what, and how it differeth from conuerſed propor. 47
Iourney from *Chicheſter* to Londō. 104
Iourney from *Chicheſter* to Rye. 131
Iourney from London to York. 145
Iourning by length of the day. 148

K

Kerſeis in queſtion. 104. 235 255
Kilderkin by aſsiſe. 67
Kindes of proportions infinite. 91

L

Land ploughed in queſtion. 148. 180
Land taxed to reparations. 124. b
Legacies diſtributed. 118. 155. b. 183
Length of a fiſh in queſtion. 219
Leſſer Inequalitie defined. 90. b. v. 187
Leſſer number diuided by the grea. 44
Leaſt commō diuidend how. 64. v. 150
Leaſt fraction vnknowne. 10. v. 22
Leaſt number which. 2. v. 4
Liberalites Arithmetike bring ſtill one truth by diuerſitie of operations. 148

Likenes in quantitie & qualitie. 94.
Liuits of proportiõ continuall. 98.b.
Line signifieng subtraction. 21.b.v.44
Lines paralles signifieng equality. 180.
Lynes between terms signifie pro. 32.
Lings in question. 252.b.
Liquid measures by assise. 65.b.
Loane of mony in question. 135.141.
Losse by a dead debitor. 156.
Losse by ill debters. 157.
Losse by tempest. 154.b.
Losse by wheat in whole lading. 144.
Losse vppon the 100.l. 145.
Losse by repentance. 239.b.
Losse or gaine by change of price. 231 232.242.b.243.b.
Losse or gayns by exchange. 266.b.

M

Maces in question. 126
Marchants rule beginning. 226
Marke of gold what. 258.b
Mariage woorth what. 117.b
Manour bought in question 208
Masons making a wall. 148
Master robbed by his seruant. 134
Maxim vsed for a Theo. or pro. vbi.
Meane by place & not by quan. 167.b
Meane by quantity & not by place. 31
Means both by place and quantity. 23
Meane proportional arith. 98.v.205
Meane proportio. Geome. 108.v.223
Meane termes progressionall. 110
Meane prices betokē the mix. 193.b
Measurs equal or like name. 192.v.302
Measures of all sortes. 66
Measures of sundry sorts reduced to an equality. 186.268.269.
Midnumb. see meane proportionall.
Mile English what. 66
Mile Italian what. 66
Miller with 3. Milles. 147
Mixt number primarilie. 7.v.16
Mixt number secondarily. 11.v.25.86
Mixt proportion what. 92.v.192
Mixture of cloues in question. 204
Mixture of corne in question 194.197
Mixture of druggs in question. 204
Mixture of hollands in question. 204
Mixture of metalls. 199.200
Mixture of wine. 198.b
Minutes in a houre. 67
Money in bag or purse. 209.209.b
Money of diuerse kindes reduced to equalitie: 178.179.265.266.267
Money exchanged in question 265
Month of 4. weeks. fol. 67. but in questions of gaynes one the 100.l. by the yeere, or of loane or interest, 30. dayes make a moneth.
Monsterous fishes length. 219
Multiplication of Integers. 22.v.47
Multiplication of fractiõs. 80.b.v.168
Multiplica. of mixt numb. 85.b.v.175
Multi. of a propor. by a number. 187
Multiplicand and mutipli. what. 23.b
Mule and her foale in question. 220
Matchtermes which. 119.b.121.
Matchtermes found how. 125
Matchterms equal omitted. 180.v.293

N

Names or sirnames of numbers. 52.b
Natural progression what. 99
Notation of numbers. 4.v.8
Notation of fractions. 8
Notation of proportions. 32
Notation of equalitie. 180.b
Number defined. 1.v.2
Number abstract what. 52.53.b
Number absolute what. 53
Numb. commensura. & incom. 57.58

Numbers contract which. 52.53
Numbers euen and odde, primes and compounds. 54.v.110.55.v.117.
Numbers euenly euen, and vneuenly euen 54.v.112.113.
Numbers figurall which. 53.b
Numbers giuen are not infinit. 2.v.5.
Nũber greatest vnexpressible. 2.v.4.5
Num. imper. abounding defecti. 55.b
Number least certainly known. 2.v.4
Numbers perfect which. 55.v.114
Numbers primes, and compounds tother 57 b.v.132.133.
Numbers proportional which. 22.b
Numb. relatiue which. 53.b.88.v.178
Numbers of like firname. 121.v.246
Numerator or top what. 8.v.20

O.

Od numbers defined. 55 b.v.116.
Ods, difference or excesse. 88.b.v.181
Oeum or product what. 22.b.v.47
Office and end of addition. 14.v.30
Office and endeauor of Mul. 22.v.47
Office and effect of diuision. 32.v.61
Office and purpose of reduc. 6.v.152.
Operation arithmetical what. 12.v.26
Operations principall are 4. only. 12
Order obseruable in all operations. 12
Outward cyphers set foly. 41.b.v.74
Oyles in question. 231

P.

Pace geometricall what. 67
Pace single what. 65.
Payment by progres. arithme. 106
Payments payd forgotten. 107
Parcels of the total which. 14.b
Pary what it signifieth. 263
Part and parts what. 44.b.45.v.79.80
Partition between brothers. 118.191
Partition between creditors. 155.b
Partition betweene clergie men. 160.b
Partition between legataries. 118.155 157.164.191.b.
Partitiõ between merchants. 154.155
Partition between partners with sundry diuersities of stocks and times. 160
Partition between Soldiers. 155.159 208.
Partitiõ of vnknowne totals. 163.164
Passage of a iourney. 104.131.145
Pasture hired by butchers. 160
Pecke by assise. 66
Peny weight sterling. 66
Peny weight in question. 135
Pepper in question. 235.b.247.b.255
Perch by assise 67
Perfect number what. 55.v.114
Pinte by assise. 67
Pioners in question. 149
Pipe or but by assise. 67
Places or degrees in numbring. 4.v.9
Place first next the right hãd. 4.v.10
Plac. in figure of the golden rul. 121
Pla. in figure of the cõpoũnd rule. 167
Pla. proper to terms of a fract. 8.v.19
Places proper to the figure of the rule of false positions. 211.b.212
Ploughing of land in question. 148.b
Plumet of a clocke. 184.b
Pottle by assise. 67
Pound weight by assise. 67.
Preposterus propounding a quest. 121
Pricking of numbers why. 3.v.8
Primarie mixt numbers which. 7.b.
Prime numbers which. 54.v.110
Price of Diaper found. 181.182
Price of 1. ell at the first hand. 139.b
Price meane, and prices extreme. 193
Price of Satyn in question. 128.182
Prise taken by ships. 155

Prisoners

Priſoners ranſome in queſtion 217
Product or ofcum. idem 22.v.47
Proofe of Addition. 16.b.v.34
Proofe of Subtraction 22.v.43
Proofe of multiplycation. 29.v.53
Proofe of diuiſion 43.b.v.76
Proofe of the goulden rule ſingle and plural both direct and con. 150.v.254
Proofe of the goulden rule compounded direct and conuerſed. 174
Proofe of the goulden rule compounded both diſcending and aſcen. 178.b
Proofe of the rule of partners. 153.b
Proofe of the rule of alligation. 195.b
Proofe of the rule of falſe poſiti. 222.b
Proofe of numbers proporti. 32.v.59
Proofe of equal fractions. 31.v.56
Proper frac. tried frō impro. 9.b.v.21
Properties of Addition. 17.v.36
Properties of ſubtraction. 21.b.v.45
Properties of Multip. begin. 29.v.54
Properties of Diui. begin. 44.b.v.77
Properties of diuers diuiſi. 45.b.v.81
Proper. of Mul. and Diui. together 48
Properties of number euen or od. 56
Pro. of num. pri. and compo. together *alias* numb. Incommen. and com. 59.b
Properties of propor. 97.b.v.202.203
Propertie of alligation. 192
Propertie of the merchants rule. 229
Proportion defined. 90.v.182.
Proportionals defined. 22.b
Proportion of equalitie. 90.v.184
Proportion inequality. 90.v.185.186.
Proportion ſimple what. 90.v.188.
Proportion mixt what. 92.v.192.
Proportion arithmeticall. 93.v.197.
Proportion Geometrical. 93.v.197
Proportion continual. 93.v.198
Propertion diſcontinual. 96.v.199
Proportion direct which. 47.96
Proportion conuerſt or reci. 48.v.201
Proportion alternate defined. 48
Proportion inuerſed defined. 48
Proportion conioynd de. 151.b.v.258
Proportion parted defined. 152.v.161
Propor. beſt expreſt and vttered. 32
Proportionality defined. 93.b.v.196
Proportionality ordinate. 97.v.203
Proportionalitie perturbate. 97.v.204
Proportionality continued. 187.v.295
Proportionality ſingle. 122.v.248
Proportionality plurall. 138.b.v.253
Propo. tions added or ſub. 89.b.150
Proportions multipli. 90.166.v.273
Primes in queſtion. 235.b

Q.

Quantitie & quali. of pro. 94.b. v.196
Quart by aſſiſe. 66.b
Quarter by aſſiſe. 66.b
Quartern weight or Tod. 66.b
Queſtions abſurd and impoſſible. 157
Queſtions ill propounded. 123
Quichenella in queſtion. 255
Quotient number what. 33

R

Rankes of termes. 7.8.v.17.18
Rankes of nūbers. 8.v.18.97.b.v.203
Rate or propor. one thing. 89.v.182
Rclayming of vnruly num. 86.v.176
Reduction generally. 65.v.152
Reduction of groſſe to ſub. 67.v.153
Reduction of ſubtil: to gro. 68.v.154
Reduction concurning. 74.b.77
Reduction cōforming. 71.v.157.159
Reduction of particles. 70.b.v.156
Reduction abbridging. 75.b.v.161
Reduction diſtinguiſhing. 76.v.163
Reduction improper. 77.v.164.165.
Reduction preuenting. 82.v.170

Reduction

Reduction exchanging, 87.v.177
Reduction to integers pro. 85.v.173
Reduction no seuerall operati. 65.127
Reysons in question. 231
Relation in quantity what. 88.v.179
Relation in qualitie what. 93.v.194
Remainer, remnant nor rest. 17.b v.37 and fol. 34. and 88.v.181.
Reperations of a church in a que. 124
Roofe couering in question. 134
Rofe curiously built in what time. 148
Rose noble or old royall valued. 67
Rule of proportion, see golden rule.
Rule of partnership. 147.158.b.162
Rule of alligation alternate. 192
Rule of alligation medial 202.b.v.308
Rule of single false positiõs. 208.
Rule of dou. false position. 211.b. 203
Rule to reduce coines to equality. 254.
Rule of gaines or losse in the 100. with time, and without time. 239.246.247
Rule of equation of time. 244.
Rule of buying and selling by the 112 pound weight. 249.b.250
Rule of bargayning by the l. 251.b.
Rules of factorage, 253
Rules of barters or trucques, 255
Rules of exchanges. 265
Rules of reduction of weights & measures to an equalitie. 268
Running for a wager. 105

S

Salt with the gaynes in question. 142
Satine in question. 128.129.182
Scarlet and violet in question. 210.b
Secõdary mixt nũbers which. 11.v.25
Setling of termes orderly. 121.b.211
Shadow shewing height of bod. 126
Shape of the numerall figures. 4
Sheepe in question. 14.b.209
Shepherds answere to a question. 209
Ship built or bought by partn. 154.b
Ships in consort. 155
Signe of addition. 17.v.35
Signe of Subtraction. 21.b.v.44
Silks bartred for woole. 257
Siluer abassed. 201
Siluer enriched. 201.b
Simple proportiõ of 3. sorts. 91.v.183
Single proportionalitie. 122.v.248
Sirnames or denomina. of numb. 52.b
Sirnames or basses of frac. what. 53
Souereignes valued. 67.b
Span in measure what. 67
Spices with their expences in que. 131
Staffe shadow in question. 126
Stealer of apples & que. thereof 218
Stealer of plate and his que. 134.b
Stipend of horsemen in que. 184.b
Stocke or principall in que. 131.b.141
Stock and gaynes shuffled. 132.b.138
Stocke wasted by trauel. 209.b
Stockfish in question. 252
Submission of mixt numb. 86.v.176
Subscription of 1. to integers maketh them fraction like. 78.v.164. 86.v.175
Subtils added to grosse or subtracted without any apparent redu. 79.v.166
Subsidie granted in question. 17.b
Sugar with gains in questi. 133.b.220
Suppositions in al compound que. 169 [illegible] v.

T

Table of gold coynes. 258.b
Table of *Pithagoras*. 23.b
Table of al maner of numbers. 51.52
Taffata in question. 136.b 236
Tayle of the fabrical figures. 122 123
Tayle terme or answering terme. 122 b.196.b.
Tare what it is. 200.b.192

Taxing of land for reparations. 124 b
Termes of a fraction. 8.v.19.20
Termes of a proportionalitie. 94
Termes of a progression. 99
Terms genderlik or matches. 96.b.119
Terms dislike, or not matches. 96.129
Terms proportiõal added. 152.v.259
Terms propor. subtracted. 153.v.262
Testament doubtfull truly perfor. 119
Testament of *Tritus* perfor. 155.b
Theefe stealing apples. 218
Theef stealing a horse & plate. 134.b
Third propor. found how. 108.v.224
Tissue in question. 240
Todde of wooll and tun weight. 66
Topte:ms or tops of fractions 8.v.19
Totall sum what. 12.b.13
Tower partly vnder erth & water 133
Towne besiedged in question. 183.b
Tractable nũbers forvntract 84 v.176
Transformatiõ of nũbers necessary. 10
Traueilers on foote in question. 104
Treyte allowed in marchandis. 130
Trenchcasting in question. 149.184
Triall of additiõ sub. &c. lok in proofe
Triall of progression dy addition. 106
Triall of the marchants rule. 235.b
Troyeweights by stature, 66
Trucquing, with the rules therof. 255,

V

Variable ans. by varietie of alligatiõ or coupling of the extr. prics. 195.v.305
Veluet lined with Buckerham. 126
Veluet lined with Taffata. 136.b
Veluet in question. 230.b
Victuals for 6. months in quest. 183.b
Vnitie. vnit. ace or 1 idem. 52
Vnitie vndiuidable by nature. 1.b
Vnitie representeth the whole or total of any entire thing diuided into parts. 133.b.146.b.147.148.157.b

W

Wager for Runing in question. 105.b
Wages of Soldiers in question. 185 b
Wall of Bricke in question. 148
Water emptied, in what time. 147.b
Water filled in what time. 133.221
Wax bought & the price in quest. 127
Way cont. 5, quarters. 66.b
Weight of bread by assise. 135.b
Weights Troy & auerdupoise. 66
Weights of a Turnebroch discen. 184
Weights of sundry nations reduced to an equalitie. 268.b
Weekes, dayes houres and minuts. 67
Wheate sold to losse. 143,b
Wheate ground in 3 milles. 147
Whole equal to al his parts. 130.v 250
Wine lost by tempest. 154.b
Wyne trucqued for furres, 258
Wyne spent by commensalls, 148.b
Wyne measures by assise, 65.b
Wooll trucqued for silkes. 257.

X

X. a fabricall figure fit for the triall of Multiplication and diuisiõ of integers. fol. 29.v.53. fol. 49. verse. 76.
X, vsed aptly in the triall of equall fraction and of proportionalls. 32.33.
X vsed in the diuision of frac. 83 v. 171
X. the fabrical figur proper to the rule of doble false positions 211. b.v.312.

Y

Yard English by assise. 67
Yeeres of age in question. 103.104
Yokefellows in nũbers which. 210.b.

Z

Z. the fittest figure for the golden rule. 121.b

FINIS, Tabulæ.

Generall Table of Number, declaring not

nely all their kindes of Number, and in what diuiſion or Subdiui-

ſion, any number whatſoeuer is comprehended: but alſo ſundrie va-

rieties of proportion ariſing by the conference or compariſon

of numbers together, aſwell in quantitie as

in qualitie:

- Absolute.
  - Solle, or by themselues
    - Euen
      - Primes as two onely.
      - Compounds
        - Euenly euen.
        - Uneuenly euen.
        - Perfect.
        - Imperfect.
          - abounding.
          - defectiue.
    - Odde
      - Primes.
      - Compounds.
  - among themselues or one to another.
    - Primes together.
    - Compounds together.
- Relatiue or comparatiue.
  - In quantitie.
    - Of Difference.
    - Of Proportion.
      - Of equalitie.
      - Of inequalitie.
  - In qualitie.
    - of proportion.
      - Arithmeticall.
        - continuall.
        - discontinuall
      - Geometricall.
        - continuall.
        - viscontinuall
          - Direct.
          - Conuerſed.
          - Alternate.
          - Inuerſed.
          - Compounded.
          - Parted.
    - Of Disproportion.
- Abstract.
- Figurall.
  - Linearie numbers.
  - Superficial numbers.
    - Triangular.
    - Circular.
    - Square.
    - Longsquare.
    - Likeflats.
    - Diametrals.
    - Multianguler, &c.
  - Solide numbers
    - Cubes.
    - Squared squares.
    - Surſolids.
    - Zenzicubes, &c.

  The arte of whith numbers is called Arithmetick figurall, & belongeth not to this Treatiſe.
- Contract.
  - To vulgar quantities, as
    - Weight.
    - Measure.
    - Time.
    - Coine.
    - Men or beasts, &c

    All which pertaine to this vulgar Arithmetick.
  - To coſsick quantities
    - Figurall whoſe proper figures are
      - 1. ſignifieng an vnit or an ace.
      - 2. ſignifieng a Roote.
      - 3. ſignifieng a Square.
      - 4. ſignifieng a Cube.
      - 5. ſignifieng a ſquared ſquare.
      - 6. ſignifieng a ſurſolide, &c.

      all which belong alſo to Arithmeticke figurall.
    - Radicall whoſe proper figures are
      - √. ſignifieng a roote ſquare.
      - √3. ſignifieng a roote Cubick.
      - √4. ſignifieng a roote Zenzizenzike.
      - √√. ſignifieng the roote ſquare of a roote ſquare.
      - √√3. ſignifieng the roote ſquare of a Cubick roote.
      - √. v. ſignifieng a roote vniuerſall, &c.

      al which appertein to Arithmetick radical.

- The greater
  - Simple.
    - Multiplex
      - Dupla, as 4 to 2. 6. to 3. 10. to 5.
      - Tripla, as 3. to 1. 6. to 2. 9. to 3.
      - Quadrupla, as 4. to 1. 8. to 2. 16. to 4.
      - Quintupla, as 5. to 1. 10. to 2. 15. to 3.
      - &c.
    - Superparticular
      - Seſquialtera, as 3. to 2. 6. to 4. 9. to 6.
      - Seſquitertia, as 4. to 3. 8. to 6. 12. to 9.
      - Seſquiquarta, as 5. to 4. 10. to 8. 15. to 12.
      - Seſquiquinta, as 6. to 5. 12. to 10. 18. to 15.
      - &c.
    - Superpartiens.
      - Superbitertias, as 5. to 3. 10. to 6. 15. to 9.
      - Supertriquartas, as 7. to 4. 14. to 8. 21. to 12.
      - Superquidriquin. as 9. to 5. 18. to 10. 27. to 15
      - Superquintuſextas, as 11. to 6. 22. to 12.
      - &c.
  - Mixed
    - Multiplex Superparticuler
      - Duplaſeſquialtera, as 5. to 2. 10. to 4.
      - Duplaſeſquitertia, as 7. to 3. 14. to 6.
      - Duplaſeſquiquarta, as 9. to 4. 18. to 8.
      - &c.
      - Triplaſeſquialtera, as 7. to 2. 14. to 4.
      - Triplaſeſquitertia, as 10. to 3. 20. to 6.
      - Triplaſeſquiquarta, as 13. to 4. 26. to 8.
      - &c.
      - Quadruplaſeſquialtera, as 9. to 2. 18. to 4.
      - Quadruplaſeſquitertia, as 13. to 3. 26. to 6.
      - Quadruplaſeſquiquarta, as 17. to 4. 34. to 8.
      - &c.
    - Multiplex Superpartiens.
      - Dupla ſuperbitertias, as 8. to 3. 16. to 6.
      - Duplaſupertriquartas, as 11. to 4. 22. to 8.
      - Duplaſuperquadriquin. as 14. to 5. 28. to 10.
      - &c.
      - Triplaſuperbitertias, as 11. to 3. 22. to 6.
      - Triplaſupertriquartas, as 15. to 4. 30. to 8.
      - Triplaſuperquadriquin. as 19. to 5. 38. to 10
      - &c.
      - Quadruplaſuperbitertias as 14. to 3. 28. to 9
      - Quadruplaſupertriquar. as 19. to 4. 38. to 8.
      - Quadrupla ſuperquadriquartas, as 24. to 5.
      - &c.
- The lesser.
  - Simple
    - Submultiplex.
      - Subdupla, as 2. to 4. 3. to 6. 5. to 10.
      - Subtripla, as 1. to 3. 2. to 6. 3. to 9.
      - Subquadrupla, as 1. to 4. 2. to 8. 3. to 12.
      - Subquintupla, as 1. to 5. 2. to 10. 3. to 15.
      - &c.
    - Subsuperparticuler
      - Subſeſquialtera, as 2. to 3. 4. to 6. 8. to 12.
      - Subſeſquitertia, as 3. to 4. 6. to 8. 9. to 12.
      - Subſeſquiquarta, as 4. to 5. 8. to 10. 12. to 15
      - Subſeſquiquinta, as 5. to 6. 10. to 12.
      - &c.
    - Subsuperpartiens.
      - Subſuperbitertias, as 3. to 5. 6. to 10. 9. to 15.
      - Subſupertriquar. as 4. to 7. 8. to 14. 12. to 21
      - Subſuperquadriq. as 5. to 9. 10. to 18. 15. to 27
      - Subſuperquintuſextas, as 6. to 11. 12. to 22.
      - &c.
  - Mixed
    - Submultiplex Superparticular.
      - Subdupla ſeſquialtera, as 2. to 5. 4. to 10.
      - Subduplaſeſquitertia, as 3. to 7. 6. to 14.
      - Subdupla ſeſquiquarta, as 4 to 9. 8. to 18.
      - &c.
      - Subtripla ſeſquialtera, as 2. to 7. 4. to 14.
      - Subtripla ſeſquitertia, as 3. to 10. 6. to 20.
      - Subtripla ſeſquiquarta, as 4. to 13. 8. to 26.
      - &c.
      - Subquadrupla ſeſquialtera, as 2. to 9. 4. to 18
      - Subquadrupla ſeſquitertia, as 3. to 13. 6. to 26
      - Subquadrupla ſeſquiquar. as 4 to. 17. 8. to 34
      - &c.
    - Submultiplex Superpartiens.
      - Subdupla ſuperbitertias, as 3. to 8. 6. to 16.
      - Subduplaſupertriquartas, as 4. to 11 8. to 22
      - Subduplaſuperquadriqu. as 5. to 14. 10. to 28
      - &c.
      - Subtripla ſuperbitertias, as 3. to 11. 6. to 22.
      - Subtripla ſupertriquart. as 4. to 15. 8. to 30.
      - Subtripla ſuperquadriqu. as 5. to 19. 10. to 38
      - &c.
      - Subquadrupla ſuperbitertias, as 3. to 14.
      - Subquadrupla ſupertriquartas, as 4. to 19.
      - Subquadrupla ſuperquadriquintas, as 5. to 24.
      - &c.

Place this Table at the beginning of the firſt Chapter, before D.

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An Analiticall Table of all the Partes, Braunches, Members and Diuisions of *Vulgar Arithmeticke*.

- The vulgar Arithmeticke treateth of parts, viz.
  - Booke. 1. The first part thereof treateth of
    - Notation or Numeration.
      - Cap. 1. of Integers which teacheth.
        - The forme or shape of the Numerall figures. verse. 8.
        - The values or signification { by their formes or shape. verse. 9. / by their places or Degrees. verse. 9.
        - Their kindes { digits. vers. 15. / articles. ver 15 / mixte. vers. 16.
      - Cap. 2. of Fractions which teacheth
        - The forme of Fractions, vers. 17.
        - Their values or significations. v. 17. { by their termes. v. 18. { Numerator or top. 19. 20. / Denominator or basis. 19. 20. } / by their kindes. v. 24. { properly, as { fractions of vnits, v. 20. / fractiõs of fractiõs, v. 23. 42 } / improperly, as { being Integers onely. v. 24. / or mixt with Integers. v. 25 } }
    - Operatiõ
      - Cap. 3. Cap. 4. Addition and Cap. 5. Subtraction { with their seuerall properties. v. 36. 45. 46. }
      - Cap. 6. Multiplicatiõ and Cap. 7. Diuision { with the properties { only of { Multiplication verse. 77. / Diuision, and } of both together. Cap. 8. / cap. 9. 10. of true integers { first { Euen, v. 109 110. / Odde, v. 116. 117. } both primes & compounds. v. 129. 130. / secondly { Primes together cap. 11. 12. / Cõpositis together, cap. 11. 12. } whereof cõmeth the finding of { the greatest common measure. v. 148. 149. / The least common Dividend. v. 150. 151 } } } }
      - Cap. 13. Reductions { of grosse to subtils. v. 153 / of subtils to grosse. v. 154 }
      - Cap. 14. of Fractions which teacheth
        - Addition and Subtraction } both { simplye / cap. 15. / mixtlye } with Reduction { Particuler v. 156. / Conforming. v. 157. 158 / Abridging. v. 161. / Distincting. v. 163. / Improper. v. 164. 165. } sometimes { Proceeding. v. 156. &c. / Concurring. v. 160. / Succeeding. v. 161. }
        - Multiplication and Diuision } both { Simple. Cap. 16. 17 / Mixtlie, Cap. 18. } with Reduction { Preuenting, v. 170. / Exchanging, v. 177. }
  - Booke. 2. The second explicateth the relation or comparison.
    - Cap. 1. In Quantitie
      - of Difference or excesse, v. 180. 181.
      - vers. 180. 182. of Rate or proportion.
        - of Equalitie. v. 183. 184.
        - vers. 183. 185. of Inequalitie
          - The greater v. 186,
            - Simple v. 188. { Multiplex. v. 189. / Superparticular. v. 190. / Superpartiens. v. 191. }
            - Mixte. v. 192. { Multiplex superparticular. v. 192. / Multiplex superpartiens. v. 193. }
          - The lesser: which by foresetting the particle Sub. argueth the rate or proportion to be contrarie. v. 187.
      - v. 196. 197 Arithmeticall
        - v. 198. 205. Continuall { In three termes only, with the properties. Cap. 3. / Cap. 4. In moe termes then three, making a progression, whose doctrine is { of the difference, v. 205. / of the termes, v. 206. } }
        - v. 199. Discontinuall { Directly, v. 200. / Conuersed, v. 201. } with the properties directly. Cap. 5.
      - v. 196. 197 Geometricall
        - ver. 198. Continuall. Cap. 6. In three termes onely teaching the Inuention { of the meane. v. 213. / of the third, v. 224 225. } proportionall.
        - Cap. 8. Discontinuall, whereof proceedeth
          - vers. 244. The Rule of proportion.
            - Vulgarly. Cap. 8.
              - Single, v. 248 { Directly { with no accõpt proceeding, ca. 8 / with an accõpt proceeding, ca. 9 } / Conuersed. Cap. 10. }
              - Plurall Cap. 11 { Direct only, Cap. 11. / Partly direct & partly conuersed. Cap. 12. }
            - Artificially
              - Cap. 13. Cõpounded { First by { Addition. v. 257 / subtract'õ. v. 260 } of proportions. / secõdly by Multipli. of proportiõs. ca. 17 }
              - Cap. 18. Continued { In two proportions giuen, v. 295 / In moe then two proportions. v. 298. }
          - The rules
            - Cap. 19. of Alligation { Alternate. v. 303. 307. / Mediall. v. 308. }
            - False positions { Single. Cap. 20. / Double. Cap. 21. }
    - v. 195. Of Proportionalitie.
      - Cap. 22. of Disproportionalitie.
    - Cap. 23. Of Incommensurable quantities.

Here the Reader is to note, that neither this Table, nor any of these 3. Bookes, make any mention of proportion Irracionall, because the same apperteineth not to this volume: for in this vulgar Arithmetick, all proportions all racionall.

Place this Table at N. iiii. Chap. 9.

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This Table briefly expreſſeth the proportion or agreement betweene certeine of the long meaſures vſed in diuers Cities and Marte townes in Europe.

100. Aunes of Roüan, make														
Aunes of Paris.	109 1/2	100. aun. of Paris.												
Elles of London.	115 1/2	105 1/2	100. Ells of Londō											
Vares of Lisbona.	115 1/2	105 1/4	100	100. vars of Lisbo.										
Aunes of Lyons.	117 1/4	107	101 1/2	101 1/2	100. aunes of Lyons									
Vares in the Isles Medera.	119 1/2	108 1/4	103 1/2	102 1/2	101 1/2	100. vare. of Mader								
Vares of Siuill.	155 1/4	142	135	135	132 1/4	130 1/2	100. vare. of Siuill.							
Elles of Danzick.	159 1/2	145 1/2	138 1/2	138 1/2	136	133 1/2	102 7/12	100. els of Dantzic.						
Elles of Vienna.	167 1/4	152 1/2	145	145	142 1/2	140 1/2	107 1/2	107 1/2	100. els of Vienna.					
Elles of Antwerp.	192 1/4	175 1/2	266 1/2	166 1/2	163 1/2	161 1/4	123 7/12	120 1/2	114 5/12	100. els of Antwerp				
Elles of Nuremberge.	200 1/2	183 1/2	174 1/2	174 1/2	171 1/4	168 1/4	129	125 1/2	120	104 1/2	100. els of Nuremb.			
Braces of Venice.	207 1/2	189 1/2	180	180	117	174 1/2	133 1/2	130	124 1/2	108	103 1/2	100. brace of venice		
Braces of Lucques.	230 1/4	210 1/2	200	200	199 1/2	193 1/2	148 1/2	144 1/2	137 5/12	120	114 1/2	111	100. brace of Lucq.	
Braces of Florence.	235 1/2	214 7/12	204 1/2	204 1/2	200 1/4	197 1/2	151 1/12	147 1/2	140 1/2	122 1/2	117 1/2	113 1/2	102	100. brac of Florē.
Elles of Franckford.	240 1/2	219 1/2	208 1/2	208 1/2	204 1/2	201 1/2	154 1/12	150 1/2	143 1/2	125	119 1/2	115 1/4	104 1/2	102	100. els of Franc.
Braes of Millaine.	365 1/2	242	230	230	126 1/2	222 1/2	170 1/2	166 1/4	158 1/2	138	132	127 1/4	115	112 1/2	110 1/2	100. brace of Millai.
Palmes of Geanes.	554 1/2	506 1/2	480 1/2	480 1/2	472 1/2	465 1/2	356 1/2	337 1/2	331 1/2	288 1/2	276	267 1/2	240 1/2	235 1/2	230 1/2	209

These two Tables of weighes and meaſures, muſt be placed at the end of this booke.

This Table sheweth briefly the proportion or agreement of the weights of diuers famous Cities and Marte townes in Europe, each to other.

The 100. f. of Vienna in Austria, make at																		
Geneua.	101 1/2	100. f. of Geneua.																
Lubeck.	108 2/3	107 1/2	100. f. of Lubeck.															
Rouan.	109	108 2/3	100	100. f. of Rouan.														
Frankeforde Lisbone.	110 1/4	109 1/3	101 1/3	101	100. f. of Frankford or Lisborn													
Noremberg.	112	110 1/3	102 1/3	102 1/3	102 1/3	100. f. of Norenb.												
Diepe.	112 1/2	111	102 2/3	102 2/3	101 1/3	100	100. f. of Diepe.											
Liebsick.	113 1/3	112	103 2/3	103 1/3	102 1/3	101 1/3	101	100 f. of Liebsick.										
Ausberge Paris and Colleyn.	114 1/2	113 1/2	105	104 1/3	103 1/3	102	102	100 2/3	100. f. of Ausberg Paris or Colleyn.									
Venice groſt.	117 2/3	116 1/3	108 1/3	107 1/3	106 1/2	105 1/3	105	104	103	100. f. gr of venice								
Antwerpe.	120 1/2	119 1/2	110 1/2	109 2/3	108 1/3	107 1/2	107 1/3	106 1/3	105 1/3	102 1/2	100. f. of Antwerp							
Stuill.	122 1/3	121 1/2	112 1/2	112	110 1/3	109 1/2	109 1/2	108 1/2	107 1/2	104 1/2	101 2/3	100. f. of Stuill.						
London.	125 1/2	123 1/3	115	114 1/2	113 1/2	111 1/3	111 1/2	110 1/2	109 1/2	106 1/2	104	101	100. f. of London.					
Lions.	132 1/2	131 2/3	121 1/2	120 2/3	119 1/2	118 1/3	118 1/2	117	115 2/3	112 1/3	110	107 2/3	118 1/2	100. f. of Lions.				
Tholonſe.	137 1/2	135 1/3	126	125 1/3	124	122 1/2	122 1/2	121 1/2	120	116 1/2	114	111 1/3	122 1/3	103 1/2	100. f. of Tholons			
Marcellis.	139	137 1/3	127 1/2	126 2/3	125 1/2	124	123 1/2	122 1/2	121 1/2	117 1/2	115 1/3	113 1/3	124 1/3	104 1/3	101 1/2	100. f. of Marcellis		
Rochell.	139 1/2	138 1/2	139 1/2	127 1/3	126 1/2	124 1/2	124 1/2	123 1/2	122 1/2	118 1/2	116	113 1/3	124 1/2	105 1/3	101 1/3	103 1/2	100. f. of Rochell.	
Dantzick.	144 1/2	143	132 1/2	131 1/2	130 1/2	129	128 1/3	127 1/2	126 1/3	122 1/2	120	117 1/3	129 1/3	109	105 1/2	104	103 1/2	100. f. of Dantzic.
Preſtlawe.	150 1/2	148 2/3	138 1/3	137 1/2	135 2/3	134 1/2	134 1/3	132 2/3	131 1/2	127 1/3	125	122 1/2	134 1/2	113 1/2	109 1/2	108 1/2	107 1/3	104 1/2
Barcellone.	160 1/2	158 1/3	147 1/3	146 1/2	144 2/3	143 1/2	143 1/2	141 1/2	140 1/2	136 1/2	133 1/2	130 1/3	143 1/2	121	116 2/3	115	114 2/3	111
Geanes & Aquila.	175 2/3	174	161 1/2	160 1/3	158 1/3	157	156 1/3	155 1/3	153 1/3	149 1/2	146	143 1/2	157 1/2	132 1/2	128	126 1/2	125 2/3	121 1/2
Venice ſmall.	186 1/2	184 1/2	171 1/3	170 1/3	168 1/2	166 1/3	166 1/2	164 1/3	164	158 1/3	155	152	166 2/3	140 1/3	135 2/3	134 1/3	133 1/3	129

106 1/2	100. f. of Barcelon								
109 1/2	102 1/2	100. f. of Gea. Aq.							
123 1/2	116 1/3	108							

